Simplified procedure for evaluating the coefficient of earth pressure at rest

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Abstract

Prediction of the in-situ state of stress in soil is one of the important parameters in a wide variety of geotechnical problems. Values of $K_o$ are used to evaluate lateral thrusts against earth retaining structures, and to predict the vertical and horizontal deformations for foundations. The typical strength-deformation characteristics of a soil, specially of an overconsolidated soil, cannot be evaluated unless the in-situ stresses are known. Simple empirical methods for evaluating the value of $K_o$ for normally consolidated and overconsolidated soils are presented for practical use. Well-documented test data for soils from the geotechnical literature world-wide are the basis of this regression analysis study. The validity of the proposed methods is supported by comparison with previous work. On the basis of the results, only the plasticity index and prior stress history are needed to predict the value of $K_o$. Evaluation based on the critical state concept of Modified Cam Clay Model is also investigated and compared with experimental data.

Although the proposed equations for the derivation of $K_o$ presented may not be perfect, they are convenient used for a preliminary work in practice.

Key words: clay, earth pressure at rest, consistency limits, constitutive equations, $K_o$ consolidation, overconsolidation ratio, plasticity, laboratory test, sand.

Introduction

The coefficient of earth pressure at rest, $K_o$, is the ratio of the effective horizontal pressure $\sigma_h'$, to effective vertical pressure, $\sigma_v'$, in a soil that currently exists under the condition of no lateral deformation: $K_o = \sigma_h'/\sigma_v'$. Laboratory experiments and in-situ measurements on undisturbed samples of a large number of holocene deposits were used to investigate the magnitude and behaviour of the values of $K_o$. It is well recognized that representative strength-deformation behaviour of a soil cannot be evaluated unless the in-situ stresses are simulated. The evaluation of in-situ stresses in the foundation is important for problems related to foundations, excavations, and numerical analyses. Thus, $K_o$ is an essential parameter in the design or numerical analysis of many conventional problems. For example, it is used to compute lateral thrusts against earth-retaining structures where the lateral movement is too small to mobilize the active state of stress (Terzaghi and Peck, 1968). It has also been used in the computation of predicted settlement in various...
situations (Lambe, 1964), in the analysis of progressive failure in clay slopes (Lo and Lee, 1973), and in the prediction of construction pore pressures in earth dams (Pells, 1973).

Geotechnical researchers developed theoretical soil mechanics and numerical techniques to overcome the geotechnical problems. Various constitutive models of cohesive soils have been developed to incorporate the influence of initial anisotropic condition on deformation and strength. The value of $K_0$ has been used in such analyses as an important soil parameter. The power of computers is now such that it is possible to carry out sophisticated numerical analyses easily. It is found that the computed deformations possible are governed by the soil parameters available. Methods to determine relevant soil parameters in the laboratory and in the field have also been improved. The selection of appropriate soil parameters from tests, however, is not an easy task.

The finance, time, facilities and required expertise for experimental testing are often not available within the constraints of practical geotechnical engineering work, particularly for small projects. In these situations, the greatest benefits of sophisticated new analytical methods are to be derived from techniques which the analysis has been reduced to a simple, convenient form (Kamei, 1985 MS and 1995; Sakajo and Kamei, 1995; Kamei and Sakajo, 1995a and b). These studies have already proposed a simplified procedure for evaluating the deformation characteristics of clay foundations. Kamei's method consists of the Sekiguchi-Ohta model (Sekiguchi, 1977; Sekiguchi and Ohta, 1977; Ohta and Sekiguchi, 1979) and Kamei's parameters (Kamei, 1985 MS), combined with some idealizing assumptions. As a result, they showed the applicability of the proposed method to the actual deformation and excess pore pressure of the foundations accompanying embankment construction was sufficiently accurate, although a number of simplifying assumptions had been made in the deformation analysis. Summary of methods for performing deformation analysis have been described in detail elsewhere (Kamei and Sakajo, 1995b).

Plasticity index is one of the key components of Casagrande's plasticity chart, which still offers a very useful means of relating the general physical behaviour of soft and sedimentary cohesive soils on a global basis. Geologically similar deposits plot in bands parallel to the A-line, which separates typical clays from inorganic silts and organic soils. Important behavioural characteristics such as the potential for being highly sensitive, anisotropic or creep susceptible are related in a general way to the location of the soil on the plasticity chart. Thus physical behaviour is related to plasticity index (PI). In addition, the PI is one of the key components of the activity chart (Skempton, 1953), which offers some measure of electrical characteristics in constitutive mineralogy. From an engineering point of view, a rough estimate of PI-values can be most easily done by physical senses, such as touch and vision (Kamei, 1996). The test is well established in practice, provides a soil sample, and a vast amount of local experience and correlation data have been collected by practitioners. There still remains a question as to the values of $K_0$-over-consolidation ratio (OCR) relationships in soil.

The evaluation of in-situ stresses in overconsolidated soils is important for problems related to foundations, excavations, and numerical analyses. The in-situ vertical effective stress at any depth can be easily determined if the unit weight of the soil and location of the water table are known, but the determination of horizontal stress is not easy. Moreover, the determination of horizontal stress
Simplified procedure for evaluating the coefficient of earth pressure at rest for an overconsolidated soil is more problematic than that of a normally consolidated soil (Garga and Khan, 1991). Measurements of in-situ horizontal stresses by field testing methods and laboratory testing methods are difficult, and introduce uncertainty as a result of changes to initial stress state. Although there are a number of laboratory methods available for the estimation of $K_0$ for normally consolidated or slightly overconsolidated soils, no techniques exist for the evaluation of $K_0$ in overconsolidated soils. Some empirical or indirect methods for the determination of in-situ stresses are available, but these are dependent on a combination of laboratory tests, such as oedometer consolidation or triaxial tests.

The purpose of this paper is to investigate the behaviour of $K_0$ during simple loading-unloading, corresponding to the virgin compression of normally consolidated state, and subsequent rebound associated with overconsolidated state. This study reviews documented and published data from in the geotechnical literature, including results compiled from over 100 different soils, and notes general trends in the correlation between the value of $K_0$ and simple soil indices. The ability of the proposed equations in predicting $K_0$-values is verified by comparison with measured $K_0$. The reliability of the analytical results for $K_0$ using the Modified Cam Clay Model are also investigated.

Review of previous studies on the values of $K_0$ and the simple soil indices

Many researchers have previously investigated the relationship between $K_0$ and the simple soil indices. It has been established that the value of $K_0$ is a function of the angle of shear resistance in terms of effective stress of soils. The relationship proposed by Jaky (1944 and 1948): $K_0 = 1 - \sin \phi$' has been accepted for common use. Although this equation was originally developed for sandy soils, it has also been used for normally consolidated clay in practical situations. In a more detailed investigation, Brooker and Ireland (1965) reported a slightly modified equation: $K_0 = 0.95 - \sin \phi$' for clays, and also established that the value of $K_0$ is a function of the PI as well as the overconsolidation ratio. Alpan (1967) also derived the following expression using the data presented by Kenny (1959): $K_0 = 0.19 + 0.233 \log \text{PI}$. Finally, Ladd et al. (1977) concluded that $K_0$-values correlated reasonably well with angle of shear resistance and plasticity index for a fairly wide range of 20 soil samples, as seen in Figs. 1 and 2. Most of the above studies have used the results of only one or two specific soils. Based on survey of laboratory data available in the literature, Massarsch (1979) described a simple relationship for estimating the value of $K_0$ against PI in normally consolidated clay: $K_0 = 0.44 + 0.0042 \text{PI}$.

When considering the strength of soils, it is necessary to distinguish clearly between peak strength and ultimate strength (Skempton, 1964). Similarly, when considering the consolidation characteristics of soils it is necessary also to distinguish clearly between undisturbed and remoulded samples. Undisturbed soils differ from remoulded soils in a number of important ways. These differences stem from the influence of the soil structure (fabric and bonding).

In attempting to deduce any correlation, however, attention must be given to the test conditions and the quality of the experimental technique for the given experimental data. In this respect, it may be more appropriate that the correlation attempt should be focused on a large body of data and
varying test conditions, rather than amassing information consisting of isolated test results presented in individual papers (Nakase et al., 1988). By reviewing laboratory data from over 170 different soils, Mayne and Kulhawy (1982) reported that $K_0$ behaviour during virgin compression, rebound, and reload can be represented approximately by simple empirical relationships. Statistical analyses were used to support the validity of the methods.

Kamei and Sano (1993) developed a simple procedure for practical use for the determination of $K_0$ from plasticity index in the normally consolidated region, based on data from 62 different cohesive soils. Several well-documented test data sets for normally consolidated cohesive soils from many studies throughout the world are the basis of regression analysis studies which investigate the correlation between $K_0$ and plasticity index. Although Ladd et al. (1977) showed a linear increase in $K_0$ with PI, no such trend exists on a global data base, as shown in Fig. 3. At this stage, they tried to distinguish Japanese soils from others. Figures 4 and 5 show the
Simplified procedure for evaluating the coefficient of earth pressure at rest

Empirical relationship between $K_0$ and plasticity index for published test data, (Kamei and Sano, 1993; after Ladd et al., 1977)

$$K_0 = 0.455 + 0.0017 \cdot PI \quad (r = 0.31)$$

Excepting Japanese Soils

Empirical relationship between $K_0$ and plasticity index for remoulded samples, excluding Japanese soils (Kamei and Sano, 1993; after Ladd et al., 1977)

$$K_0 = 0.483 + 0.0031 \cdot PI \quad (r = 0.57)$$

Empirical relationship between $K_0$ and plasticity index for undisturbed samples, excluding Japanese soils (Kamei and Sano, 1993; after Ladd et al., 1977)

$$K_0 = 0.386 + 0.0049 \cdot PI \quad (r = 0.84)$$

Fig. 3. Empirical relationship between $K_0$ and plasticity index for published test data, (Kamei and Sano, 1993; after Ladd et al., 1977)

Fig. 4. Empirical relationship between $K_0$ and plasticity index for remoulded samples, excluding Japanese soils (Kamei and Sano, 1993; after Ladd et al., 1977)

Fig. 5. Empirical relationship between $K_0$ and plasticity index for undisturbed samples, excluding Japanese soils (Kamei and Sano, 1993; after Ladd et al., 1977)
relationship between $K_0$ and PI for remoulded and undisturbed foreign soils, respectively. A distinct trend of increasing $K_0$ with increasing PI is apparent, with high correlation coefficients of 0.57 and 0.84. In contrast, Japanese soils show an essentially constant value with PI, expressed by $K_0 = 0.45 \pm 0.05$. Thus a range, rather than a linear regression line, should be used for Japanese soils. As a result, Kamei and Sano (1993) showed the relationship between $K_0$ and PI for foreign soils and Japanese soils separately, as shown in Figs. 3 through 6. This contrasting behaviour is a product of differences in the shear strength and clay mineralogy.

Fig. 7 shows the relationship between the angle of shear resistance and PI in water front areas in Japan, with the relationship in Scandinavian clays reported by Bjerrum and Simons (1960) shown for comparison. From this comparison, Kamei and Sano (1993) indicated that the angle of shearing resistance and PI depended on the region. Values of $K_0$ may reflect both the geometrical

![Figure 6](image6.jpg)

Fig. 6. Empirical relationship between $K_0$ and plasticity index for Japanese soils (Kamei and Sano, 1993)

![Figure 7](image7.jpg)

Fig. 7. Relationship between friction angle and plasticity index
Simplified procedure for evaluating the coefficient of earth pressure at rest

arrangement of the clay particles with respect to each other (i.e., the clay fabric), and the nature and extent of the electrochemical interparticle forces that prevail within the clay-water electrolyte system (Abdelhamid and Krizel, 1976). X-ray diffraction results (see Fig. 8) of some of the Japanese soils show that quartz and plagioclase are major components (Kamei, 1985 MS). This explains the higher angle of shear resistance of Japanese soils, compared with foreign soils which have lower angle of shear resistance. Finally, Kamei (1985 MS) pointed out that the key to the success of the analysis depended largely on the mineralogy and region of the soils. For practical application, these results demonstrate the need for careful data analysis to take account of the sources and mineralogy of soils to improve their accuracy and applicability.

Overconsolidation state due to rebound results in higher values of $K_0$ than the $K_{0nc}$ values obtained during normal consolidation. The $K_0$-OCR relationship was first reported by Kjellman (1936) for a pure sand. The most comprehensive investigation of $K_0$ was reported by Hendron (1963) for sands and by Brooker and Ireland (1965) for clays. Another empirical approach has been proposed by many researchers. In this, the variation of $K_{0oc}$ with OCR can be expressed simply as a function of the effective stress friction angle as hypothesized by Schmidt (1966). This approach has a distinct advantage, since only one soil parameter is required for predicting both normally consolidated and overconsolidated values of $K_0$. The simplest relationship proposed is that given by Schmidt (1966) for $K_{0oc}$ during unloading:

$$K_{0oc} = K_{0nc} (OCR)^\alpha$$

where the overconsolidation ratio $OCR = \sigma_{v'm}/\sigma_{v}'$, $\sigma_{v}'$ is the maximum effective vertical stress, $\sigma_{v}'$ is the present effective overburden pressure, and $\alpha$ = an exponent defined as the at-rest rebound parameter of the soil. This approach has subsequently been used by others.

![Fig. 8 X-ray diffraction results of tested Japanese clays (Kamei, 1985 MS)](image)
Predictions of the value of $K_0$ for normally consolidated cohesive soils using Modified Cam Clay Model

The definition of the earth pressure at rest, $K_{onc}$, is given by the following equation:

$$K_{onc} = \frac{(3-\eta)(3+2\eta)}{3}$$  \hspace{1cm} (1)

where $\eta$ is the stress ratio $\eta = q/p'$ where $q = \sigma_1 - \sigma_3$ and $p' = (\sigma_1' + 2\sigma_3')/3$. During normal consolidation, $K_{onc}$ has a constant value, consequently $\eta =$ constant. Schofield and Wroth (1968) determined that the values of $K_{onc}$ predicted by the original Cam Clay Model are larger than those measured in practice. Thus, in this study, the Modified Cam Clay model proposed by Roscoe and Burland (1968) is used in order to evaluate the value of $K_{onc}$ based on Eq. (1).

Modified Cam Clay is as follows:

$$\frac{de_p}{de_r} = 2\eta_0/(M^2 - \eta_0^2)$$  \hspace{1cm} (2)

where $M = q/p'$ is the stress ratio at the critical state condition, and $e_q$ and $d e_p$ are shear and plastic volumetric strains, respectively. Using following relations (Yamaguchi, 1897) with $\eta = \eta_0$ in Eq. (2), in which $\eta_0$ is the stress ratio at $K_0$-condition:

$$\frac{de_q}{de_r} \cdot \frac{de_r}{de_p} \cdot \frac{de_p}{de_q} = 1 = \frac{2}{3} \cdot \frac{1}{(1 - \kappa/\lambda)} \cdot \frac{(M^2 - \eta_0^2)}{2\eta_0}$$  \hspace{1cm} (3)

where $\lambda$ and $\kappa$ are compression and swelling indices, respectively and $e_v$ is volumetric strain. This equation finally leads to the following 2nd order equation in terms of the stress ratio $\eta_0$:

$$\eta_0^3 + 3\Lambda\eta_0 - M^2 = 0$$  \hspace{1cm} (4)

where $\Lambda = 1 - \kappa/\lambda$. Although Eq. (4) has two solutions, only one solution is acceptable, which is

$$\eta_0 = \frac{-3\Lambda + \sqrt{9\Lambda^2 + 4M^2}}{2}$$  \hspace{1cm} (5)

Substituting Eq. (5) into Eq. (1), the value of $K_{onc}$ is finally formulated as

$$K_{onc} = \frac{(3-\eta_0)(3+2\eta_0)}{3}$$  \hspace{1cm} (6)

In order to check whether the above equations can be more generally applied, the calculated values of $K_{onc}$ are compared with experimental data for 12 different soils from Kamei (1985MS). The correlations between calculated $K_{onc}$ and the test results are shown in Fig. 9. This shows that the values of $K_0$ calculated using the Modified Cam Clay Model are about 10% larger than those observed in the experiments. Here, the relationship $K_{onc} = 0.9 K_{onc}$ is proposed, and this is also plotted. For the equation of Jaky (1944 and 1948), on the other hand, the calculated values of $K_0$ are about 20% smaller than those of the experimental results. A larger data base may confirm the correlation.
Simplified procedure for evaluating the coefficient of earth pressure at rest

\[ Ko = 1 - \sin \theta \]

Modified Cam Clay Model

Present Paper

\[ Ko = I - \sin \theta \]

\[ Ko = Ko^*(OCR) \]

where the overconsolidation ratio \( OCR = \sigma_{vm}/\sigma_v \), \( \sigma_{vm} \) is the maximum effective vertical stress, \( \sigma_v \) is the present effective overburden pressure, and \( \alpha = \) an exponent defined as the at-rest rebound parameter of the soil. The value of \( \alpha \) is also the slope of the relationship between log (\( K_{oc} \)) and log (OCR). Based on Hendron’s data for four sands, Schmidt (1967) concluded that \( \alpha \), with values in the range of 0.3~0.5, is independent of the initial density of sand. Additional data were interpreted by Mayne and Kulhawy (1982), who proposed that \( \alpha = \sin \theta \) for clays as well as sands.

Fig. 10 shows the relationship between rebound parameter (\( \alpha \)) and PI for clays. No distinct trend between \( \alpha \) and PI is observed, with \( \alpha \) essentially constant at all values of PI. It can be expressed by \( \alpha = 0.431 \). This result agrees with that for sands reported by Schmidt (1967). No significant difference, therefore, was observed between the values of \( K_0 \) obtained for soil samples and overconsolidation ratio during the unloading phase.

Figures 11 and 12 show the relationships between rebound parameter (\( \alpha \)) and \( K_{oc} \) for clays and sands, respectively. The regression lines shown in the figures can be expressed by:

\[ \alpha = 1.23 - 1.45 K_{oc} (\text{clays}) \]
\[ \alpha = 1.93 - 3.32 K_{oc} (\text{sands}) \]

which have sample correlation coefficients of 0.572 and 0.698, respectively. Finally, the following equations for predicting \( K_0 \)-OCR relations can be obtained.

\[ K_{oc} = K_{oc} (OCR)^{\alpha} \]

Fig. 9. Relationship between the measured and predicted \( K_0 \) of twelve marine clays

**\( K_0 \)-OCR Relationships**

As mentioned earlier, the simplest relationship proposed is that given by Schmidt (1966) for \( K_{oc} \) during unloading:

\[ K_{oc} = K_{oc} (OCR)^{\alpha} \]
Fig. 10. Relationship between rebound parameter ($\alpha$) and PI for cohesive soils

$$K_{0c} = K_{0c}(OCR)^{0.43}$$

Figures 13 through 15 show the measured and the predicted $K_0$-values of four Japanese clays, four foreign clays, and two foreign sands during unloading, respectively. In this method, the
determination of the value of $K_0$ requires a knowledge of the value of $K_{oc}$ as well as of OCR. As shown in the figures, the $K_0$-value increases slowly from its $K_{oc}$-value until it exceeds unity at an OCR of about 5, after which it increases more rapidly with an increase above OCR of about 10. The $K_0$-values calculated from Eq. (8) showed good agreement with those measured directly. Eq. (7) was found to give relatively larger, although still acceptable $K_0$-values. To this end, increase in $K_0$-values as a result of overconsolidation history can be estimated using either equation.

The applicability of the proposed empirical equations for prediction of $K_0$ is verified by comparison with measured results of $K_0$-values during unloading. It should be emphasized that even if the constitutive parameters are estimated only by plasticity index, the numerical simulation gives acceptable results when compared with the triaxial test results. It is therefore concluded that the use of the proposed empirical equations in geotechnical engineering practice is viable. Ad-
mittedly, these equations may be most suitable for preliminary work, owing to the idealizing assumptions and uncertainty in the data. The applicability of the present equations to other soils will require further research.

**Conclusions**

By reviewing laboratory data from over 100 different soils, it is established that the value of $K_0$ during virgin compression and rebound can be represented approximately by simple empirical relationships. The following conclusions can be advanced:

1) The values of $K_0$ calculated using Modified Cam Clay model are about 10% larger than those of the observed experimental values. Direct application of this equation to evaluate lateral
earth pressure will require further research.

2) Comparisons of \( K_0 \)-OCR relationships in soils confirms the usefulness of the proposed empirical equations. On the basis of the findings, only the plasticity index and prior stress history(OCR) are needed to predict approximate values of \( K_0 \).

The applicability of the results to all kinds of soils is not, as yet, entirely clear. It can be stated with reasonable certainty, however, that other soils possessing similar qualities are likely to behave in a similar manner.

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Simplified procedure for evaluating the coefficient of earth pressure at rest


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