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Regge Residues Predicted from a Uniquely Determined π-π Dual Amplitude

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We present and discuss Regge residues predicted from a uniquely determined π-π dual amplitude.

The dual resonance model can be successfully applied to the high-energy soft hadronic reactions.[1] However, because of the freedom in the choice of satellite terms, the resulting fits are rather inconclusive.

Recently, a uniquely determined $\pi^-\pi^+ \to \pi^-\pi^+$ dual Born amplitude has been obtained by Munakata, Sakamoto and the author.[2] (This work is referred to as I.) In I, we have started with the most general Veneziano-type amplitude in terms of the exchange-degenerate $\rho - f$ Regge trajectory, which has just the degree of freedom to provide an arbitrary residue at each of parents and their daughters, and we have restricted it by two conditions. One is the local duality scheme[3] involving the harmonic-oscillator spectrum of $SU(6)\otimes O(3)_L$ multiplets. The other is an asymptotic convergence condition. As shown in I, the obtained amplitude has good properties: (a) It contains neither negative-norm states nor tachyons in a domain of ($\alpha' m_\pi^2$(the Regge slope times the square of the poin mass), α_0 (the Regge zero-intercept)) around the physical m_π, α' and α_0. (b) When m_π, α' and α_0 are taken to be their physical values and the overall multiplying factor is adjusted by observed $\rho \to 2\pi$ width, it predicts the partial decay widths of low-lying resonances, which are consistent with available experiments.

The author[4] has discussed also high-energy behaviours of the uniquely determined amplitude. (This work is referred to as II.) There is another problem in the fits to experiments. One must deal with the necessary unitarization procedure. To solve the problem explicitly will be difficult. However, a method for testing the essences of the Veneziano-type models has been presented by Froggatt, Nielsen and Petersen,[5] using only the imaginary part of the amplitude. It is based on the observation that the general (s, t) Veneziano forms has two-dimensional inverse Laplace transforms with a very characteristic support property. Because of its use of only the imaginary part of the amplitude, considering explicit unitarity corrections is not necessary. In II, we have used the method presented by Froggatt, Nielsen and Petersen. The support property of the two-dimensional inverse Laplace transform of the general (s, t) Veneziano-type amplitude is equivalent to the behaviour of the single inverse Laplace or Mellin transform of the fixed-t general Veneziano-type amplitude.[5] And, one can test the amplitude $F(s, t)$ by calculating

$$\tilde{G}(y, t)=\frac{\alpha'}{\pi}\int_{cut} ds y^{\alpha' s}\,\mathrm{Im}\,F(s, t), \tag{1}$$

and comparing it with corresponding phenomenological amplitudes. Here, α' is the slope of the Regge trajectory, by which the Veneziano-type amplitude $F(s, t)$ is written. It is noted that the calculation requires only the value of the imaginary part of the scattering amplitude along the branch cuts. In order to eliminate the contribution of the diffraction scattering to $\tilde{G}(y, t)$ explicitly, it is better to compare the model with experiments for the $I_t=1$ amplitude defined by

$$\tilde{G}_{I_t=1}(y, t)=\tilde{G}_{\pi^-\pi^+\to\pi^-\pi^+}(y, t)-\tilde{G}_{\pi^+\pi^+\to\pi^+\pi^+}(y, t)\,. \tag{2}$$

As phenomenological amplitudes, we have used ones obtained also by Froggatt, Nielsen and Petersen.[5] The phenomenological amplitudes in the forward directions of $0\geqq t\geqq -1.0$ $(\mathrm{GeV}/c)^2$ are essentially uniquely determined by data, analyticity and phase-shift analysis (unitarity) from threshold up to a dipion mass of 1.8 GeV. The uniquely determined amplitude has been compared with phenomenological amplitudes, normalizing it at the ρ pole. It is found that the uniquely determined amplitude is successful in describing the low-lying resonances and the high-energy behaviours simultaneously.

In this paper, we present Regge residues predicted from the uniquely determined amplitude for any future use.

The uniquely determined $\pi^-\pi^+\to\pi^-\pi^+$ dual amplitude is

$$F(s, t)=-\lambda(1-\beta^2)\sum_{n=1}^{\infty}\frac{1}{(n-1)!(2n-1+\beta)(2n-3+\beta)}$$
$$\times\left(\frac{\Gamma(n-\alpha_s)\Gamma(n-\alpha_t)}{\Gamma(n-\alpha_s-\alpha_t)}+\frac{(1-\beta)}{2}\frac{2\Gamma(n-\alpha_s)\Gamma(n-\alpha_t)}{\Gamma(n+1-\alpha_s-\alpha_t)}\right). \tag{3}$$

Here, $\alpha_s\equiv\alpha' s+\alpha_0$ is the $\rho-f$ trajectory, and λ is a constant, and β is $\beta\equiv 2-3\alpha_0-4\alpha' m_\pi^2$.

The asymptotic form of Eq. (3) in the forward direction is

$$F(s, t)\xrightarrow[s\to\infty,\ t\ \mathrm{fixed}]{}\lambda\Gamma(1-\alpha_t)e^{-i\pi\alpha_t}I(\alpha_t)\alpha_s^{\alpha_t}, \tag{4}$$

where

$$I(\alpha_t)=-\frac{(1-\beta^2)}{\Gamma(1-\alpha_t)}\sum_{n=1}^{\infty}\frac{\Gamma(n-\alpha_t)}{(n-1)!(2n-1+\beta)(2n-3+\beta)}$$

$$=F\left(1-\alpha_t, \frac{\beta-1}{2}, \frac{\beta+3}{2}; 1\right)=\frac{\Gamma\left(\frac{3+\beta}{2}\right)\Gamma(1+\alpha_t)}{\Gamma\left(\frac{1+\beta}{2}+\alpha_t\right)},\quad (\mathrm{Re}\,(3+\beta)>0,\ \mathrm{Re}\,(1+\alpha_t)>0). \tag{5}$$

In Eq. (5), $F(a, b, c; z)$ is Gauss's hypergeometric function. Eq. (4) has an extra factor $I(\alpha_t)$ in contrast to the asymptotic form of the Lovelace-Shapiro-Veneziano[6] and Frampton[7] amplitudes.*) (The Frampton amplitude possesses the same asymptotic form as the Lovelace-Shapiro-Veneziano one does, because its satellite terms involve only daughters.) The effects of the new factor of Eq. (5) is shown in Fig. 1 together with the corresponding factor of the Lovelace-Shapiro-Veneziano and Frampton models, $I(\alpha_t)=1$, in the region $0 \geqq t \geqq -1$ $(\mathrm{GeV}/c)^2$ where the Born amplitude may be dominant. Here, m_π, α' and α_0 are taken to be their physical values

$$m_\pi = 0.140\ \mathrm{GeV}, \quad \alpha' = 0.888\ (\mathrm{GeV})^{-2}, \quad \alpha_0 = 0.475. \tag{6}$$

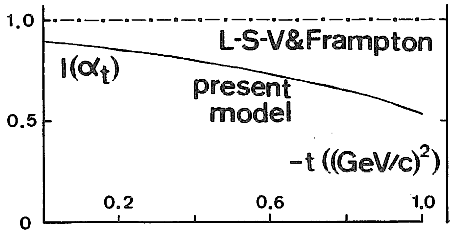

Fig. 1. The effects of the new factor $I(\alpha_t)$.

We decompose the asymptotic form in Eq. (4) into two parts

$$F(s, t) \xrightarrow[s\to\infty,\ t\ \text{fixed}]{} A_\rho(s, t) + A_f(s, t), \tag{7}$$

where

$$\begin{aligned} A_\rho(s, t) &= \lambda \Gamma(1-\alpha_t) \frac{-1+e^{-i\pi\alpha_t}}{2} I(\alpha_t) \alpha_s^{\alpha_t} \\ &= i\gamma_\rho(t) e^{-i\pi\alpha_t/2} (s/s_0)^{\alpha_t}, \end{aligned} \tag{8}$$

$$\begin{aligned} A_f(s, t) &= \lambda \Gamma(1-\alpha_t) \frac{1+e^{-i\pi\alpha_t}}{2} I(\alpha_t) \alpha_s^{\alpha_t} \\ &= \gamma_f(t) e^{-i\pi\alpha_t/2} (s/s_0)^{\alpha_t}, \end{aligned} \tag{9}$$

$$\gamma_\rho(t) = -\lambda \Gamma(1-\alpha_t) (\alpha' s_0)^{\alpha_t} I(\alpha_t) \sin\frac{\pi\alpha_t}{2}, \tag{10}$$

$$\gamma_f(t) = \lambda \Gamma(1-\alpha_t) (\alpha' s_0)^{\alpha_t} I(\alpha_t) \cos\frac{\pi\alpha_t}{2}. \tag{11}$$

Here, s_0 is defined to be $s_0 = 1\ (\mathrm{GeV})^2$.

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*) The Lovelace-Shapiro-Veneziano amplitude has odd daughters, the existence of which is doubted from the quark model and experiments. While, the Frampton one ($A_4^{(1)}$ in his work) contains neither odd daughters nor, presumably, ghosts. But, his principle to construct the Veneziano-type amplitude does not lead to a unique amplitude.

With λ adjusted by observed $\rho \to 2\pi$ width 154 MeV,[8] $\lambda = -0.743$, predictions for the ρ and f Regge residue functions are shown in Figs. 2 and 3.

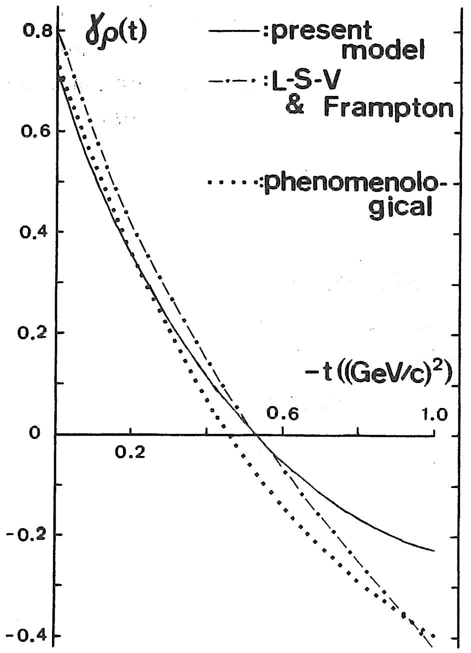

Fig. 2. Predicted ρ Regge residues from the present model and other models. Phenomenological one is also shown.

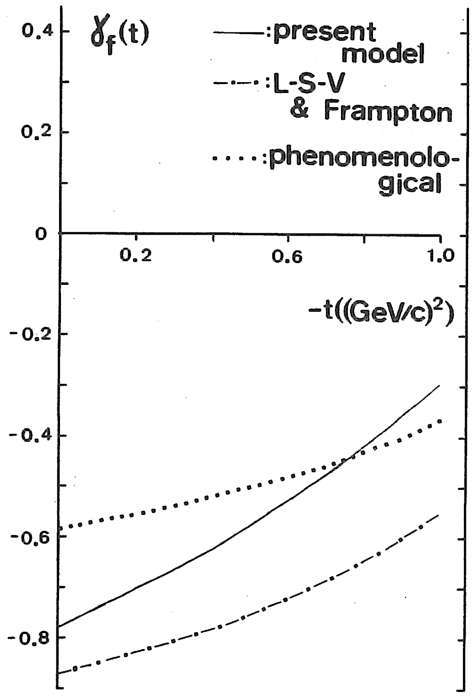

Fig. 3. Predicted f Regge residues from the present model and other models. Phenomenological one is also shown.

In Figs. 2 and 3, predictions from the Lovelace-Shapiro-Veneziano and Frampton models are also shown, normalizing the models at the ρ pole. Phenomenological residues in Figs. 2 and 3 are taken from a phase-shift analysis between 1.0 and 1.8 GeV based on the fixed momentum transfer analyticity, done by Froggatt and Petersen.[9] They are only extrapolations of experimental data through the analyticity. But, the experimental amplitude at 1.8 GeV is not too different from the asymptotic form with them.[9]

It is interesting that the predicted ρ Regge residues from the present model and other models are near to phenomenological one. As for both of the ρ and f Regge residues, the predictions from the present model are in better agreement with phenomenological amplitudes than other models. This is due to the new factor $I(\alpha_t)$.

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