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A Possible Description of the π and ρ Mesons as Nambu-Goldstone Bosons

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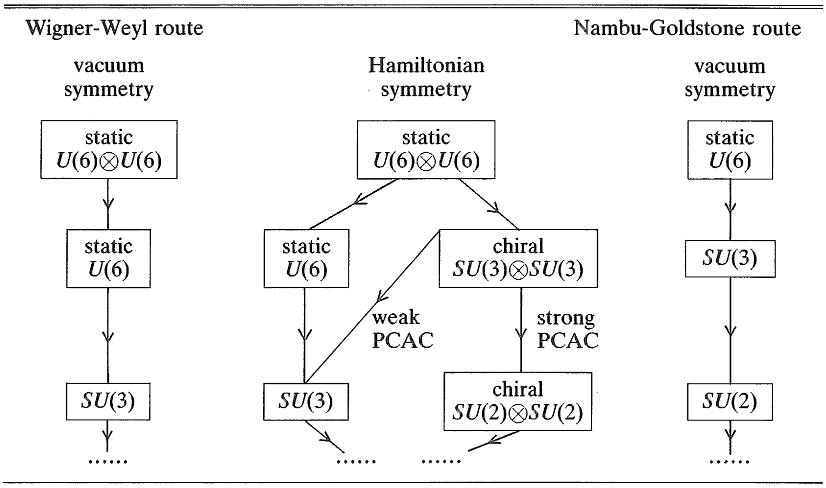
A dynamical system, composed of the local duality scheme, the most general Veneziano-type formula (the global duality) and an asymptotic convergence condition and being able to determine Born amplitudes for some meson-meson scattering, is studied in connection with the Caldi-Pagels idea to resolve the ρ-π puzzle that how can the ρ and π be members of a quark model $U(6)$ $\underline{36}$ and the π be a Nambu-Goldstone boson satisfying the PCAC.

There remains a long-standing problem in the hadron dynamics, called as the $\rho-\pi$ puzzle: How can the ρ and π be members of a quark model $U(6)$ $\underline{36}$ and the π be a Nambu-Goldstone boson satisfying the partial conservation of the axial-vector current (PCAC)?

An idea for a resolution of the above puzzle has been proposed by Caldi and Pagels.[1] They have adopted the quark model and abstract commutation relations from QCD. Further, they have assumed that in the static limit the ρ and the π and their $U(3)$ partners transform like members of a $(6, \bar{6})\oplus(\bar{6}, 6)$ representation of the Feynman-Gell-Mann-Zweig chiral $U(6)\otimes U(6)$ algebra.[2] Their starting point is to imagine a non-relativistic world with a Hamiltonian symmetry chiral $U(6)\otimes U(6)$. The vacuum symmetry is spontaneously broken to $U(6)$ or $SU(6)$ and this is the classificatory group for hadrons at rest. The π and ρ along with their $U(3)$ partners are true Goldstone bosons in this non-relativistic world. In the relativistic world, in which the $U(6)$ vacuum symmetry is necessarily broken, the ρ will be massive — however, it remembers its origin as a Goldstone state. The pseudoscalars can remain strictly massless true Goldstone states in this relativistic world with a chiral $SU(3)\otimes SU(3)$ Hamiltonian symmetry. The breaking of chiral $SU(3)\otimes SU(3)$ then proceeds as in the Gell-Mann, Oakes and Renner model.[3] Their idea is shown in Table I, constrasted to the Wigner-Weyl route. And, the removal of hadron mass degeneracies in the model is shown in Fig. 1.

However, the Caldi-Pagels idea can not be embodied by the field theory. As is well known, it is impossible to construct an interacting relativistic field theory with the $U(6)\otimes U(6)$ symmetry.[4] In order to give a concrete form to the Caldi-Pagels idea, one must find the other representation of the hadron interactions.

Table I. Group diagram for the Caldi-Pagels Nambu-Goldstone route contrasted to the Wigner-Weyl route.



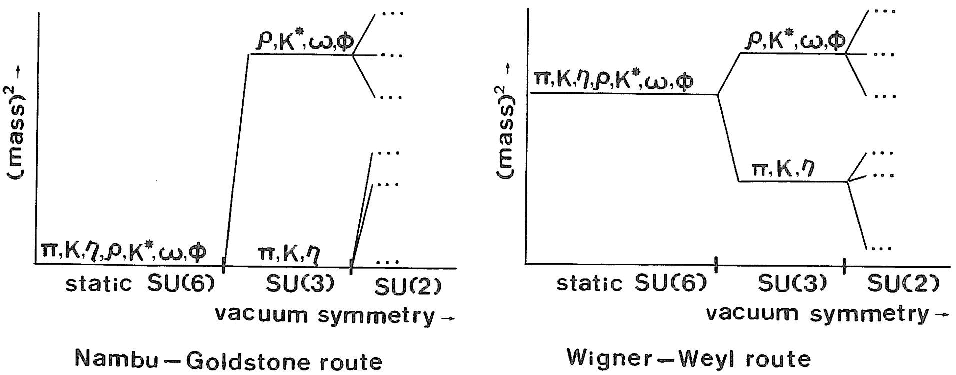


Fig. 1. Level diagrams of the ground-state pseudoscalar and vector mesons in the two routings shown in Table I.

As is also well known, a relativistic theory can yield a non-relativistic world as a very limited range of phenomena. Therefore, it is worth-while to search a relativistic model for hadrons which has a static limit as in the Caldi-Pagels idea.

Recently, Munakata, Sakamoto and the author have obtained a uniquely determined $\pi\pi\to\pi\pi$ amplitude.[5] We are now examining that does this uniquely determined $\pi\pi\to\pi\pi$ Born amplitude or the dynamical system yielding it have a static limit where the π and the ρ are Goldstone bosons. We have made some discussions in this direction.[6,7] In this note, we shall make a further argument which will suggest that the dynamical system being able to determine $\pi\pi$ Born amplitude etc.

uniquely is promising to embody the Caldi-Pagels idea.

We start with the dynamical system composed of the local duality scheme[8] involving the harmonic-oscillator spectrum of $SU(6)\otimes O(3)_L$ multiplets, the most general Veneziano-type amplitude representing the global duality that Σ (s-channel poles) $=\Sigma$ (t-channel poles)[9] and an asymptotic convergence condition. As shown in Ref. 5), starting with the most general Veneziano-type amplitude with just the degree of freedom to provide an arbitrary residue at each of parents and their gaughters, and imposing the constraints of the local duality scheme and an asymptotic convergence condition, we can obtain a uniquely determined $\pi^-\pi^+\to\pi^-\pi^+$ Born amplitude.

The dynamical system seems to be in an intimate connection with the PCAC. The obtained $\pi^-\pi^+\to\pi^-\pi^+$ Born amplitude has the property that i) in the limit $m_\pi=0$ and $\alpha_0=1$ (that is, in the limit $m_\pi=m_\rho=0$), it satisfies Adler's PCAC consistency condition, where α_0 is the zero-intercept of the (exchange-degenerate) $\rho-f$ Regge trajectory, ii) it contains neither negative-norm states nor tachyons in a domain ($\alpha' m_\pi^2$, α_0) around the physical m_π, α' (the Regge slope of the $\rho-f$ trajectory) and α_0 which contains the point of $m_\pi=0$ and $\alpha_0=1$, and iii) when m_π, α' and α_0 are taken to be their physical values, and the overall multiplying factor is adjusted by $\rho\to 2\pi$ width, the predicted partial widths of low-lying resonances are in good agreement with available experiments.[5]

Here, we shall study a $\pi^-\rho^+\to\rho^-\pi^+$ Born amplitude predicted by the above dynamical system. The Caldi-Pagels picture of the vector mesons implies the PCTC (partial conservation of tensor current).[10] We shall discuss a resulting $\pi^-\rho^+\to\rho^-\pi^+$ amplitude with respect to the PCTC in addition to the PCAC.

For the present purpose, we construct only an invariant amplitude $A(s, t)$ in the decomposition of the $\pi^-\rho^+\to\rho^-\pi^+$ amplitude

$$F(\pi^-\rho^+\to\rho^-\pi^+)=(\varepsilon\varepsilon')A(s, t)+\cdots, \tag{1}$$

where ε and ε' are the polarization vectors of the incoming and outgoing ρ mesons. The invariant amplitude $A(s, t)$ is $s-t$ corssing even. The exchanged states relevant to it are shown in Fig. 2, assuming the exchange-degeneracy of the ω and A_2 trajectories. As we are interested in the limit $m_\pi=m_\rho=0$, we construct the invariant amplitude $A(s, t)$ in the case $m_\pi=m_\rho\equiv\mu$.

As similarly as in Ref. 5), preparing the most general Veneziano-type amplitude in terms of the $\omega-A_2$ trajectory $\alpha(s)$, which has just the degree of freedom to provide an arbitrary residue at each of parents and their daghters including odd daughters,

$$A(s, t)=\sum_{n=1}^{\infty}\sum_{k=n}^{2n}\lambda_{n,k}\frac{\Gamma(n-\alpha(s))\Gamma(n-\alpha(t))}{\Gamma(k-\alpha(s)-\alpha(t))}, \tag{2}$$

setting all the contributions of the odd daughters to be zero, and imposing the constraints of the local duality scheme and the convergence condition for $s\to\infty$, small $|t|$, we obtain a uniquely determined $A(s, t)$ amplitude.

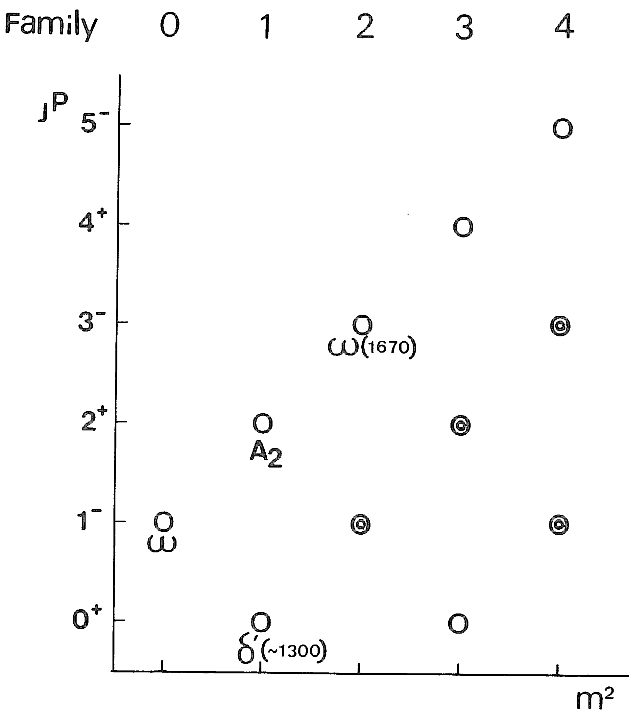


Fig. 2. The exchanged states relevant to the invariant amplitude $A(s, t)$ of the reaction $\pi^-\rho^+\to\rho^-\pi^+$. They are classified into the resonance families. The double circles imply that there are two states with different quark-orbital angular momenta at their places.

The local duality scheme[8] for a $0^-1^-\to0^-1^-$ process with one exotic channel (the exotic u-channel) is as follows: For each of the invariant amplitude, we postulate the set of local duality relations

$$\sum_{a\in N} R_a^{(s)}(t)|_{t=m_{N'}^2} = \sum_{b\in N'} R_b^{(t)}(s)|_{s=m_N^2}, \quad N, N'=0, 1, 2,\cdots. \tag{3}$$

Here, $R_a^{(s)}(t)$ ($R_b^{(t)}(s)$) is the residue of the considered invariant amplitude at the s-(t-) channel resonance a (b) in the narrow-width approximation, and N (N') is the s-(t-) channel resonance family. The resonance family N is defined to be a set of resonances with a fixed total number N of quanta of harmonic-oscillator excitations. In the relations (3), the mass-degeneracy for each resonance family is assumed so as to be accommodated to the Veneziano-type amplitude.

In Fig. 2, we assign N to the s-(t-) channel resonances.

The resulting amplitude is

$$A(s, t)=-\lambda(1-\beta^2)\sum_{n=1}^{\infty}\frac{1}{(n-1)!(2n-1+\beta)(2n-3+\beta)}$$
$$\times\left[\frac{\Gamma(n-\alpha(s))\Gamma(n-\alpha(t))}{\Gamma(n-\alpha(s)-\alpha(t))}+\frac{(1-\beta)}{2}\frac{\Gamma(n-\alpha(s))\Gamma(n-\alpha(t))}{\Gamma(n+1-\alpha(s)-\alpha(t))}\right], \qquad (4)$$

where λ is an overall multiplying constant and β is

$$\beta=2-3\alpha_0-4\alpha'\mu^2. \qquad (5)$$

The amplitude (4) satisfies Adler's soft-pion PCAC consistency condition[11)]

$$A(s=t=m_\rho^2,\ u=m_\pi^2;\ (\text{mass})^2 \text{ of one of the two pions}=0)=0, \qquad (6)$$

in the limit

$$\mu(=m_\pi=m_\rho)=m_\omega \longrightarrow 0. \qquad (7)$$

Adler-like soft-ρ-meson PCTC consistency condition may be written as

$$A(s=t=m_\pi^2,\ u=m_\rho^2;\ (\text{mass})^2 \text{ of one of the two } \rho\text{-mesons}=0)=0 \qquad (8)$$

analogously to Adler's PCAC consistency condition for soft-pseudoscalar meson, and this condition is also satisfied by the amplitude (4) in the limit (7).

Here, we note three things.

A) If one considers the $1/N_c$ expansion of QCD, the conceptual link between the colour gauge theory and the dual models is provided.[12)] Especially, it is expected that QCD gives, in the leading order, something like the tree approximation of a planar dual model.[12)] The tree approximation of a planar dual model is what we are now studying. Therefor, to expect an intimate connection between the Caldi-Pagels picture and the present dynamical system.

B) It is reasonable to postulate $m_\rho=m_\omega$ in the present dynamical system, because the planar duality (planar diagram) necessarily leads to the ideally mixed nonet mesons.

C) The present dynamical system provides amplitudes (in terms of the Mandelstam variables s, t and u) in a relativistically invariant way, and, as seen in the resulting $\pi^-\pi^+\to\pi^-\pi^+$, with the physical values for m_π, α' and α_0 (thus, the physical values for m_π, m_ρ and etc.) as input, the resulting amplitude is consistent with observed patial decay widths. Further, the present system or the resulting amplitudes have a limit $m_\pi=m_\rho=m_\omega=0$ which is a non-relativistic world as a very limited range of phenomena.

The above satisfaction of the PCAC and PCTC consistency conditions at the limit (7) may suggest that the limit (7) corresponds to the non-relativistic world in the Caldi-Pagels idea, and the π and ρ mesons are described as Goldstone bosons in the present dynamical system.

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