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Relativistic Two-Body Bound States in a Renormalizable Field Theory

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Abstract

We study relativistic bound states of two kinds of the massive Dirac particles with a current-current interaction in 2-dimensional space-time. We have two types of the solutions; normal and abnormal solutions. The former solution shows that the binding energy of the particle-particle pair from the positron theory is smaller than that from the single electron theory.

1 Introduction

Several years ago, Glöckle, Nogami and Fukui (GNF) proposed a two dimensional quantum mechanical model consisting of two kinds of the massive Dirac particles which interact with each other through a δ-function potential. [1] This model satisfies all the requirements of quantum mechanics and special relativity. They derived exact two-body bound state solutions and clarified the structures of relativistic composite states. Extension of this model to the case of more than two bodies has been done and n-body bound state solutions have been found. [2, 3, 4]

The GNF model is, however, based on the single electron theory instead of the positron theory, and their Hamiltonian is not positive-definite. Hence, their equation allows a bound state solution composed of the particles with positive and negative masses. They have considered that this solution is a counterpart of a solution of a particle-anti-particle bound state in the positron theory. In order to overcome this defect of negative energy, Munakata, Ino and Nagamura (MIN) introduced the Bethe-Salpeter (BS) equation with a Fermi-type interaction. [5] They have shown that GNF equation is derived from the BS equation by using the retarded propagators instead of the Feynman propagators. They have also found that the BS equation contains divergence when the Feynman propagators are made use of, i.e. when the model is treated positron-theoretically, and that all the bound states disappear. MIN have considered that this is because the effect of pair-creation of the particle and anti-particle makes the interaction so weak that the bound state cannot exist.

After MIN, Glöckle, Nogami and Toyama have modified the δ-function

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potential to an analytic function with a finite range to investigate behaviors of the solutions for different interaction ranges numerically. [6] They have found that there is a critical value of the interaction range for a finite coupling constant g and that the interaction range must be larger than this critical value so that a bound state may be composed. Contrary to the explanation of MIN above, they conjectured that for the δ-function potential the non-zero g is too strong no matter how small it may be.

Motivated by their observation, Ino has shown that the BS equation above can be dealt with rationally by a kind of renormalization of the coupling constant and he has given bound state solutions in the positron theory. [7]

In this article we investigate the model field-theoretically. The GNF equation or the corresponding BS equation can be derived from the massive Thirring-like model consisting of the Dirac fields with different species. In the next section we start with Lagrangian of two kinds of the Dirac fields with a Fermi-type current-current interaction. A model of the Dirac fields with a Fermi-type interaction diverges generally but it is renormalizable in two dimensional space-tyme. We renormalize the theory by the conventional procedure and derive the Green function for a particle-particle scattering process by the chain approximation. In section 3 we search poles of the 4-point Green function which corresponds to the particle-particle bound states. We also consider the particle-anti-particle bound states. The last section is devoted to discussion.

2 Four-Point Green Function

Lagrangian of our model is given by

$$\mathscr{L} = \sum_{i=a,b} \bar{\psi}_{0i}(i\partial\!\!\!/ - m_0)\psi_{0i} + \frac{g_0}{2} j^a_{0\mu} j^\mu_{0b}, \tag{1}$$

where a and b denote the species of the Dirac fields. Current $j^i_{0\mu}$ is given as

$$j^i_{0\mu} = \bar{\psi}_{0i}\gamma_\mu\psi_{0i}. \tag{2}$$

Here we do neither discuss the case $m_0 = 0$ [8] nor treat the mass term as a perturbation to the massless theory, [9] though these seem to have rich structures. It is seen that only two and four-point Green functions diverge in $1 + 1$ dimensional space-time, and the model is renormalizable. Then following the conventional renormalization procedure, we rewrite Lagrangian (1) by renormalized variables with counter-terms as

$$\begin{aligned} \mathscr{L} = {} & \bar{\psi}(i\partial\!\!\!/ - m)\psi + \frac{g}{2} j^a_\mu j^\mu_b + \frac{g}{2}(Z_1 - 1) j^a_\mu j^\mu_b + (Z_2 - 1)\bar{\psi} i\partial\!\!\!/ \psi \\ & - \bar{\psi}\{(Z_2 - 1)m - Z_2 \delta m\}\psi, \end{aligned} \tag{3}$$

where Z_1, Z_2 and δm are the coupling constant, the wave function and the mass renormalization parameters, respectively. As is seen later, in the higher order terms of perturbation, other types of interactions, i.e. scalar and pseudoscalar type interactions as $(\bar{\psi}_a\psi_a)(\bar{\psi}_b\psi_b)$ and $(\bar{\psi}_a\gamma_5\psi_a)(\bar{\psi}_b\gamma_5\psi_b)$ are induced. But these interactions do not cause any other divergences. Therefore, we do not need any other counter-terms than in Eq. (3). These induced interactions, however, force us a different definition of the renormalized coupling constant from the conventional one. We can see that the self-energy diagram diverges logarithmically and its divergent part does not depend on its external momentum. This means that wave function renormalization is finite and in the followings we neglect the wave function renormalization. We also regard that the mass renormalization is finished. These mean that we neglect the last two terms of Eq. (3).

Now we consider the 4-point Green function which is defined by

$$G(x_a, x_b, y_b, y_a) = \langle 0|\mathrm{T}\{\psi_a(x_a)\psi_b(x_b)\bar{\psi}_b(y_b)\bar{\psi}_a(y_a)\}|0\rangle \tag{4}$$

in the Heisenberg representation, where $x_a^0, x_b^0 > y_a^0, y_b^0$. We neglect all the counter-terms in Eq. (3) at the moment and by chain approximation we find that G satisfies the following BS equation:

$$\begin{aligned} G(x_a, x_b, y_b, y_a) &= S_F^a(x_a - y_a)S_F^b(x_b - y_b) \\ &\quad + \int d^2z S_F^a(x_a - z)S_F^b(x_b - z)\frac{ig}{2}\gamma_a\cdot\gamma_b G(z, z, y_b, y_a), \end{aligned} \tag{5}$$

where $\gamma_a\cdot\gamma_b = \gamma_\mu^a\gamma_b^\mu$. (See Fig. 1) Here $\gamma_i^\mu (i = a, b)$ denotes γ-matrix operated on the i field.

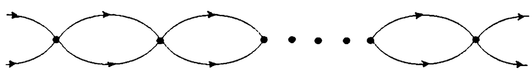

Fig. 1. Chain diagram of the particle-particle scattering process.

The homogeneous BS equation for the bound state is obtained by omitting the first term in the above equation as [10]

$$\Psi_P(x_a, x_b) = \int d^2z S_F^a(x_a - z)S_F^b(x_b - z)\frac{ig}{2}\gamma_a\cdot\gamma_b\Psi_p(z, z), \tag{6}$$

where the BS amplitude $\Psi_P(x_a, x_b)$ is defined as

$$\Psi_P(x_a, x_b) = \langle 0|\mathrm{T}\{\psi_a(x_a), \psi_b(x_b)\}|P\rangle \tag{7}$$

with $|P\rangle$ denoting the two-body bound state of a and b-particles with total momentum P^μ. In momentum space we put

$$\Psi_P(x_a, x_b) = \int \frac{d^2 p}{(2\pi)^2} e^{-ipx} \Phi_P(p) e^{-iPX}, \tag{8}$$

where $x_\mu = x_{a\mu} - x_{b\mu}$, $X_\mu = \frac{1}{2}(x_{a\mu} + x_{b\mu})$, $P^\mu = p_a^\mu + p_b^\mu$ and $p^\mu = \frac{1}{2}(p_a^\mu - p_b^\mu)$. Eq. (6) is rewritten into

$$\Phi_P(p) = -\frac{\not{p}_a + m}{p_a^2 - m^2 + i\varepsilon} \cdot \frac{\not{p}_b + m}{p_b^2 - m^2 + i\varepsilon} \frac{ig}{2} \gamma_a \cdot \gamma_b \int \frac{d^2 k}{(2\pi)^2} \Phi_P(k). \tag{9}$$

This equation, however, contains divergence, which is seen by integration of the both sides of the above equation over p. The right-hand side diverges logarithmically. This divergence originates from a loop integration of the chain diagram. We can eliminate this divergence by renormalization.

Now we consider the chain expansion of G given by Eq. (5). We convert G into $\tilde{G}$ in momentum space as

$$\begin{aligned}(2\pi)^2 i\delta(p_a + p_b - q_a - q_b)\tilde{G}(p_a, p_b, q_b, q_a) \\ = \int \prod dx dy e^{ip_a x_a} e^{ip_b x_b} e^{-iq_a y_a} e^{-iq_b y_b} G(x_a, x_b, y_b, y_a),\end{aligned} \tag{10}$$

where $\prod dx dy \equiv d^2 x_a d^2 x_b d^2 y_a d^2 y_b$. The 1-loop term of $\tilde{G}$ is

$$\tilde{G}(p_a, p_b, q_b, q_a)^{(1)} = S_F^a(p_a) S_F^b(p_b) \frac{ig}{2} \gamma_a \cdot \gamma_b I(P) \frac{ig}{2} \gamma_a \cdot \gamma_b S_F^a(q_a) S_F^b(q_b), \tag{11}$$

where

$$I(P) = \int \frac{d^2 k}{(2\pi)^2} S_F^a(k) S_F^b(P - k) = -\int \frac{d^2 k}{(2\pi)^2} \frac{(\not{k} + m)_a}{k^2 - m^2} \cdot \frac{(\not{P} - \not{k} + m)_b}{(P - k)^2 - m^2}. \tag{12}$$

Here m^2 is an abbreviation of $m^2 - i\varepsilon$. The term above diverges logarithmically as mentioned before. We regularize it by analytic continuation to n-dimensional momentum space to obtain

$$\begin{aligned}\frac{g^2}{4} I^{(n)}(P) = & -\frac{g^2}{4} \mu^{2-n} \int_0^1 dx \left[(x - x^2) \frac{\not{P}_a \not{P}_b}{\mu^2} + x(\not{P}_a + \not{P}_b) \frac{m}{\mu^2} + \frac{m^2}{\mu^2} \right. \\ & \left. - \frac{1}{2} \gamma_a \cdot \gamma_b \left\{ (x - x^2) \frac{P^2}{\mu^2} - \frac{m^2}{\mu^2} \right\} \frac{2}{2 - n} \right] \\ & \times \frac{i(-\pi)^{n/2}}{(2\pi)^n} \frac{\Gamma(2 - n/2)}{\Gamma(2)} \frac{1}{\left\{ (x - x^2) \frac{P^2}{\mu^2} - \frac{m^2}{\mu^2} \right\}^{2 - n/2}},\end{aligned} \tag{13}$$

where μ is a renormalization parameter with the same dimension as mass m. The pole term in the above equation, then, is given as

$$\frac{ig^2}{16\pi}\frac{\gamma_a\cdot\gamma_b}{(2-n)}, \tag{14}$$

which yields

$$\frac{ig^2}{4\pi(2-n)}S_F^a(p_a)S_F^b(p_b)\gamma_a\cdot\gamma_b S_F^a(q_a)S_F(q_b), \tag{15}$$

to $\tilde{G}^{(1)}$, where we use identity $(\gamma_a\cdot\gamma_b)^3=4\gamma_a\cdot\gamma_b$. This is canceled by the counter-term $\frac{g}{2}(Z_1-1)j_\mu^a j_b^\mu$ in Lagrangian (3). The finite part of $I(P)$ is given by

$$\begin{aligned} I_r(P) &= \frac{i}{8\pi}\gamma_a\cdot\gamma_b\int_0^1 dx\left\{\frac{(x-x^2)P^2}{(x-x^2)P^2-m^2}+\ln\frac{m^2-(x-x^2)P^2}{\mu^2}\right\} \\ &\quad +\frac{i}{4\pi}\int_0^1 dx\frac{m^2}{(x-x^2)P^2-m^2}, \end{aligned} \tag{16}$$

where we have taken into account that I_r is eventually sandwiched by $\gamma_a\cdot\gamma_b$'s and have made use of identities;

$$\gamma^a\cdot\gamma_b\not{P}_a\not{P}_b\gamma_a\cdot\gamma_b=2\gamma_a\cdot\gamma_b P^2=\frac{1}{2}(\gamma_a\cdot\gamma_b)^3P^2, \tag{17}$$

$$\gamma_a\cdot\gamma_b(\not{P}_a+\not{P}_b)\gamma_a\cdot\gamma_b=0. \tag{18}$$

The last term of Eq. (16) corresponds to induced interaction of scalar and pseudo-scalar types, as mentioned before, because this term contributes to $\tilde{G}^{(1)}$ of (10) as

$$\text{const.}\,(\gamma_a\cdot\gamma_b)^2=2\,\text{const.}\,(1-\gamma_a^5\gamma_b^5), \tag{19}$$

where $\gamma^5=\gamma^0\gamma^1$. This term, however, does not cause any other divergence to $\tilde{G}$ than those from the terms with $\gamma_a\cdot\gamma_b$ even in a higher order chain diagram, and hence we don't need any other counter-terms but $\frac{g}{2}(Z_1-1)j_a\cdot j_b$. We put the renormalization point at $P^2=m^2/\alpha\,(\alpha>1/4)$ where we make the term with $\gamma_a\cdot\gamma_b$ in I_r vanish. We write I_r as

$$I_r(P)=i\gamma_a\cdot\gamma_b A(P)+iB(P), \tag{20}$$

where

$$A(P)=\frac{1}{4\pi}\left[\frac{\frac{m^2}{P^2}-\frac{1}{2}}{\sqrt{\frac{m^2}{P^2}-\frac{1}{4}}}\tan^{-1}\frac{1}{2\sqrt{\frac{m^2}{P^2}-\frac{1}{4}}}-\frac{2\alpha-1}{\sqrt{4\alpha-1}}\tan^{-1}\frac{1}{\sqrt{4\alpha-1}}\right], \tag{21}$$

$$B(P)=-\frac{1}{4\pi}\frac{m^2}{P^2}\frac{2}{\sqrt{\frac{m^2}{P^2}-\frac{1}{4}}}\tan^{-1}\frac{1}{2\sqrt{\frac{m^2}{P^2}-\frac{1}{4}}}. \tag{22}$$

In the above there is a cut for $P^2 \geq 4m^2$ as is expected. Renormalization of $\tilde{G}$ is, therefore, performed by replacing I_r for I.

Let us introduce a truncated chain function $\mathscr{G}(P)$ defined by

$$\mathscr{G}(P)=\frac{i}{2}g\gamma_a\cdot\gamma_b+\frac{i}{2}g\gamma_a\cdot\gamma_b I_r\frac{i}{2}g\gamma_a\cdot\gamma_b+\frac{i}{2}g\gamma_a\cdot\gamma_b I_r\frac{i}{2}g\gamma_a\cdot\gamma_b I_r\frac{i}{2}g\gamma_a\cdot\gamma_b+\cdots, \tag{23}$$

which is shown graphically in Fig. 2.

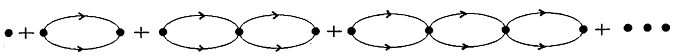

Fig. 2. Truncated diagram $\mathscr{G}$ defined in the text for the particle-particle bound states.

We should note that (23) gives different definition of the renormalized coupling constant from that of the conventional procedure. The renormalized coupling constant is defined conventionally as the value of 4-point Green function at the renormalization point or in this case here $\mathscr{G}(P^2=m^2/\alpha)$ may be put equal to $\frac{i}{2}g\gamma_a\cdot\gamma_b$. However, in (23) I_r does not vanish at $P^2=m^2/\alpha$ and there remain scalar and pseudo-scalar type terms.

Eq. (23) can be summed up as

$$\mathscr{G}(P)=\frac{i}{2}\frac{1}{\left(\frac{1}{g}+2A\right)^2-B^2}\left[\gamma_a\cdot\gamma_b\left(\frac{1}{g}+2A\right)-\frac{1}{2}(\gamma_a\cdot\gamma_b)^2 B\right]. \tag{24}$$

The Green function $\mathscr{G}$ should not depend on the renormalization point explicitly. Therefore, $\frac{1}{g}+2A$ is independent of α, and this defines α-dependence of the renormalized coupling constant g. We put

$$\frac{1}{g_r}\equiv\frac{1}{g}-\frac{1}{2\pi}\frac{2\alpha-1}{\sqrt{4\alpha-1}}\tan^{-1}\frac{1}{\sqrt{4\alpha-1}}. \tag{25}$$

$\mathscr{G}$ in (24) is rewritten as

$$\mathscr{G} = \frac{i}{2} \frac{1}{\left(\frac{1}{g_r} + 2A'\right)^2 - B^2} \left[\gamma_a \cdot \gamma_b \left(\frac{1}{g_r} + 2A' \right) - \frac{1}{2} (\gamma_a \cdot \gamma_b)^2 B \right], \tag{26}$$

where

$$A' = \frac{1}{4\pi} \frac{\frac{m^2}{P^2} - \frac{1}{2}}{\sqrt{\frac{m^2}{P^2} - \frac{1}{4}}} \tan^{-1} \frac{1}{2\sqrt{\frac{m^2}{P^2} - \frac{1}{4}}}. \tag{27}$$

The function $\mathscr{G}$ also satisfies the BS-type equation as

$$\mathscr{G} = \frac{i}{2} g_r \gamma_a \cdot \gamma_b + \frac{i}{2} g_r \gamma_a \cdot \gamma_b I_r \mathscr{G}, \tag{28}$$

and for the bound state we have

$$\Phi_r' = \frac{i}{2} g_r \gamma_a \cdot \gamma_b I_r \Phi_r'. \tag{29}$$

To compare this equation with the unrenormalized BS equation (9), we operate $(\not{p}_a - m)(\not{p}_b - m)$ on the both sides of (9) and obtain

$$\Phi_P' \equiv (\not{p}_a - m)(\not{p}_b - m)\Phi_P(p) = -\frac{i}{2} g \gamma_a \cdot \gamma_b \int \frac{d^2k}{(2\pi)^2} \Phi_P(k). \tag{30}$$

This shows that Φ_P' does not depend on p. We rewrite $\Phi_P(k)$ on the right-hand side of the above equation by Φ_P' to obtain

$$\Phi_P' = \frac{i}{2} g \gamma_a \cdot \gamma_b I(P) \Phi_P'. \tag{31}$$

Thus, the renormalized BS equation (29) is obtained by the substitution of I_r and g_r for I and g to the unrenormalized BS equation (31), respectively.

3 Bound States

The function $\mathscr{G}$ of (26) has poles at

$$\left(\frac{1}{g_r} + 2A'\right)^2 - B^2 = 0, \tag{32}$$

i.e.

$$g_r(2A' + B) = -1, \tag{33}$$

or

$$g_r(2A' - B) = -1, \tag{34}$$

which correspond to the bound states of the particle-particle pair. These are obtainable also from the renormalized BS equation (29). We have two solutions for the bound states. As is shown below, the first one (33) is a normal solution and the other (34) is an abnormal one. We discuss them separately.

(i) *Normal solution* (33)

Substituting (27) and (22) into (33) we get an eigenvalue equation for the bound state mass $\sqrt{P^2}$,

$$\frac{1}{\sqrt{\frac{m^2}{P^2} - \frac{1}{4}}} \tan^{-1} \frac{1}{2\sqrt{\frac{m^2}{P^2} - \frac{1}{4}}} = \frac{4\pi}{g_r}. \tag{35}$$

The term on the left-hand side of (35) increases monotonously with respect to P^2 and diverges for $P^2 \to 4m^2$. Then g_r vanishes at $P^2 = 4m^2$. Therefore, this is a normal solution of the particle-particle bound state. We see that the above solution corresponds to that of Ino. [7] We find that for a small value of g_r, Eq. (35) gives

$$P^2 = 4m^2 - \frac{g_r^2 m^2}{4}, \tag{36}$$

which corresponds to the consquence from the single-electron theoretical treatment. [1, 5] We find that even the single-electron theoretical treatment is still valid at least when g_r is very small. We investigate the behavior of square mass P^2 of the bound state numerically more in detail, comparing with those of GNF and MIN. See Fig. 3. We see that binding energy in the positron theory is smaller than those in the single-electron theory. Then the conjecture of MIN mentioned before is partly correct.

(ii) *Abnormal solution* (34)

We get from (34)

$$\sqrt{\frac{m^2}{P^2} - \frac{1}{4}} \tan^{-1} \frac{1}{2\sqrt{\frac{m^2}{P^2} - \frac{1}{4}}} = -\frac{\pi}{g_r}. \tag{37}$$

The left-hand side of the above decreases from $1/2$ to 0 monotonously with $0 < P^2 < 4m^2$. Therefore, g_r diverges for $P^2 \to 4m^2$ and this is an abnormal

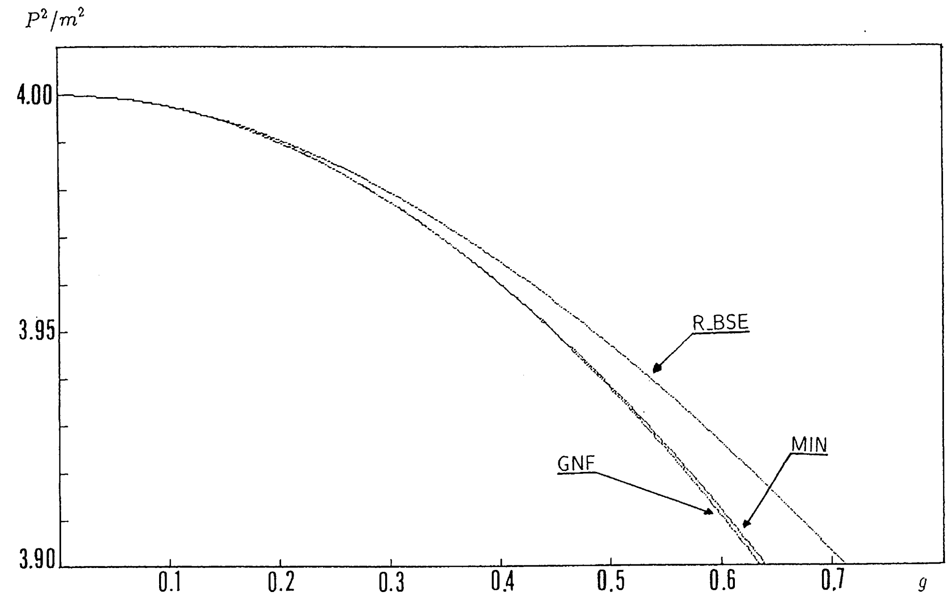

Fig. 3. Behavior of the normal bound state mass $\sqrt{P^2}$ for coupling constant g_r. Our consequence from the renormalized BS equation (R-BSE) is shown here with those of the single electron theory (GNF: $g = 4\tan^{-1}\sqrt{4m^2/P^2 - 1}$ [1] and MIN: $g = 4\sqrt{4m^2/P^2 - 1}$ [5]). It is seen that the binding energy from the positron theory is smaller than those from single electron theory.

solution. It is found that there exists a solution only for $g_r < -2\pi$. It is known that the homogeneous BS equation generally contains such abnormal solutions and there are some arguments about the existence of the abnormal solutions in the real world. [10] In our case here, we consider that the details of solution (34) or (37) may be out of the validity of our approximation because it exists only for large $|g_r|$ and our argument here is based on perturbation theory.

Next we consider the bound state of a particle-anti-particle pair. We put

$$G_A(x_a, x_b, y_b, y_a) = \langle 0|T\{\psi_a(x_a)\bar{\psi}_b(x_b)\psi_b(y_b)\bar{\psi}_a(y_a)\}|0\rangle, \tag{38}$$

with $x_a^0, x_b^0 > y_a^0, y_b^0$. The chain diagram of the above Green function is shown in Fig. 4.

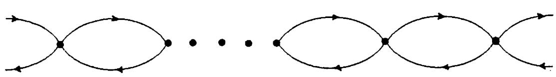

Fig. 4. Chain diagram of the particle-anti-particle scattering process.

Then we can obtain the expressions of the Green functions in momentum space from the corresponding particle-particle Green functions by replacing all γ_b's with their transposed ones and putting the signs of b-particle momenta opposite. For instance, the 1-loop term of $\tilde{G}_A$, which corresponds to (11), is given by

$$\tilde{G}_A(p_a, p_b, q_b, q_a)^{(1)} = -S_F^a(p_a)S_F^{bT}(-p_b)\frac{ig}{2}\gamma_a \cdot \gamma_b^T I_A(P)\frac{ig}{2}\gamma_a \cdot \gamma_b^T S_F^a(q_a)S_F^{bT}(-q_b), \tag{39}$$

where

$$I_A(P) = \int \frac{d^2k}{(2\pi)^2} S_F^a(k)S_F^{bT}(k-P). \tag{40}$$

By the similar procedure to the case of the particle-particle bound state, we obtain the renormalized 4-point Green function for the particle-anti-particle pair as

$$\mathscr{G}_A(P) = \frac{i}{2}\frac{1}{\left(\frac{1}{g_r} - 2A'\right)^2 - B^2}\left[\gamma_a \cdot \gamma_b^T\left(\frac{1}{g_r} - 2A'\right) - \frac{1}{2}(\gamma_a \cdot \gamma_b^T)^2 B\right]. \tag{41}$$

Here A' and B are given by (27) and (22). $\mathscr{G}_A$ has poles at

$$g_r(2A' - B) = 1, \tag{42}$$

and

$$g_r(2A' + B) = 1. \tag{43}$$

These show that the sign of g_r is opposite to that of the particle-particle case. This means, therefore, that in the validity of our approximation, i.e. for a small value of g_r, the bound states of particle-particle and particle-anti-particle pairs cannot exist at the same time.

4 Discussion

In this article we calculate the four-point Green function for the massive Dirac fields with the current-current interaction in the 2-dimensional space-time. We see from Eq. (36) that even the single-electron theoretical treatment has its validity for the normal solution when coupling constant is small.

As for the abnormal solutions, however, the situation is very different. In the GNF model the abnormal solution has been regarded as a counterpart of the bound state solution of the particle-anti-particle pair, because it is composed of the positive and the negative energy particles. There does not exist, of course, the anti-particle in the single-electron theory, where conserved charge $Q_i = \int dx^1 j_i^0$ is positive-definite. In the positron theory, on the other hand, Q_i is not positive-definite and the particle and the anti-particle states are distinguishable from each other by the signs of their eigenvalues of Q_i. The abnormal solution (34) has obviously $Q_a = 1$, $Q_b = 1$, and hence it is another solution of the particle-particle bound state, though the sign of coupling constant is opposite to that of the normal solution and is same as that of the particle-anti-particle case.

As mentioned before, the definition of the renormalized coupling constant here is different from the conventional one. This is because vector type interaction induces scalar and pseudo-scalar type interactions through the higher order perturbations though the latter interactions are not included in original Lagrangian. When we start with Langragian which contains scalar and pseudo-scalar type interactions as well as vector type one, such discrepancy would not occur.

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