Mem. Fac. Sci. Shimane Univ.,
20, pp. 37–42 Dec. 20, 1986

A Description of $q\bar{q}$ and $q\bar{q}q\bar{q}$ Mesons and High-energy Forward $\pi^-\pi^+\to\pi^0\pi^0$ Reaction

Taketoshi INO

Department of Physics, Shimane University, Matsue 690
(Received September 6, 1986)

We have recently presented a model for $\pi\pi\to\pi\pi$ scattering, involving the $q\bar{q}$ and $q\bar{q}q\bar{q}$ mesonic resonances. The model has a dual term involving the $q\bar{q}$ mesons and a short-range correction term doing the $q\bar{q}q\bar{q}$ mesons. Here, we discuss how does the correction term modify Regge asymptotic behaviours of the dual term.

Experimental informations on the scalar mesons[1)~5)] indicate that the meson spectrum can not be understood only by the quark-model $q\bar{q}$ states. In fact, in addition to well-established scalar mesons $S(975)$, $\delta(980)$, $\varepsilon(1300)$ and $\kappa(1350)$,[1)] there are promising candidates for the scalar mesons, $\varepsilon(\sim 800)$,[2),3)] $\delta'(\sim 1300)$[4)] and an $s\bar{s}$ component.[5)] It may be reasonable to assign the $\varepsilon(1300)$ and $\kappa(1350)$ and $\delta'(\sim 1300)$ and an $s\bar{s}$ component in Ref. 5) to the $q\bar{q}$ $^3P_0$ states, because their masses are near to the masses of the $f(1270)$, $K^*(1430)$, $A_2(1320)$ and $f'(1525)$[1)] (which are assigned to the $^3P_2$ states) and those of the $D(1285)$, $Q_A(1340)$,*) $A(1270)$ and $E(1420)$[1)] (to the $^3P_1$ ones).

The remaining low-lying $S(975)$, $\delta(980)$ and $\varepsilon(\sim 800)$ do not, however, possess corresponding $q\bar{q}$ states.

An interesting understanding of the low-lying $\varepsilon(\sim 800)$, $S(975)$ and $\delta(980)$ is to assign them to the lightest cryptoexotic $qq\bar{q}\bar{q}$ nonet, supposing the existence of other components κ (and $\bar{\kappa}$) with mass$\simeq$900 MeV.[6)] Following Jaffe[6)] who has studied the S-wave $qq\bar{q}\bar{q}$ states in a semi-classical approximation to the MIT bag model, the states are analogous to the usual S-wave $q\bar{q}$ mesons and have large widths in mesonic channels, because they preferentially decay by just falling apart into two $q\bar{q}$ mesons. (They are not baryonium-like states,[7)] and they are often denoted by $q\bar{q}q\bar{q}$, which we also use hereafter.) As has been also discussed by Jaffe, the lowest $q\bar{q}q\bar{q}$ nonet lies, in mass, lower than the $q\bar{q}$ $^3P_0$ nonet. The lightness of the lowest $q\bar{q}q\bar{q}$ nonet is mainly due to the short-range spin-spin force mediated by one-gluon exchange.

Recently we have made a discussion about the remaining problem: How is the scattering amplitude involving the $q\bar{q}$ and $q\bar{q}q\bar{q}$ mesons written?[8)] We have paid attention to that the $q\bar{q}q\bar{q}$ states, i.e., the S-wave $qq\bar{q}\bar{q}$ states are quite different from

---

*) The observed $I=1/2$, $S=1$ axial-vector mesons are the $Q\,(1280)$ and $Q\,(1400)$.[1)] The Q_A is one of the two G eigenstates.

the $l(\geqq 1)$ $qq-\bar{q}\bar{q}$ (diquark-antidiquark) states.[7),9)] The $q\bar{q}q\bar{q}$ states are analogous to the usual S-wave $q\bar{q}$ mesons, as already stated above. While, the $l(\geqq 1)$ $qq-\bar{q}\bar{q}$ ones are corresponding to elongated bag structures.[7),9)] Since their couplings to mesonic channels are suppressed by centrifugal barrier effects, only their couplings to baryon-antibaryon channels will be important.[7),9)] Therefore, as far as the meson-meson scattering is considered, only the $q\bar{q}q\bar{q}$ states are concerned in the short-range correction to the dual amplitude involving the $q\bar{q}$ resonances. As for the details of the correction term for the $0^-0^-\to 0^-0^-$ scattering, only the 0^+ $q\bar{q}q\bar{q}$ states contribute practically to the term. (The 2^+ $q\bar{q}q\bar{q}$ states can not dissociate into two 0^- mesons.[6),*)]) This is very important. The correction term never spoils the high-energy behaviour of the dual term. However, the correction term somewhat changes high-energy behaviours of the dual term. In this paper, we discuss high-energy behaviours of the amplitude composed of the dual and short-range correction terms.

First, for the $\pi\pi\to\pi\pi$ scattering, we state details of the present model. For the $\pi^-\pi^+\to\pi^-\pi^+$ scattering, the present model is illustrated by the quark diagram in Fig. 1. A uniquely determined $\pi^-\pi^+\to\pi^-\pi^+$ dual resonance amplitude recently obtained by Munakata, Sakamoto and the author[10)] is taken as the dual term. It is obtained, starting with the most general Veneziano-type amplitude in terms of the exchange-degenerate $\rho-f$ Regge trajectory, which has just the degree of freedom to provide an arbitrary residue at each of parents and their daughters, and restricting the most general Veneziano-type amplitude by two conditions; the local duality scheme[11)] involving the harmonic-oscillator spectrum of $SU(6)\otimes O(3)_L$ multiplets, and an asymptotic convergence condition. It is

$$F(s,\,t)=-\lambda(1-\beta^2)\sum_{n=1}^{\infty}\frac{1}{(n-1)!\,(2n-1+\beta)(2n-3+\beta)}\times\left(\frac{\Gamma(n-\alpha_s)\Gamma(n-\alpha_t)}{\Gamma(n-\alpha_s-\alpha_t)}+\frac{(1-\beta)}{2}\frac{\Gamma(n-\alpha_s)\Gamma(n-\alpha_t)}{\Gamma(n+1-\alpha_s-\alpha_t)}\right). \quad (1)$$

Here, $\alpha_s\equiv\alpha' s+\alpha_0$ is the $\rho-f$ trajectory, and λ is a constant, and β is $\beta\equiv 2-3\alpha_0-4\alpha' m_\pi^2$. As for the correction term, we consider only the lightest multiplet for simplicity, although there are four 0^+ $q\bar{q}q\bar{q}$ multiplets.[6)] (It is noted that the heavier multiplets ($\underline{9}^*$ and $\underline{36}^*$) couple only weakly to 0^-0^- channels.[6)]) The lightest multiplet is a nonet, to which the $\varepsilon(\sim 800)$, $\kappa(\sim 900)$, $\delta(980)$ and $S(975)$ are assigned. We note also that the $\delta(980)$ and $S(975)$ components of the lightest nonet, whose quark contents are $\{u\bar{d}s\bar{s}$ etc.$\}$ and $s\bar{s}(u\bar{u}+d\bar{d})/\sqrt{2}$, are forbidden to decay into $\pi\pi$ by the OZI rule. We tentatively write down the $\varepsilon(\sim 800)$ exchange amplitude in the narrow-width approximation which is used to construct the dual term,

$$\frac{g^2_{\varepsilon(\sim 800)\pi\pi}m^2_{\varepsilon(\sim 800)}}{m^2_{\varepsilon(\sim 800)}-s}+\frac{g^2_{\varepsilon(\sim 800)\pi\pi}m^2_{\varepsilon(\sim 800)}}{m^2_{\varepsilon(\sim 800)}-t}\,. \quad (2)$$

*) The decay products of the $q\bar{q}q\bar{q}$ mesons must be S-wave $q\bar{q}$ mesons in relative S waves.[6)]

The whole amplitude $A(s, t)$ for $\pi^-\pi^+\to\pi^-\pi^+$ scattering in the present model is Eq. (1) plus Eq. (2). With input

$$m_\pi=0.140\text{ GeV},\quad \alpha'=0.888\text{ (GeV)}^{-2},\quad \alpha_0=0.475,\quad \lambda=-0.743, \tag{3}$$

and $m_{\varepsilon(\sim 800)}=0.8$ GeV and the kinematics in $\pi\pi$ scattering, the present model reproduces experiments on partial decay widths and S-wave scattering lengths fairly well and satisfies Adler's PCAC consistency condition[12] and the Adler-Weisberger sum rule.[13] (It is noted that λ is adjusted by observed $\rho\to 2\pi$ width.)

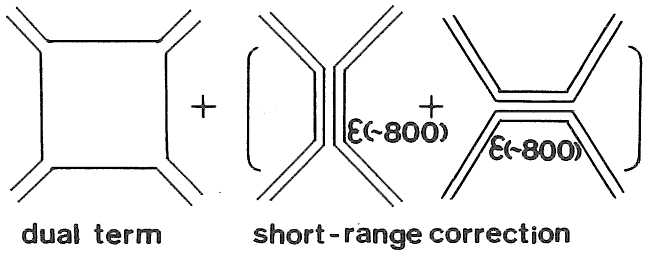

Fig. 1. The present model for $\pi^-\pi^+\to\pi^-\pi^+$ scattering. The dual term involves $q\bar{q}$ resonances, and its Regge behaviour reflects the long-range confining force. The correction to it involves the $J^P=0^+$ $qq\bar{q}\bar{q}$ states, i.e., the S-wave 0^+ $qq\bar{q}\bar{q}$ states and has a fixed and short force range.

Now, we discuss high-energy behaviours of the amplitudes in the present model. The charge-exchange $\pi^-\pi^+\to\pi^0\pi^0$ amplitude in the present model is

$$A(\pi^-\pi^+\longrightarrow\pi^0\pi^0)=\frac{1}{2}[F(t, u)-F(s, t)-F(s, u)]-\frac{g^2_{\varepsilon(\sim 800)\pi\pi}m^2_{\varepsilon(\sim 800)}}{m^2_{\varepsilon(\sim 800)}-s}, \tag{4}$$

and its dual term has the asyptotic form

$$\frac{1}{2}[F(t, u)-F(s, t)-F(s, u)]$$

$$\xrightarrow[s\to\infty,\ t\text{ fixed}]{}\frac{1}{2}[\lambda\Gamma(1-\alpha_t)I(\alpha_t)\alpha_s^{\alpha_t}-\lambda\Gamma(1-\alpha_t)e^{-i\pi\alpha_t}I(\alpha_t)\alpha_s^{\alpha_t}]\,. \tag{5}$$

Here,

$$I(\alpha_t)=-\frac{(1-\beta^2)}{\Gamma(1-\alpha_t)}\sum_{n=1}^{\infty}\frac{\Gamma(n-\alpha_t)}{(n-1)!\,(2n-1+\beta)(2n-3+\beta)}$$

$$=F\left(1-\alpha_t,\ \frac{\beta-1}{2},\ \frac{\beta+3}{2};1\right)=\frac{\Gamma\left(\frac{3+\beta}{2}\right)\Gamma(1+\alpha_t)}{\Gamma\left(\frac{1+\beta}{2}+\alpha_t\right)},\ (\text{Re}\,(3+\beta)>0,\ \text{Re}\,(1+\alpha_t)>0). \tag{6}$$

In Eq. (6), $F(a, b, c; z)$ is Gauss's hypergeometric function.

In Fig. 2, we compare the high-energy behaviour of amplitude (4) with that of its dual term, at a dipion mass of 1.8 GeV and in the forward direction $0 \geqq t \geqq -1\ (\mathrm{GeV}/c)^2$ where the Born amplitude may be dominant.

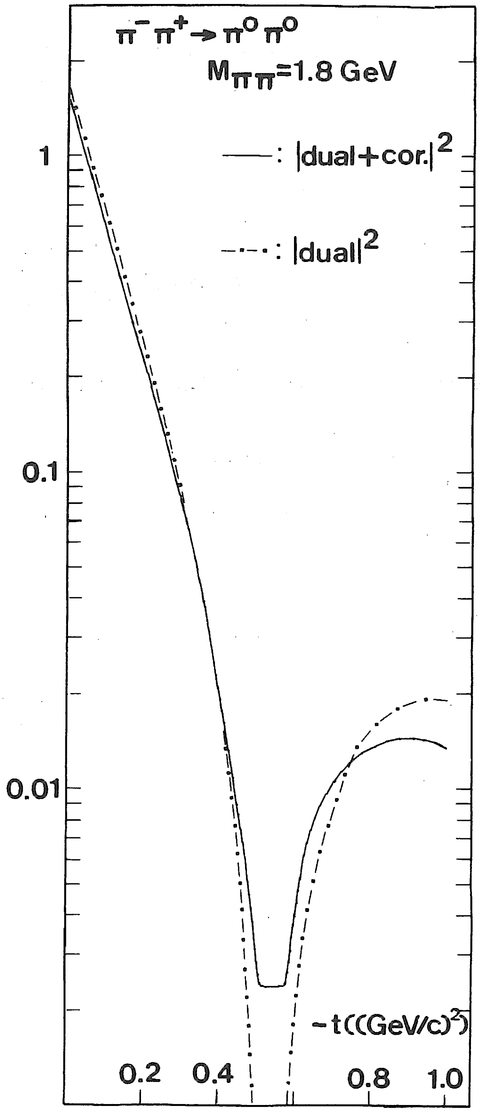

Fig. 2. Comparison of the $\pi^-\pi^+\to\pi^0\pi^0$ am-plitude in the present model with its dual term at a high energy.

Similar comparisons are made for the $\pi^-\pi^+\to\pi^-\pi^+$ and $\pi^+\pi^+\to\pi^+\pi^+$ scattering in Figs. 3 and 4. As for the $\pi^+\pi^+\to\pi^+\pi^+$ amplitude in the present model, it is

$$A(\pi^+\pi^+ \longrightarrow \pi^+\pi^+) = F(t,\, u) + \frac{g^2_{\varepsilon(\sim 800)\pi\pi} m^2_{(\varepsilon\sim 800)}}{m^2_{\varepsilon(\sim 800)} - t} + \frac{g^2_{\varepsilon(\sim 800)\pi\pi} m^2_{\varepsilon(\sim 800)}}{m^2_{\varepsilon(\sim 800)} - u}\,. \tag{7}$$

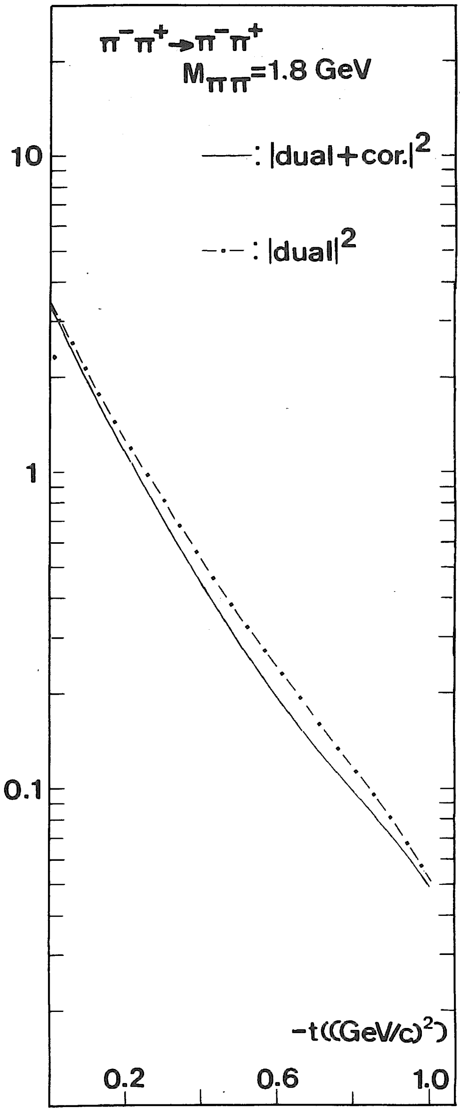

Fig. 3. Comparison of the $\pi^-\pi^+\to\pi^-\pi^+$ amplitude in the present model with its dual term at a high energy.

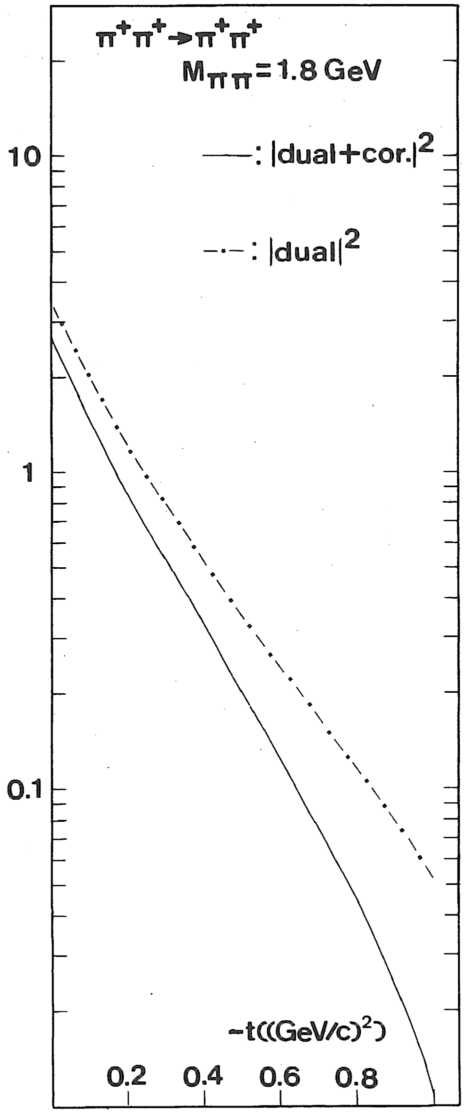

Fig. 4. Comparison of the $\pi^+\pi^+\to\pi^+\pi^+$ amplitude in the present model with its dual term at a high energy.

From Figs. 2, 3 and 4, we see the following:
(a) In the charge-exchange $\pi^-\pi^+\to\pi^0\pi^0$ process, the high-energy behaviour of the dual term is a little modified by the $\varepsilon(\sim 800)$ contribution. At higher energies, such a feature is retained, because the $\varepsilon(\sim 800)$ appears only in the s-channel. Therefore, it will be difficult to find effects of the $\varepsilon(\sim 800)$ on this process at high energies, although the diffraction scattering is absent from this process.
(b) In the $\pi^+\pi^+\to\pi^+\pi^+$ scattering, the high-energy behaviour of the dual term is considerably modified by the $\varepsilon(\sim 800)$ exchange. Such modification becomes larger at higher energies. However, the diffraction scattering seems to dominate at higher energies ($M_{\pi\pi}>2$ GeV), judging from experiments on $\pi^-\pi^+\to\pi^-\pi^+$ scattering at the dipion masses up to 3.5 GeV.[14)]

References

1) Particle Date Group Tables 1984, Rev. Mod. Phys. **56** (1984), S1.
2) N. M. Cason et al., Phys. Rev. **D28** (1983), 1586.
3) G. Mennessier, Z. Phys. **C16** (1983), 241.
4) A. D. Martin et al., Phys. Lett. **74B** (1978), 417.
5) A. Etkin et al., Phys. Rev. **D25** (1982), 2446.
6) R. L. Jaffe, Phys. Rev. **D15** (1977), 267.
7) L. Montanet, G. C. Rossi and G. Veneziano, Phys. Rep. **63C** (1980), 149.
8) T. Ino, to be published.
9) R. L. Jaffe, Phys. Rev. **D17** (1978), 1444.
10) T. Ino, Y. Munakata and J. Sakamoto, Prog. Theor, Phys. **73** (1985), 143.
11) T. Ino, Prog. Theor. Phys. **61** (1979), 1863; **62** (1979), 1177; **71** (1984), 864.
12) S. L. Adler, Phys. Rev. **137** (1965), B1022.
13) S. L. Adler, Phys. Rev. **140** (1965), B736.
W. I. Weisberger, Phys. Rev. **143** (1966), 1302.
14) C. Bromberg et al., Phys. Rev. D29 (1984), 588.