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Nucleon and Pion Form Factors at High Momentum Transfers in a Unified Approach Based on the Quark Model

Taketoshi INO

Department of Physics, Shimane University, Matsue 690
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Employing a new prescription for relativizing static form factors which has been recently proposed by taking account of solvable relativistic two- and three-body models, we show that experimental data on the nucleon and pion electromagnetic form factors can be reproduced well by the Gaussian internal wave functions for nucleon and pion.

It has been shown by several authors[1,2,3] that considering the Lorentz contraction efect of the extended or composite nucleon, the Gaussian inner orbital wave function can produce nucleon electromagnetic form factors very close to experimental data. The calculations in those works have been based on some devised prescriptions for relativizing static form factors (or static density distributions) given by static (or non-relativistic) models of hadron structure. The prescriptions in those works have all differed, and therefore the details of those works have also differed. In the work by Stanley and Robson[3] who have intended to find a reasonable prescription, the pion electromagnetic form factor has also been studied on the same footing as for the nucleon form factors, and the prescription proposed by Mitra and Kumari[4] (as a modified version of that by Licht and Pagnamenta[2]) has been insisted on. In the work[3] quarks have been assumed to have an effectively-finite size, differently from in the other works.[1,2]

In this paper, we study the nucleon and pion electromagnetic form factors by employing a new prescription recently proposed by Munakata, Sakamoto, Yamamoto, Nakamae and the author.[5] (The work of Ref. 5) is referred to as I hereafter.) As static wave functions for the nucleon and the pion, the harmonic-oscillator wave functions are taken. The quarks are assumed to be of zero size. First, we state the prescription proposed in I and compare it with other prescriptions, taking an interest in basic hypotheses for invention of prescriptions. From a clear reason, the prscription in I is insisted on. Next, the detail of the present approach for the form factors is stated, and calculated form factors are compared with experiments. Lastly, the result is discussed.

A solvable relativistic two-body model has been presented in $1+1$ dimensional space-time by Glöckle. Nogami and Fukui[6] (GNF). The GNF model is described in terms of the two-body Dirac equation with an interaction in the form of the delta function. The GNF equation is not manifestly covariant, but its relativistic covariance is guaranteed by the existence of the boost operator. The bound-state solution in the GNF model exhibits exact Lorentz contraction, when it is boosted.*) GNF have used the bound-state solution to calculate the form factor in electron-bound state scattering. Extending the GNF model to a relativistic N-body model where N constituent Dirac particles interact through pairwise direct instantaneous interactions in $1 + 1$ dimensions and the boost operator is explicitly constructed, the authors of I have recently presented an exact three-body bound-state solution,[8] and employed this solution in order to calculate the form factor in electron-three-body bound state scattering (in I). In addition to giving relativistic form factors of bound states, the GNF model and this relativistic N-body model provide an explicit relation between a relativistic form factor of a bound state and its static approximation.

In I, the relation

$$F(Q^2) = \{1 + Q^2/(4E_0^2)\}^{-N/2} F_{\text{static}}(Q^2/\{1 + Q^2/(4E_0^2)\}) \qquad (N = 2, 3) \tag{1}$$

is presented. Here, $F(Q^2)$, Q^2 and E_0 are a relativisti form factor, the squared momentum transfer ($Q^2 = (p' - p)^2 - (E_{p'} - E_p)^2$) and the rest mass of the relevant bound or composite state respectively, and the last factor on R. H. S. is obtained from a static form factor $F_{\text{static}}(q^2)$ ($q^2 = (p' - p)^2$) by replacing q^2 with $Q^2/\{1 + Q^2/(4E_0^2)\}$. The relation (1) works as a prescription for relativizing static form fators. As it does not depend on the assumed form of interaction, it can be applied to composite systems of Dirac particles interacting through confining potentials. Although it is obtained in $1 + 1$ dimensions, it is supposed to be usable also in the real world, because it represents the Lorentz contraction effect (occurring only in the direction of motion of the composite state).**) It is noted that the Lorentz contraction is described, in the wave function of the GNF model and the N-body model stated above, as an intrinsic coupling between the structure of a composite system (in terms of the relative coordinates) and the overall translational motion of the system,[6,8,9] and therefore the prescription (1) is not applicable to shell-model-like static models, in which the total wave function does not have a well-defined total momentum.

This prescription (1) is different from other ones.[1,2,4,10] As far as we are concerned with the GNF relativistic model and the relativistic N-body model stated above, we can not understand other prescriptions. We are interested somewhat only in the Licht-Pagnamenta prescription,[2] which has been invented,

*) Munakata, Nagamura and the author[7] have shown that the GNF model is rewritten in a manifestly covariant form of Bethe-Salpeter equation with the Fermi-type interaction, when the single-electron-theoretical treatment is adopted.

**) The factor $\{1 + Q^2/(4E_0^2)\}^{-1/2}$ comes from the Lorentz-contraction factor E_0/E_p. This is easily known by the consideration of electron-bound state scattering in the Breit frame where one has $p' = -p$.

from the idea of regarding composite particles as clusters, to be $F(Q^2)=\{1+Q^2/(4E_0^2)\}^{(1-N)/2}F_{\text{static}}(Q^2/\{1+Q^2/(4E_0^2)\})$. As for the N-dependence of the Pagnamenta relation, it is natural. The difference between the relation (1) and the Licht-Pagnamenta one is supposed to come from the thing that the relativistic normalization of the wave function is taken into account in the relation (1) but not in the Licht-Pagnamenta one.

Now, we stusy the nucleon and pion electromagnetic form factors, using the prescription (1). A needed static model is prepared from te quark model, the dynamics of which comes from the colour gauge theory. The colour gauge couplings produce a long-range spin-independent force, which leads to the appearance of $SU(6)_q \otimes SU(6)_{\bar{q}} \otimes O(3)_L$ supermultiplets of hadrons. In the calculation of hadron masses, the short-range force arising from one-gluon exchange is taken into account perturbatively. As for the calculation of static density distributions relevant to the present problem, the long-range spin-independent force is essential, since it predominantly determines the density distributions. Explicit forms of static density distributions (or static wave functions) are constructed on the harmonic-oscillator basis. The harmonic-oscillator model, which predicts squared masses being proportional to the total numbers of quanta of harmonic-oscillator excitations and so is successful in the classification of hadrons, is on an intimate connection with the potential $k^2|\vec{r}|$ predicted by the lattice gauge theory.*) Finally, static form factors are relativized considering the Lorentz contraction effect. In using the prescription, E_0 is regarded as an effective static mass being near to the mass determined only by the long-range spin-independent force. The effective static mass is close to the observed rest mass for the nucleon, but it is not so for the pion.

The above situation is, in outline, the same as in other works.[1,2,3] We state details of the present approach. (a) As the effective static mass which is used in the prescription (1), we take the mass determined only by the long-range spin-independent force. We use the following values:

$$E_{0,N}=1.09\text{ GeV},\quad E_{0,\pi}=0.611\text{ GeV}. \tag{2}$$

(These values are related to the combinations of observed m_N, m_Δ, m_π and m_ρ being $(m_N+m_\Delta)/2$ and $(m_\pi+3m_\rho)/4$.) We make a note about the effective static mss of the pion. In the work by Stanley and Robson[3] who have intended a parameter-free approach by using only a Hamiltonian which simultaneously describes mesons and baryons, $q\bar{q}$ interactions allowing for annihilation of $q\bar{q}$ into two or three gluons are taken into account in adition to the long-range and

*) When the potential is incorporated into the equation for composite states of confined quarks which is presented by Lichtenberg et al.,[11] such a model is provided, as far as quarks of small current masses are concerned. (The constituent quark masses are obtained from the current masses and the expectation value of the long-range spin-independent potential in the state considered.[11])

one-gluon-exchange potentials. About the mass splitting among neutral 0^- mesons and that among neutral 1^- mesons, we imagine such qq^- interactions. In the result, the effective pion mass in Eq. (2) is near to the one in that work.*) (b) The quarks are as sumed to be of zero size, differently from the work by Stanley and Robson. (c) In the work by Fujimura, Kobayashi and Namiki[1] who have first studied the nucleon electromagnetic form factors considering the Lorentz contraction effect, two cases for the static density distribution have been argued. One is the case where vector-meson clouds around the mucleon are not taken into account. The other is the case where each quark has the vector-meson cloud dominated by the ρ and ω mesons and the density distribution of the cloud undergoes the Lorentz contraction. In the present work, we argue mainly the latter which is reasonable.

Using the effective masses (2), we write the assumed static form factors for the nucleon and the pion to be

$$F_{\mathrm{static},N(\pi)}(\vec{q}^{\,2}) = (1 + \vec{q}^{\,2}/m_V^2)^{-1} e^{-C_{N(\pi)}\vec{q}^{\,2}/E^2_{0,N(\pi)}}, \tag{3}$$

where m_V^2 is the mean square mass of ρ and ω mesons for the nucleon and the square mass of ρ meson for the pion. Relativistic form factors obtained by employing the prescription (1) to the static form factors (3) are compared with experimental data on G_M^p/μ_p[12] (adopted as a representative of the nucleon form factors) and F_π[13] in Figs. 1 and 2, taking C_N and C_π to be $C_N = 0.728$ and

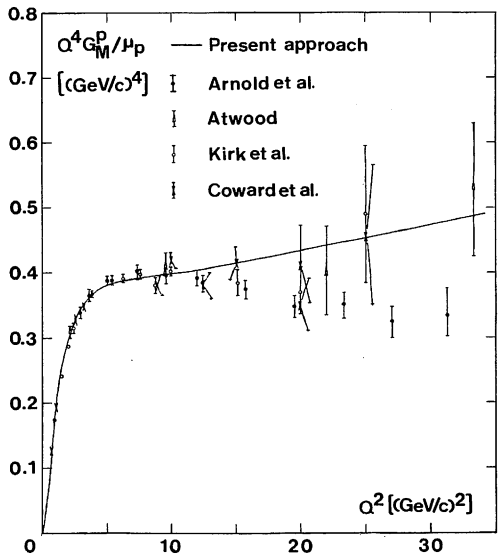

Fig. 1. Comparison of the theoretical $Q^4G_M^p/\mu_p$ with experiments.

*) The effective pion mass is estimated to be 0.638 GeV in that work.

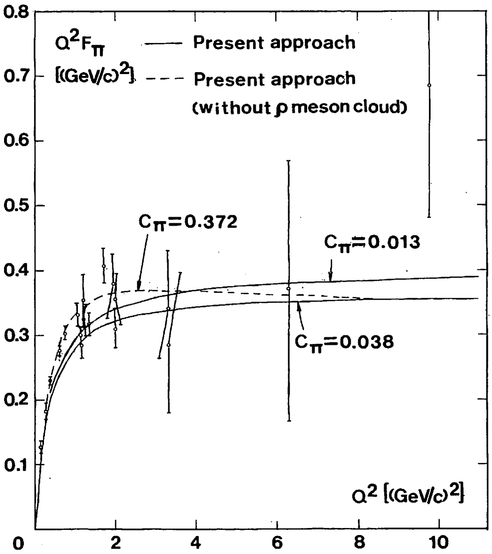

Fig. 2. Comparison of the theoretical Q^2F_π with experiments.

$C_\pi = 0.013$ or 0.038. The fit with $C_\pi = 0.013$ is to all data, while the fit with $C_\pi = 0.038$ is to data at $Q^2 > 2$ $(\mathrm{GeV}/c)^2$, to which C_π is sensitive.

We discuss the result for G_M^p/μ_p first and that for F_π next.

(I) The theoretical curve for $Q^4 G_M^p/\mu_p$ can reproduce experimental data in a range of Q^2 from zero to about 15 $(\mathrm{GeV}/c)^2$ well. As for at higher momentum transfers ($Q^2 \gtrsim 20$ $(\mathrm{GeV}/c)^2$), more precise experiments are desired in order to make a rigorous comparison.

The case of ignoring the vector-meson cloud does not provide a good fit to experimental data on G_M^p/μ_p.

(II) The theoretical curves for Q^2F_π with $C_\pi = 0.013$ and 0.038 are close to experimental data in a range of Q^2 from about 1.5 to about 6.5 $(\mathrm{GeV}/c)^2$. It is remarked that the asymptotic behaviour of the present-approach prediction for F_π is the same as that of the non-perturbative QCD prediction.[14] (The asymptotic value of Q^2F_π predicted by the non-perturbative QCD is in a good agreement with experimental data above 2 $(\mathrm{GeV}/c)^2$ except for the only one datum at 9.77 $(\mathrm{GeV}/c)^2$.[15] It is hoped that precise experiments are performed at Q^2 above 7 $(\mathrm{GeV}/c)^2$.) It is noted that the present approach (taking account of also the ρ meson cloud) is consistent also with experimental data at low momentum transfers, when one takes the effective mass $E_{0,\pi}$ to be ~ 0.69 GeV. Considering further an upheaval in the density near the center due to the short-range attractive interaction, such a value for $E_{0,\pi}$ may be effectively produced.

The case of ignoring the ρ meson cloud provides a good fit to experimental data, though it requires a rather large value for C_π ($C_\pi = 0.372$).

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