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A Note on the Factorization Property of the Resonance Coupling Constants

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We note that all the resonance couplings can be factorized in the recently proposed dynamical system, which involves the harmonic-oscillator spectrum of $SU(6)\otimes O(3)_L$ multiplets and provides a uniquely determined dual Born amplitude for each of the meson-meson scattering.

QCD has been supposed to be the fundamental theory of strong interaction. It may unify the main models for hadrons; the Regge pole model, the collective models, the quark model, the current algebra, the dual models, the string model, the parton model and the bag models. However, starting with the QCD Lagrangian, we must invent a good scenario and follow a long chain of unknown manipulations and various approximations in order to deduce the actual properties of resonances and scattering amplotudes. To fill this large gap, it will be worth-while to unify the main old models for hadrons semi-phenomenologically in a motivation from QCD. We are now attempting to make such a trial.[1)]

It will be an interesting idea to start with the dual models[2)] and accommodate them to the other old models. If one considers the $1/N_c$ expansion[3)] of QCD, the conceptual link between the colour gauge theory and the dual models is provided.[4)] We are now studying a dynamical system which represents some kind of planar duality.[1)] The dynamical system involves the harmonic-oscillator spectrum of $SU(6)\otimes O(3)_L$ multiplets, and it can provide a uniquely determined dual Born amplitude for each of the meson-meson scattering. Its tools, which determine the meson-meson systems, are (i) the local duality scheme,[5)] (ii) the most general representation of the global duality, that is Σ(s-channel poles)$=\Sigma$(t-channel poles), and (iii) an asymptotic convergence condition. As the representation of the global duality, we take the most general Veneziano-type amplitude which provides an arbitrary residue at each of the relevant resonance poles. It is attempted to describe properties of the leading-order term of the $1/N_c$ expansion of QCD, which may be the following; planarity, crossing, infinitely narrow resonances, factorization at the resonance pole, no-ghost, real linearly-rising Regge trajectories, global duality and high degeneracy of the levels considerably responsible for the ghost cancellation.[4)]

In this note, we shall point out that in the above dynamical system, all the resonance coupling constants are factorized.

We discuss the $\pi\pi\to\pi\pi$, $\pi\pi\to K\bar{K}$ and $K\bar{K}\to K\bar{K}$ scattering. First, we state the uniquely determined dual Born amplitude for $\pi^-\pi^+\to\pi^-\pi^+$ scattering. Expanding the amplitude in terms of the s-channel resonances, we define resonance coupling constants in the conventional manner. Next, we give the s-channel pole expansions of the $\pi^-\pi^+\to K^-K^+$ and $K^-K^+\to K^-K^+$ amplitudes, and discuss the factorization property of the resonance coupling constants.

A uniquely determined dual Born amplitude for $\pi^-\pi^+\to\pi^-\pi^+$ scattering

The $\pi^-\pi^+\to\pi^-\pi^+$ amplitude is $s-t$ crossing even. The resonance families and their members relevant to the $\pi^-\pi^+\to\pi^-\pi^+$ scattering and the appearance pattern of them are shown in Fig. 1.

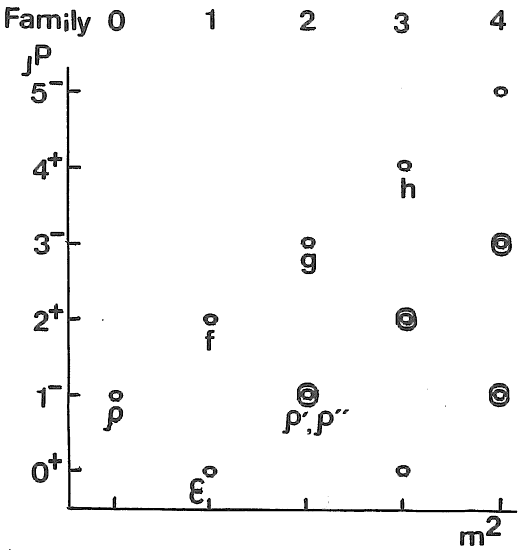

Fig. 1. The resonance families and their members in the π^-—π^+ scattering. The double circles imply that there are two states with different quark-orbital angular momenta at their places.

The uniquely determined dual Born amplitude for the $\pi^-\pi^+\to\pi^-\pi^+$ scattering obtained in our dynamical system[1)] is

$$F(s,t)=-\lambda_{1,1}(1-\beta^2)\sum_{n=1}^{\infty}\frac{1}{(n-1)!(2n-1+\beta)(2n-3+\beta)}$$
$$\times\left[\frac{\Gamma(n-\alpha_s)\Gamma(n-\alpha_t)}{\Gamma(n-\alpha_s-\alpha_t)}+\frac{(1-\beta)}{2}\frac{\Gamma(n-\alpha_s)\Gamma(n-\alpha_t)}{\Gamma(n+1-\alpha_s-\alpha_t)}\right], \tag{1}$$

where $\alpha_s=\alpha' s+\alpha_0$ is the exchange-degenerate $\rho-f$ Regge trajectory, $\lambda_{1,1}$ is a constant, and β is

$$\beta=2-3\alpha_0-4\alpha' m_\pi^2. \tag{2}$$

Expanding amplittude (1) with (2) in terms of the s-channel resonances,

$$F(s, t)=\sum_{J=1}^{\infty}\frac{R_J(q_{s,J}^2, \cos\theta_s)}{J-\alpha_s}, \tag{3}$$

one can define the squared coupling constant for the resonance having spin j and being associated with a parent of spin J, g_{J-j}^2, by the equation

$$R_J(q_{s,J}^2, \cos\theta_s)=\begin{cases}\sum_{j=\mathrm{odd}\geqq 1}^{J} g_{J-j}^2\cdot(q_{s,,J}^2)^j\cdot P_j(\cos\theta_s) & \text{for odd } J,\\ \sum_{j=\mathrm{even}\geqq 0}^{J} g_{J-j}^2\cdot(q_{s,J}^2)^j\cdot P_j(\cos\theta_s) & \text{for even } J.\end{cases} \tag{4}$$

Here, $q_{s,J}^2$ is the squared c.m. momentum in the s-channel fixed as $s=m_J^2$,

$$q_{s,J}^2=\frac{m_J^2-4m_\pi^2}{4}, \tag{5}$$

and θ_s is the c.m. scattering angle in the channel. Excepting the overall multiplying factor $\lambda_{1,1}$, g_{J-j}^2 depends on $(\alpha')^j$, J, j, $\alpha' m_\pi^2$ and α_0 (or $\alpha' m_\rho^2=1-\alpha_0$). It is noted that in a domain (of the $(\alpha' m_\pi^2, \alpha' m_\rho^2)$ plane) involving the point for the physical m_π, α' and α_0, all the squared resonance coupling constants and all the squared masses of the scattering particle and resonances are positive.[6)]

In order to avoid the complexity, we give $R_J(q_{s,J}^2, \cos\theta_s)$, instead of g_{J-j}^2,

$$R_1(q_{s,J}^2, \cos\theta_s)=-\lambda_{1,1}\frac{(\tilde{\alpha}_t-\tilde{\alpha}_u)}{2},$$

$$R_3(q_{s,J}^2, \cos\theta_s)=-\lambda_{1,1}\frac{3(\tilde{\alpha}_t-\tilde{\alpha}_u)\{(\tilde{\alpha}_t-\tilde{\alpha}_u)^2-b_3\}}{2(3+\beta)\,(5+\beta)}, \tag{6}$$

$$R_5(q_{s,J}^2, \cos\theta_s)=-\lambda_{1,1}\frac{5(\tilde{\alpha}_t-\tilde{\alpha}_u)\{(\tilde{\alpha}_t-\tilde{\alpha}_u)^2-b_5\}\{(\tilde{\alpha}_t-\tilde{\alpha}_u)^2-2^2\}}{2(3+\beta)(5+\beta)(7+\beta)(9+\beta)},$$

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$$R_2(q_{s,J}^2, \cos\theta_s)=-\lambda_{1,1}\frac{2\{(\tilde{\alpha}_t-\tilde{\alpha}_u)^2-b_2\}}{2(3+\beta)},$$

$$R_4(q_{,,J}^2, \cos\theta_s)=-\lambda_{1,1}\frac{4\{(\tilde{\alpha}_t-\tilde{\alpha}_u)^2-b_4\}\{\tilde{\alpha}_t-\tilde{\alpha}_u)^2-1^2\}}{2(3+\beta)(5+\beta)(7+\beta)}, \tag{7}$$

$$R_6(q_{s,J}^2, \cos\theta)=-\lambda_{1,1}\frac{6\{(\tilde{\alpha}_t-\tilde{\alpha}_u)^2-b_6\}\{(\tilde{\alpha}_t-\tilde{\alpha}_u)^2-1^2\}\{(\tilde{\alpha}_t-\tilde{\alpha}_u)^2-3^2\}}{2(3+\beta)(5+\beta)(7+\beta)(9+\beta)(11+\beta)},$$

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where

$$\tilde{\alpha}_t = \alpha'\tilde{t} + \alpha_0,\ \tilde{\alpha}_u = \alpha'\tilde{u} + \alpha_0,\ \tilde{t} = -2q_{s,J}^2(1-\cos\theta_s),\quad \tilde{u} = 4m_\pi^2 - m_J^2 - \tilde{t}, \tag{8}$$

$$b_J = \frac{(J-1)\{(J+\beta-1)^2 - J\}}{J}. \tag{9}$$

As for the general expression for $R_J(q_{s,J}^2, \cos\theta_s)$, see Ref. 1).

Amplitndes for $\pi^-\pi^+ \to K^-K^+$ and $K^-K^+ \to K^-K^+$ scattering

The s-channel pole expansion of the $\pi^-\pi^+ \to K^-K^+$ amplitude indicated the present dyanmical system is given by the following substitution in Eqs. (3), (6) and (7)[7]

$$\begin{aligned} &\lambda_{1,1} \longrightarrow \lambda_{1,1}^{\pi K}, \\ &\tilde{\alpha}_t - \tilde{\alpha}_u = 4\alpha' q_{s,J}^2 \cos\theta_s \longrightarrow 4\alpha' q_{s,J} p_{s,J} \cos\theta_s, \\ &\beta \longrightarrow \beta^{\pi K} = 2 - \alpha_0 - 2\alpha_0^Y - 2\alpha'(m_\pi^2 + m_K^2), \end{aligned} \tag{10}$$

where

$$p_{s,J}^2 = \frac{m_J^2 - 4m_K^2}{4}, \tag{11}$$

and α_0^Y is the zero-intercept of the exchange-degenerate $K^* - K^{**}$ trajectory. Here we assume the universality of the Regge slope.

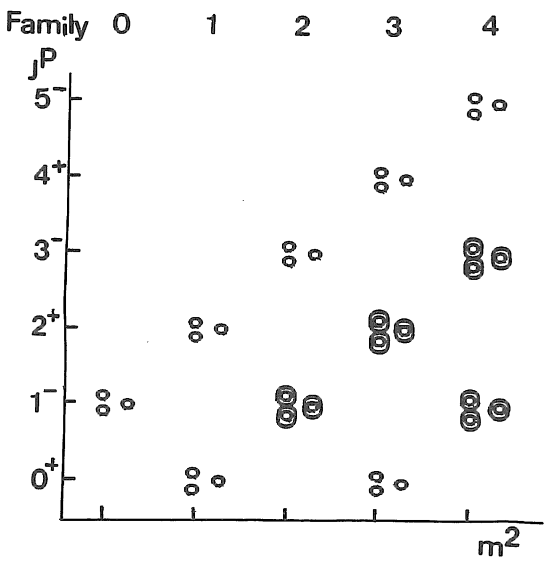

Fig. 2. The resonance families and their members in the $K^- - K^+$ scattering. The double circles imply that there are two states with different quark-orbital angular momenta at their places.

We assume (i) the ideal nonet scheme for mesonic resonances, (ii) the $SU(3)$ invariance for $1^- - 0^- - 0^-$ vertices, and (iii) the OZI decoupling rule, in order to discuss the $K^-K^+ \to K^-K^+$ process. Then, we have the resonance families and their members relevant to the process as shown in Fig. 2. (We note that the amplitude for the process is $s-t$ crossing even.) And, in order to examine the factorization property of the couplings for the resonances in Fig. 1, the normalization factor is adjusted.

Excepting the normalization factor, a part of the s-channel pole expansion of the $K^-K^+ \to K^-K^+$ amplitude, which is concerned with the resonances in Fig. 1, is given by the substitution in Eqs. (3), (6) and (7),[7]

$$\begin{aligned} \tilde{\alpha}_t - \tilde{\alpha}_u = 4\alpha' q_{s,J}^2 \cos\theta_s &\longrightarrow 4\alpha' p_{s,J}^2 \cos\theta_s, \\ \beta &\longrightarrow \beta^{KK} = 2 - \alpha_0 - 2\alpha_0^1 - 4\alpha' m_K^2, \end{aligned} \tag{12}$$

where α_0^1 is the zero-intercept of the exchange-degenerate $\phi - f'$ trajectory. Here we again assume the universality of the Regge slope.

All the coupling constants for the resonances in Fig. 1 are factorized, when and only when

$$\beta = \beta^{\pi K} = \beta^{KK}, \tag{13}$$

that is,

$$2(\alpha' m_\pi^2 + \alpha_0) = 2(\alpha' m_K^2 + \alpha_0^Y) = (\alpha' m_K^2 + \alpha_0) + (\alpha' m_K^2 + \alpha_0^1). \tag{14}$$

The relations (14) imply

$$m_{K^*}^2 - m_\rho^2 = m_\phi^2 - m_{K^*}^2 = m_{K^{**}}^2 - m_f^2 = m_{f'}^2 - m_{K^{**}}^2 = m_K^2 - m_\pi^2, \tag{15}$$

that is, the broken $SU(6)$ mass relations. This is consistent with the present basic assumption of planarity. Therefore, we can conclude that all the resonance couplings can be factorized in the present dynamical system.

As for the double circles in Fig. 1 which imply that there are two states with different quark-orbital angular momenta, no difficulties are brought about. The factorization of the sum of two contributions,

$$(f_1^2 + g_1^2)(f_2^2 + g_2^2) = (f_1 f_2 + g_1 g_2)^2, \tag{16}$$

is equivalent to those of each couplings and

$$f_1 / f_2 = g_1 / g_2. \tag{17}$$

And the relation (17) is reasonably supposed to be realized in the nature.

In conclusion, the present dynamical system provides unique meson-meson amplitudes so as to allow the factorization of all the resonance couplings. And, when the $SU(3)$ invariance is assumed for the lowest lying resonance vertices, the invariance is expected also for all the resonance vertices.

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