Janossy densities

for chiral random matrices

+ application to 2c QCD

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H. Fuji, I. Kanamori, SMN arXiv:1903.07176 (45 pp) = JHEP 09 (2019) ***

SMN arXiv:1606.00376 (LATTICE proceedings)

RMT in Sub-Atomic Physics and Beyond (in Honor of Jac Verbaarschot's 65th Birthday)

2019.8.5-9 ECT*, Trento, Italy

Janossy densities §2

for chiral random matrices

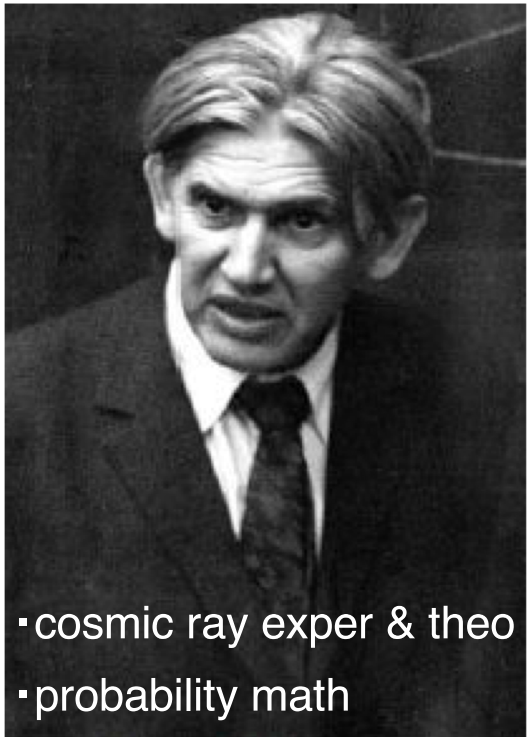
§4 Dirac spectrum of

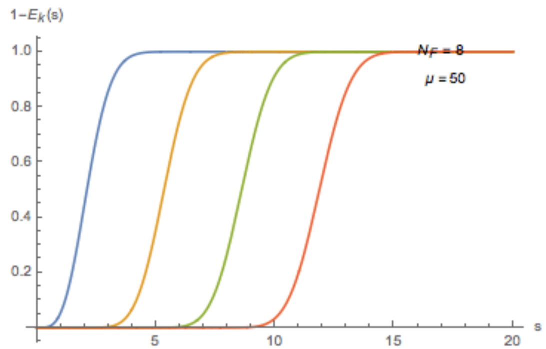
+ application to 2c QCD

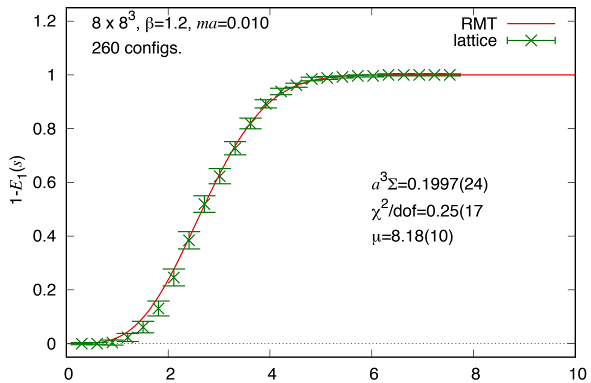
§1 Introduction

§3 Ordered EV statistics

§5 Summary







personal recollection on Jac :

invited me & my wife to Thanksgiving dinner

at his house, for the 1st time - Nov. 2000

S : why don't you eat it?

(Oh, he must have prepared it solely to treat us, despite he's vegan…)

he cut & served a big turkey for us,

but didn't start eating it himself

J : because I'm a little worried about Salmonella

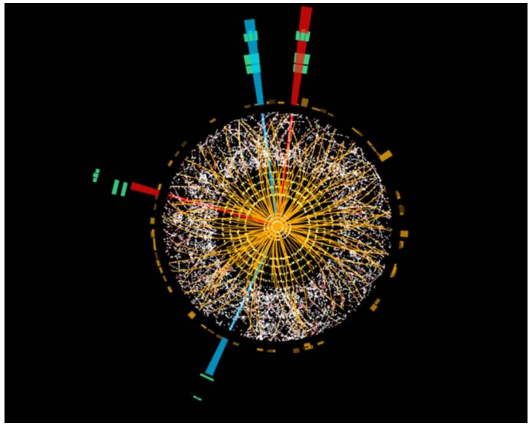
Happy Birthday, Jac

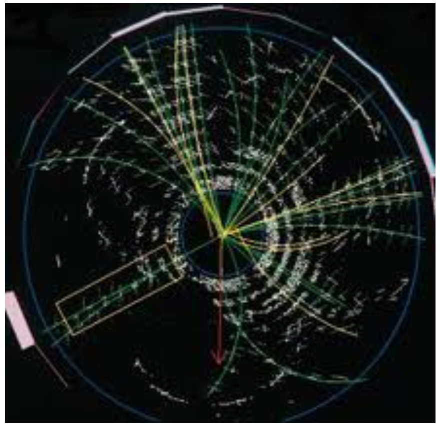
1. Introduction

Hierarchy problem of SM

Higgs boson M=125GeV ≈ Top quark m=173GeV

LHC@CERN 2012; 2013 Tevatron@FNAL 1995; 2008



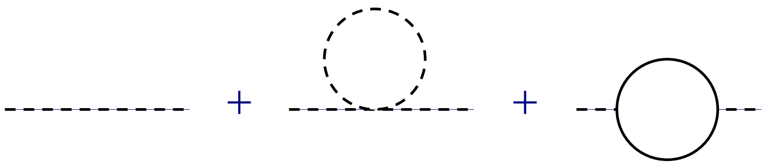


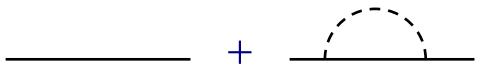
radiative correction of masses

$$M^2 = M_0^2 + \# \Lambda^2 + \# m_0^2 \log \frac{\Lambda}{m_0} + \cdots$$

$$m = m_0 + \# m_0 \log \frac{\Lambda}{m_0} + \cdots$$

Weisskopf '34



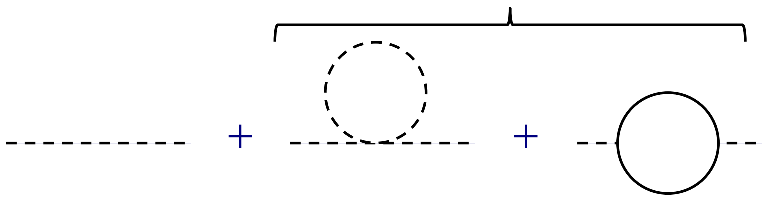


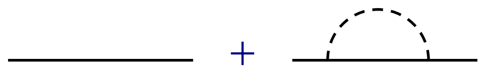
extremely-fine tuning needed to account for M_H=125GeV << Λ~10^{17}GeV

Hierarchy problem of SM

Solution 1: SUSY $(H\ ,\ \psi_H)$ degenerate m_0 ⇨ degenerate m

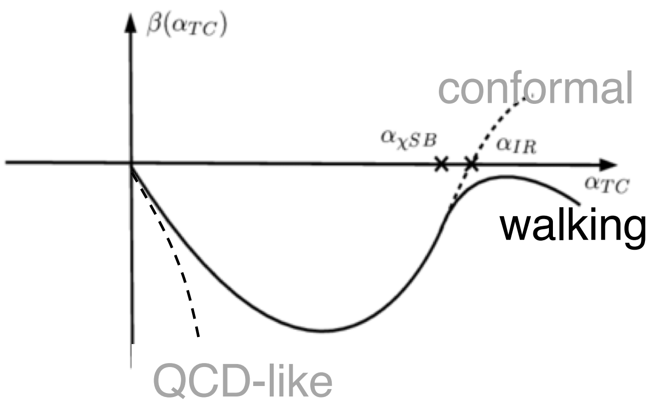
$$m^2 = m_0^2 + \underset{\substack{\uparrow \\ \text{O}(\Lambda^2)\text{ cancelled}}}{0} + \# m_0^2 \log\frac{\Lambda}{m_0} + \cdots \qquad\qquad m = m_0 + \# m_0 \log\frac{\Lambda}{m_0} + \cdots$$



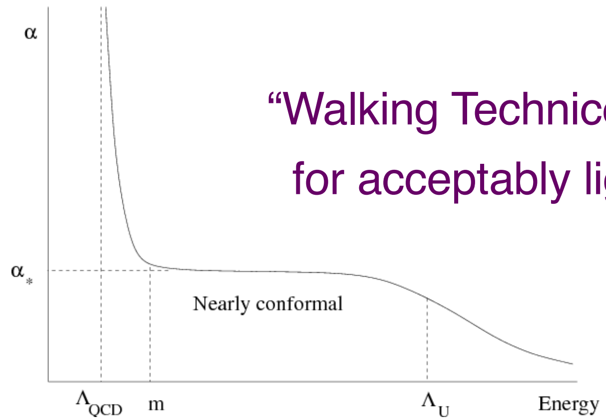


Solution 2: Higgs is ~~fundamental~~ ⇨ composite of 2 fermions in (new) gauge theory

$H \sim \bar{t}\, t$ (top condensation) or $\bar{Q}\, Q$ (technicolor)



⇨



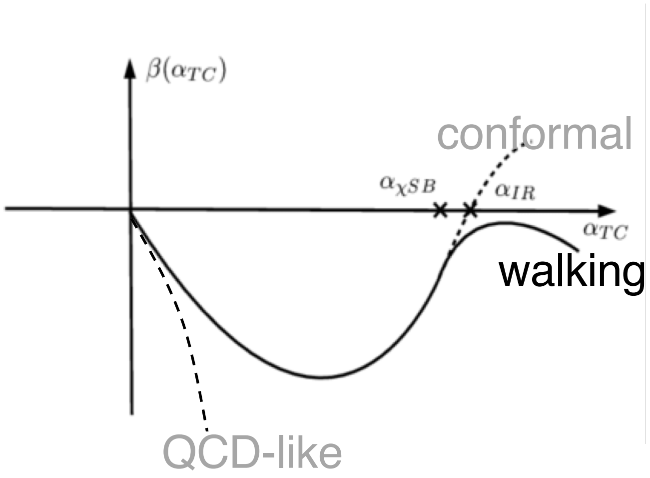
"Walking Technicolor"

for acceptably light m_H

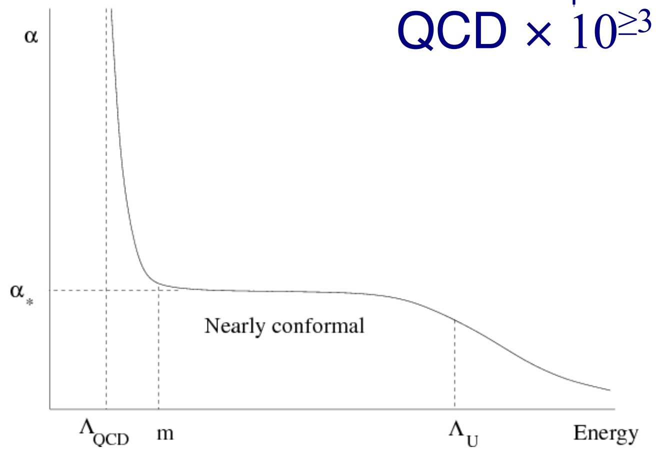
Technicolor proposal

Solution 2: Higgs is ~~fundamental~~ ⇨ composite of 2 fermions in new gauge theory

QCD × $10^{\geq 3}$ scale-up



⇨



$$\beta(\alpha) = -\underbrace{\left(\frac{11}{3}N - \frac{2}{3}n_F - \frac{4}{3}N\ n_{Ad}\right)}_{\text{small but }>0}\alpha^2 - \underbrace{\left(\frac{34}{3}N^2 - \left(\frac{13}{3}N - \frac{1}{N}\right)n_F - \frac{32}{3}N^2 n_{Ad}\right)}_{<0}\alpha^3 - \cdots$$

- $N = 3\ ,\ n_F = 12$ Nagoya KMI group; Kuti; Hasenfratz
- $N = 2\ ,\ n_{Ad} = 2$ "Minimal WTC" Catterall-Sannino
- $N = 2\ ,\ n_F = 8$ NCTU group, Helsinki group...

Chiral symm. broken like QCD, or
Conformal (unsuited for WTC) ?
…tested for $\mathrm{SU}(2)$ GTs on lattice

Chiral Random Matrices

[Shuryak-Verbaarschot '93]

$$\langle \cdots \rangle_{\mathrm{chRM}} = \frac{1}{Z(m)} \int dH \; e^{-\mathrm{tr}\, H^2} \prod_{f=1}^{n_f} \det\left(H + i m_f\right) \cdots \quad , \quad H = \begin{array}{c} \overbrace{}^{N+\nu} \\ \left.\left[\begin{array}{c|c} 0 & M \\ \hline M^+ & 0 \end{array}\right]\right\} N \end{array}$$

$$M_{ab} \in \begin{cases} \mathbf{R} : \beta = 1 \\ \mathbf{C} : \beta = 2 \\ \mathbf{H} : \beta = 4 \end{cases}$$

HS transf.

$N \to \infty \;,\; m_f \to 0$

$\mu_f = N m_f$ fixed

captures all
global & discrete symmetries
of a gauge theory

$$\int_{U(N_F)} dU \; \left(\det U\right)^{\nu} \exp\left\{\mathrm{Re}\,\mathrm{tr}\,\mathrm{diag}(\mu_f) U\right\} = \mathrm{cst.} \frac{\det\left[\mu_i^{j-1} I_{\nu+j-i}(\mu_i)\right]_{i,j=1}^{N_F}}{\Delta(\mu_f^2)}$$

$\updownarrow$

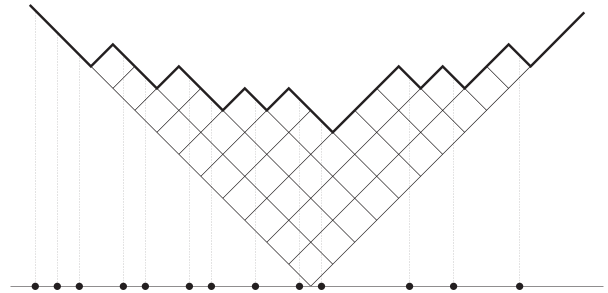
effective chL: $V\Sigma\, \mathrm{Re}\,\mathrm{tr}\,\mathrm{diag}(M_f)\, U$

if a gauge theory is in **chSB phase**,

- Dirac EVs $\{V\lambda_k\}$ measured at the same scaled quark masses $\{V M_f\}$ on different V collapse onto chRM results
- extrapolation to $M_f \to 0$ retains the best-fit $\Sigma \neq 0$

2. Janossy Density in DPP

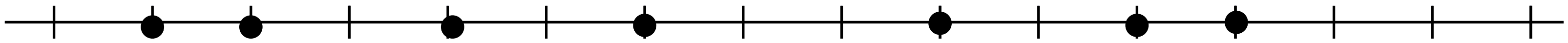
Determinantal point process



$$\mathrm{Prob}_N(n_1,\ldots,n_N) = \frac{1}{N!}\det\left[K(n_i,n_j)\right]_{i,j=1}^{N} \qquad n_i \in \mathbf{Z}$$

$$\mathbf{K} = \left[K(n,m)\right]_{n,m\in\mathbf{Z}} \ : \text{projective } \mathbf{K}\cdot\mathbf{K} = \mathbf{K}\ ,\ \mathrm{tr}\,\mathbf{K} = N,\ \mathbf{K}^{\dagger}=\mathbf{K}$$

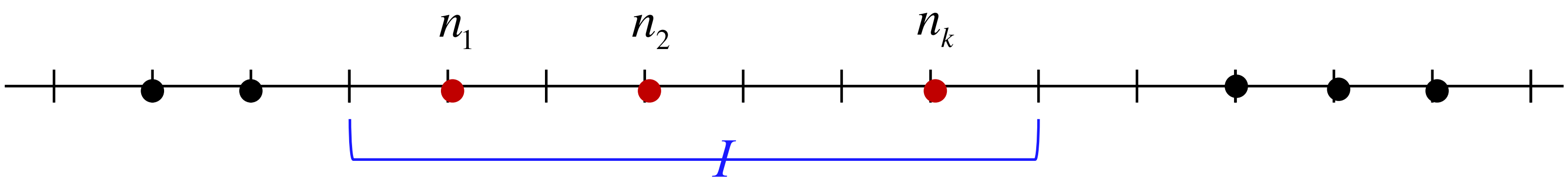
- Plancherel measure {YT}
- directed percolation
- continuous ⇨ β=2 RM



Janossy density

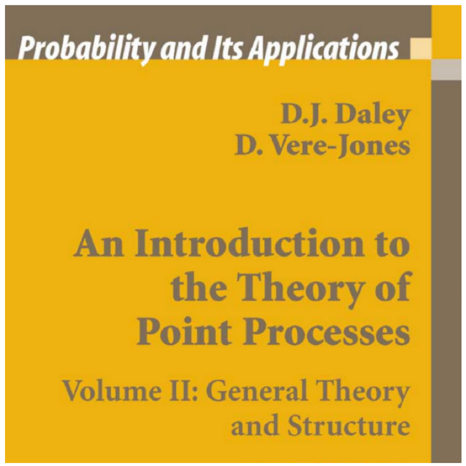
for Det point process $R_k(n_1,\ldots,n_k) = \det\left[K(n_i,n_j)\right]_{i,j=1}^{k}$

$J_{k,I}(n_1,\ldots,n_k) = \text{Prob of}$



no pts in I except for k designated pts

[Daley-Vere-Jones '88, p.140]

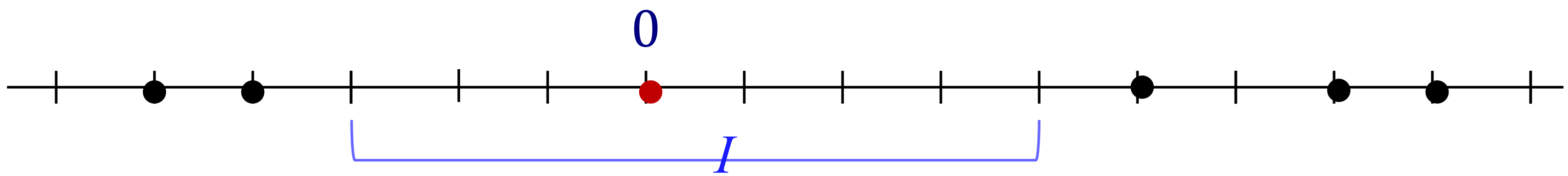


$$\mathbf{K}_I := \left[K(n,m)\right]_{n,m\in I}$$

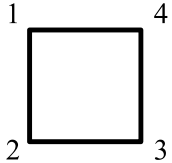
$$J_{k,I}(n_1,\ldots,n_k) = \det\underbrace{\left(1-\mathbf{K}_I\right)}_{\text{all pts in } I}\cdot\det\underbrace{\left[\left\langle n_i\middle|\mathbf{K}_I\left(1-\mathbf{K}_I\right)^{-1}\middle|n_j\right\rangle\right]_{i,j=1}^{k}}_{\text{designated pts}}$$

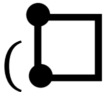
2 Janossy density in DPP

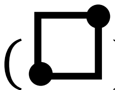
Sketch of Proof



$$
\begin{aligned}
& J_{1,I}(0) \\
= \quad & R_1(0) - \sum_{n \in I} R_2(0,n) + \frac{1}{2!} \sum_{n,n' \in I} R_3(0,n,n') - \cdots \\
= \quad & K(0,0) - \sum_{n \in I} \begin{vmatrix} K(0,0) & K(0,n) \\ K(n,0) & K(n,n) \end{vmatrix} + \frac{1}{2!} \sum_{n,n' \in I} \begin{vmatrix} K(0,0) & K(0,n) & K(0,n') \\ K(n,0) & K(n,n) & K(n,n') \\ K(n',0) & K(n',n) & K(n',n') \end{vmatrix} \cdots \\
= \quad & \langle 0 | \mathbf{K}_I | 0 \rangle - \left\{ \langle 0 | \mathbf{K}_I | 0 \rangle \ \mathrm{tr} \mathbf{K}_I - \left\langle 0 | \mathbf{K}_I^2 | 0 \right\rangle \right\} \\
& + \frac{1}{2!} \left\{ \langle 0 | \mathbf{K}_I | 0 \rangle \left(\mathrm{tr} \mathbf{K}_I \right)^2 - \langle 0 | \mathbf{K}_I | 0 \rangle \ \mathrm{tr} \mathbf{K}_I^2 - 2 \left\langle 0 | \mathbf{K}_I^2 | 0 \right\rangle \ \mathrm{tr} \mathbf{K}_I + 2 \left\langle 0 | \mathbf{K}_I^3 | 0 \right\rangle \right\} - \cdots \\
= \quad & \langle 0 | \mathbf{K}_I | 0 \rangle \left\{ 1 - \mathrm{tr} \mathbf{K}_I + \frac{1}{2!} \left(\mathrm{tr} \mathbf{K}_I \right)^2 - \cdots \right\} \left\{ 1 - \frac{1}{2} \mathrm{tr} \mathbf{K}_I^2 + \cdots \right\} \cdots \\
+ \quad & \left\langle 0 | \mathbf{K}_I^2 | 0 \right\rangle \left\{ 1 - \mathrm{tr} \mathbf{K}_I + \cdots \right\} \cdots \\
+ \quad & \left\langle 0 | \mathbf{K}_I^3 | 0 \right\rangle \left\{ 1 - \cdots \right\} \cdots + \cdots \\
= \quad & \left\langle 0 \left| \mathbf{K}_I + \mathbf{K}_I^2 + \mathbf{K}_I^3 + \cdots \right| 0 \right\rangle \det(\mathbf{1} - \mathbf{K}_I) = \left\langle 0 \left| \mathbf{K}_I (\mathbf{1} - \mathbf{K}_I)^{-1} \right| 0 \right\rangle \det(\mathbf{1} - \mathbf{K}_I)
\end{aligned}
$$

Example: 2 Fermions on 

Prob() = 1/8

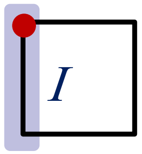
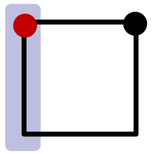
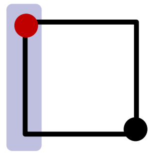
Prob() = 1/4

$\Leftrightarrow$

$$\mathrm{Prob}(n,m) = \frac{1}{2!}\begin{vmatrix} K(n,n) & K(n,m) \\ K(m,n) & K(m,m) \end{vmatrix}$$

$$\mathbf{K} = \frac{1}{4}\begin{pmatrix} 2 & 1-i & 0 & 1+i \\ 1+i & 2 & 1-i & 0 \\ 0 & 1+i & 2 & 1-i \\ 1-i & 0 & 1+i & 2 \end{pmatrix} = \mathbf{K}\cdot\mathbf{K}$$

tr $\mathbf{K} = 2$

$J_{1,I}(1) = \mathrm{Prob}($  $)$ =  +  $= \frac{1}{8} + \frac{1}{4}$

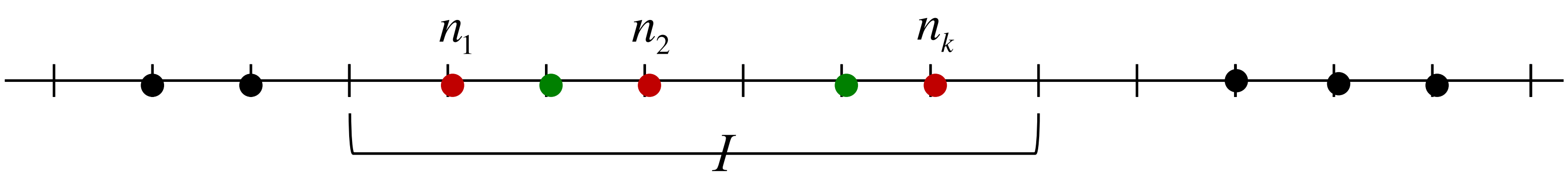
$$= \det(\mathbf{1} - \mathbf{K}_I)\cdot\langle 1|\mathbf{K}_I(\mathbf{1}-\mathbf{K}_I)^{-1}|1\rangle$$

$$= \left|\mathbf{1} - \frac{1}{4}\begin{pmatrix} 2 & 1-i \\ 1+i & 2 \end{pmatrix}\right| \cdot \begin{pmatrix} 3 & 2-2i \\ 2+2i & 3 \end{pmatrix}_{1,1} = \frac{1}{8}\cdot 3$$

Janossy density

Det. point process: $R_k(n_1,\ldots,n_k) = \det\left[K(n_i,n_j)\right]_{i,j=1}^{k}$

$J_{p,k,I}(n_1,\ldots,n_k)$ = Prob of



exactly p pts ● in I except for k designated pts ●

$$\mathbf{K}_I := \left[K(n,m)\right]_{n,m\in I}$$

$$J_{p,k,I}(n_1,\ldots,n_k) = \frac{1}{p!}\left(-\partial_\xi\right)^p \det\left(1-\xi\mathbf{K}_I\right)\cdot \det\left[\left\langle n_i \middle| \mathbf{K}_I\left(1-\xi\mathbf{K}_I\right)^{-1} \middle| n_j\right\rangle\right]_{i,j=1}^{k}\Bigg|_{\xi=1}$$

- Alternative expression by $\begin{vmatrix} A & B \\ C & D \end{vmatrix} = |D|\,|A - CD^{-1}B|$

$$J_{k,I}(n_1,\ldots,n_k) = \det(\mathbf{1}-\mathbf{K}_I)\cdot\det\left[\langle n_i|\mathbf{K}_I(\mathbf{1}-\mathbf{K}_I)^{-1}|n_j\rangle\right]_{i,j=1}^{k}$$

$$= (-)^k \det\begin{vmatrix} \overbrace{-\left[\langle n_i|\mathbf{K}_I|n_j\rangle\right]_{i,j=1}^{k}}^{\text{designated pts }\bullet} & \overbrace{-\left[\langle m|\mathbf{K}_I|n_j\rangle\right]_{j=1,\ldots,k}^{m\in I}}^{\text{all pts in } I} \\ -\left[\langle n_i|\mathbf{K}_I|m\rangle\right]_{m\in I}^{j=1,\ldots,k} & \mathbf{1}-\mathbf{K}_I \end{vmatrix}\begin{matrix} \}\ \bullet \\ \}\ I \end{matrix}$$

unifies 2 well-known formulas in DPP / RMT

$$R_k(n_1,\ldots,n_k) = \det\left[K(n_i,n_j)\right]_{i,j=1}^{k}$$: k-point Corr. Function

$$E_{0,I} = \det(\mathbf{1}-\mathbf{K}_I)$$: Gap Probability

- Generalizable to qdet : Pfaffian PP = RMT at $\beta = 1, 4$

- Alternative expression by $\begin{vmatrix} A & B \\ C & D \end{vmatrix} = |D|\,|A - CD^{-1}B|$

$$J_{k,I}(n_1,\dots,n_k) = \det(\mathbf{1}-\mathbf{K}_I)\cdot\det\left[\langle n_i|\mathbf{K}_I(\mathbf{1}-\mathbf{K}_I)^{-1}|n_j\rangle\right]_{i,j=1}^{k}$$

$$= (-)^k \det\begin{vmatrix} \overbrace{-\left[\langle n_i|\mathbf{K}_I|n_j\rangle\right]_{i,j=1}^{k}}^{\text{designated pts }\bullet} & \overbrace{-\left[\langle m|\mathbf{K}_I|n_j\rangle\right]_{j=1,\dots,k}^{m\in I}}^{\text{all pts in } I} \\ -\left[\langle n_i|\mathbf{K}_I|m\rangle\right]_{m\in I}^{j=1,\dots,k} & \mathbf{1}-\mathbf{K}_I \end{vmatrix}\begin{matrix}\}\ \bullet \\ \}\ I\end{matrix}$$

- Continuous distribution : Fredholm Det, approx. by $I \sim \{y_1,\dots,y_m\}$

$$K(m,m') \to \sqrt{\Delta y_a}\,K(y_a,y_b)\sqrt{\Delta y_b}$$

$$J_{k,I}(x_1,\dots,x_k) = \lim_{\Delta y_a\to 0}$$

$$(-)^k \det\begin{vmatrix} -\left[K(x_i,x_j)\right]_{i,j=1}^{k} & -\left[\sqrt{\Delta y_a}\,K(y_a,x_i)\right]_{y_a\in I}^{i=1,\dots,k} \\ -\left[K(x_i,y_b)\sqrt{\Delta y_b}\right]_{y_b\in I}^{j=1,\dots,k} & \mathbf{1}-\left[\sqrt{\Delta y_a}\,K(y_a,y_b)\sqrt{\Delta y_b}\right]_{y_a,y_b\in I} \end{vmatrix}$$

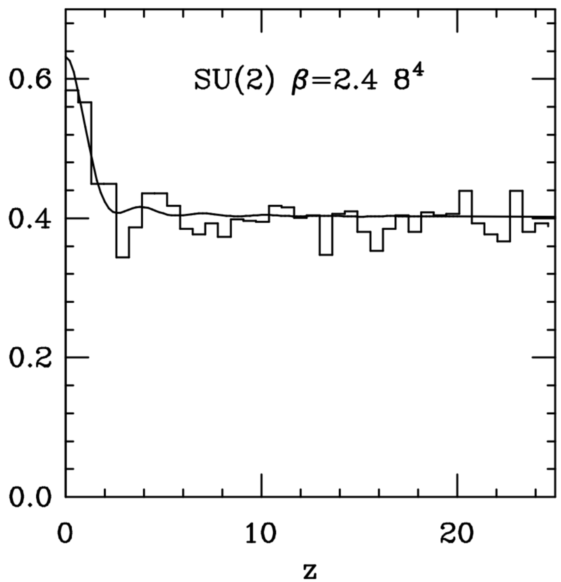
not explicit in [Borodin-Soshnikov '03][Forrester-Witte '07][Forrester-Witte-Bornemann '12]

3. Ordered EV Statistics

Individual EV distributions

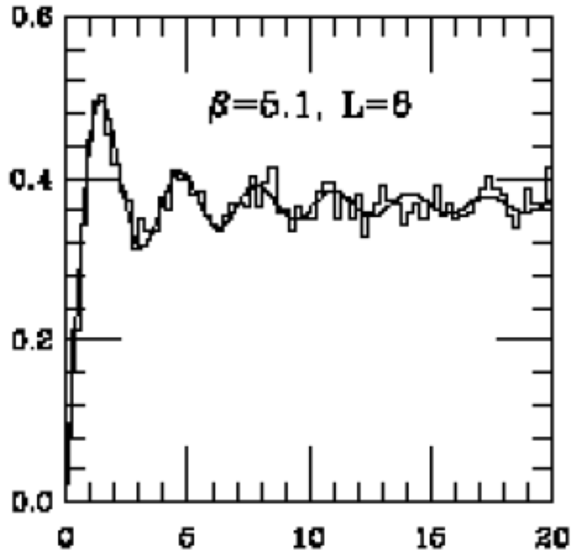
quotes from U. Heller's presentation: quenched chRMT vs LGT

chGOE



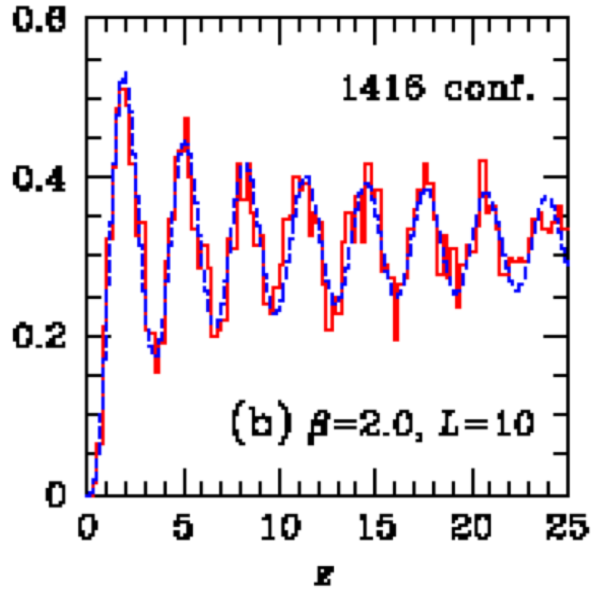
[Edwards-Heller-Narayanan 99]

chGUE



[Damgaard-Heller-Krasnitz 99]

chGSE



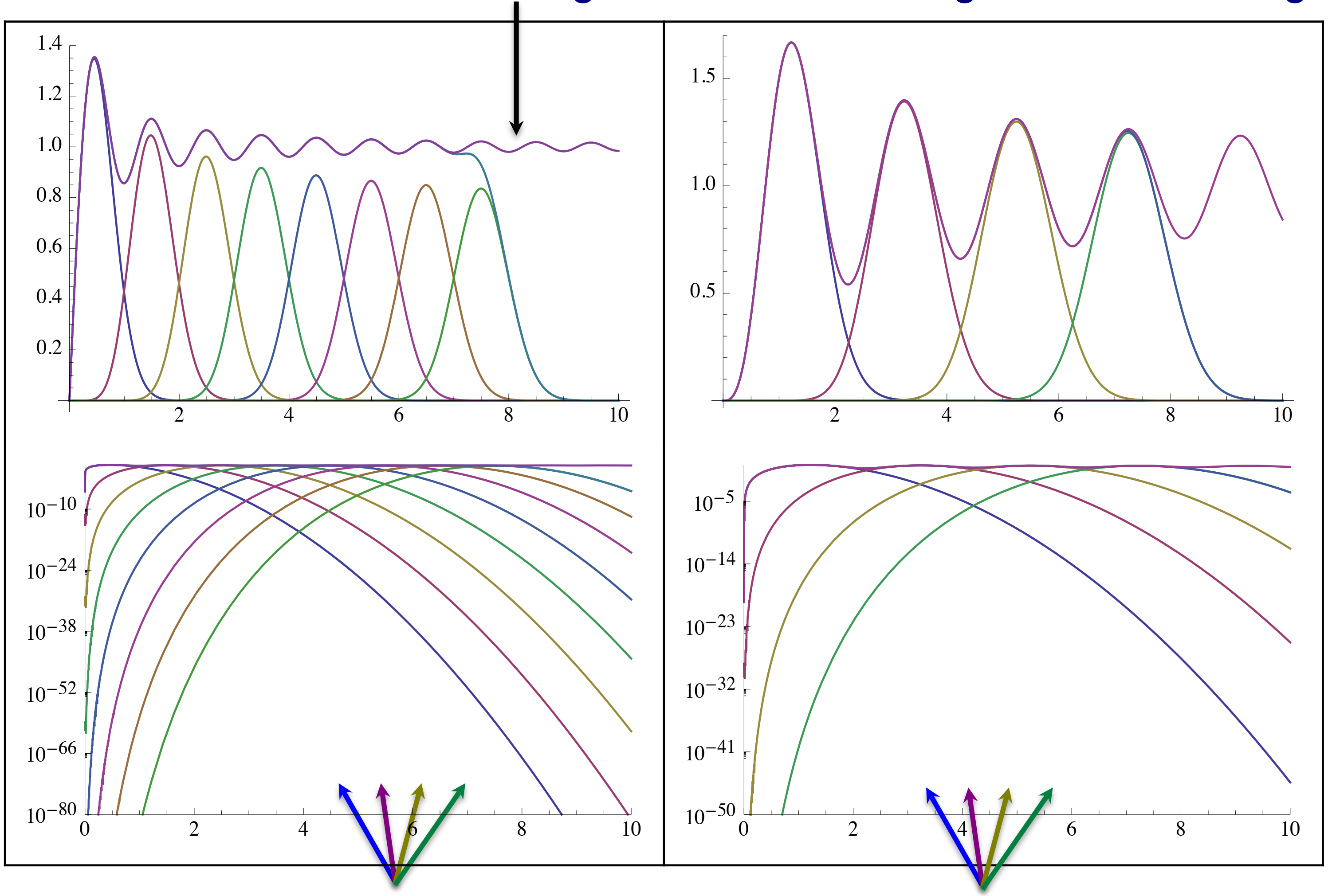
[Berbenni-Bitsch-Meyer-Schaefer-Verbaarschot-Wettig 98]

separation of peaks in $\mathrm{Spec}(\not{D})$ is highly desirable for precise fitting

Individual EV distributions

1st ~8th EV distributions of chGUE , chGSE

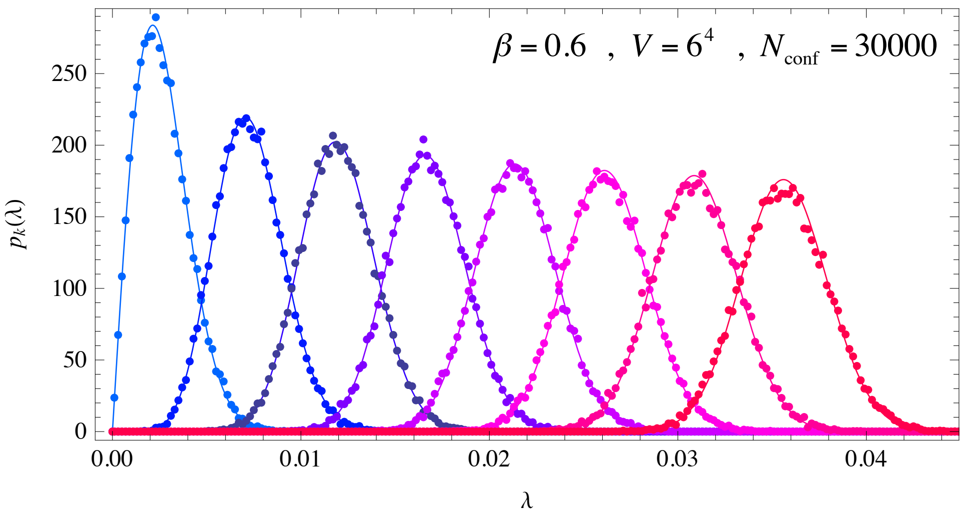
flattening oscillation ⇒ larger error for fitting

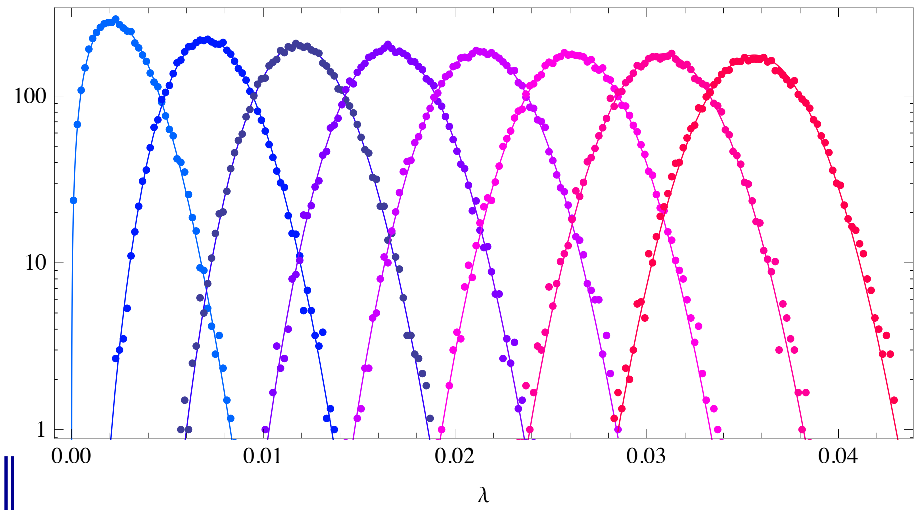


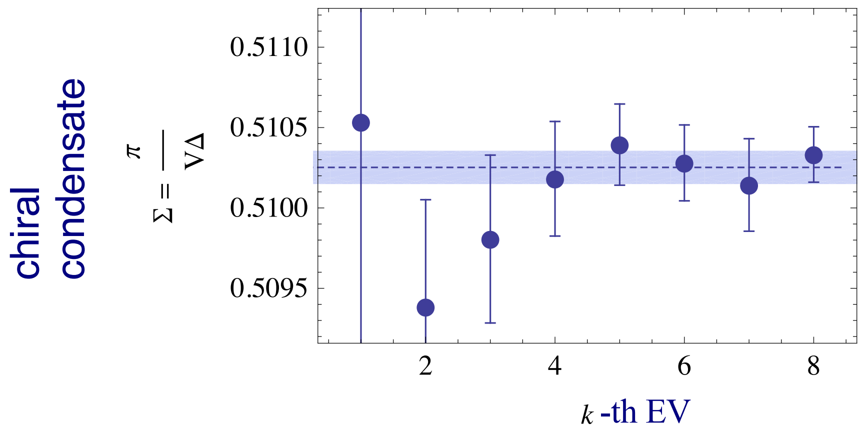
quasi-Gaussian ⇒ precise fitting possible

Chiral condensate from Individual EV distributions

exercise 1 : quenched U(1) Dirac spectrum vs chGUE





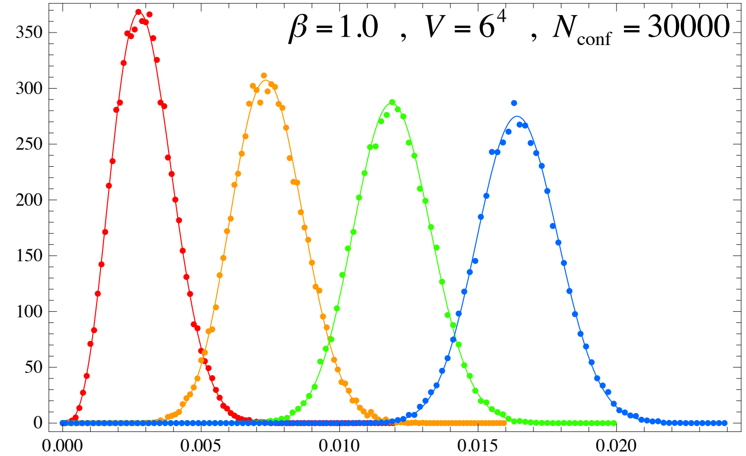


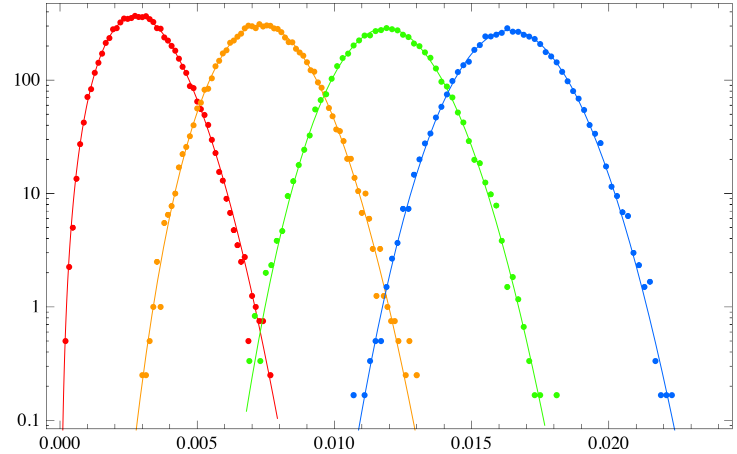
[SMN '16]

$$\Sigma a^3 = 0.51025(10)$$

Chiral condensate from Individual EV distributions

exercise 2 : quenched SU(2) Dirac spectrum vs chGSE

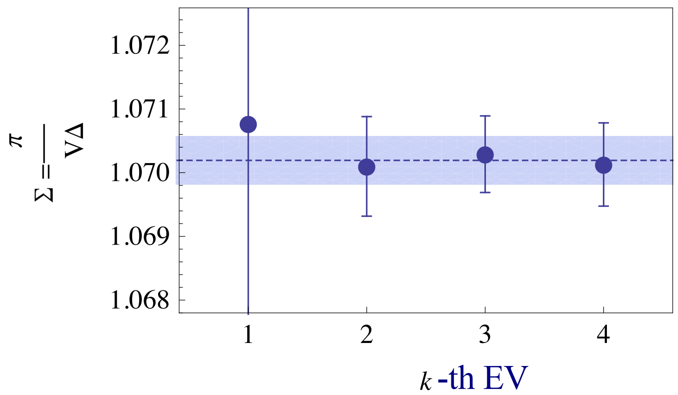




$\Downarrow$

[SMN '16]

chiral condensate



$$\Sigma a^3 = 1.07019(38)$$

Σ within O(10^{-4}) error!

Shifting method

[Damgaard-SMN '01]

$$\lambda_i \geq 0 \quad \in \ \mathrm{Spec}(H^2)$$

$$dH\ \mathrm{e}^{-\mathrm{tr}\,H^2} \prod_f \det\left(H + i m_f\right) \ \propto \ \prod_{i=1}^{N} \left(d\lambda_i\ \lambda_i^{\beta(\nu+1)/2-1}\ \mathrm{e}^{-\lambda_i} \prod_f \left(\lambda_i + m_f^2\right)\right) \prod_{i>j}^{N} \left|\lambda_i - \lambda_j\right|^{\beta}$$ … JPD of EVs

$$P_k(\lambda_1,\ldots,\lambda_k) = \boxed{\int_{\lambda_k}^{\infty}\cdots\int_{\lambda_k}^{\infty} d\lambda_{k+1}\cdots d\lambda_N \left(JPD_{N_F}(\lambda_1,\ldots,\lambda_N;\{m\};\nu)\right)}$$ … JPD of first k EVs

$$\tilde{\lambda}_i \equiv \lambda_i - \lambda_k \quad \Downarrow$$

$$= C(\{m\}) \int_0^{\infty}\cdots\int_0^{\infty} d\tilde{\lambda}_{k+1}\cdots d\tilde{\lambda}_N \left(JPD_{\tilde{N}_F}(\tilde{\lambda}_1,\ldots,\tilde{\lambda}_N;\{\tilde{m}\};\tilde{\nu})\right) \xrightarrow{N\to\infty} \text{ratio of Bessel det's}$$

$$\Downarrow$$

$$p_k(\lambda_k) = \int_0^{\lambda_k}\cdots\int_0^{\lambda_k} d\lambda_1\cdots d\lambda_{k-1} P_k(\lambda_1,\ldots,\lambda_k) \xrightarrow{N\to\infty} \boxed{(k\text{-}1)\text{-fold integral of ratio of Bessel det's}}$$

for this trick to work, the exponent

$$\beta\frac{\nu+1}{2}-1 \in \begin{cases} \mathbf{N} & (\beta=1,2) \\ 2\mathbf{N} & (\beta=4) \end{cases} \Rightarrow \begin{array}{l} \times\ \text{chGOE},\ \nu = 0,2,4\ldots \\ \times\ \text{chGSE},\ N_F = 0,2,4\ldots \end{array}$$

practically unfeasible for $k \geq 5$

authors	year	β	k	ν	dynamical n_F	cons
Edelman	88, 91	1	1	0~3	0	finite N
Forrester	93	2	1	∀	0	Fredholm det
		4	1	∀	0	Jack poly.
Tracy-Widom	94	2	∀	∀	0	Painleve
Forrester-Hughes	94	2	1+2	∀	0	shifting
Nagao-Forrester	98	4	1	∀	0+odd massless	skew OP
Wilke-Guhr-Wettig	98	2	1	∀	∀	shifting
Berbenni-Bitsch-Meyer-Wettig	98	4	1	0	0 +evn massless	finite N
		4	1	0	1	finite N
Nagao-SMN, Damgaard-SMN	00, 01	2	∀	∀	∀	shifting
		4	∀	∀	∀pairs +odd	massless
		1	∀	odd	∀	
Forrester	00, 06	4, 1	∀	∀	0	Painleve
Wirtz-Akemann-Guhr-Kieburg-Wegner	14	1	1	evn	0	skew OP
SMN	16	2	∀	∀	∀	Nystrom
Fuji-Kanamori-SMN	19	4	∀	∀	∀pairs	Nystrom

Nystrom-type approx to Fredholm Det

Gauss-Legendre Quadrature : $\{x_1,\ldots,x_M\}\in I,\ \ \{\Delta x_1,\ldots,\Delta x_M\}>0$

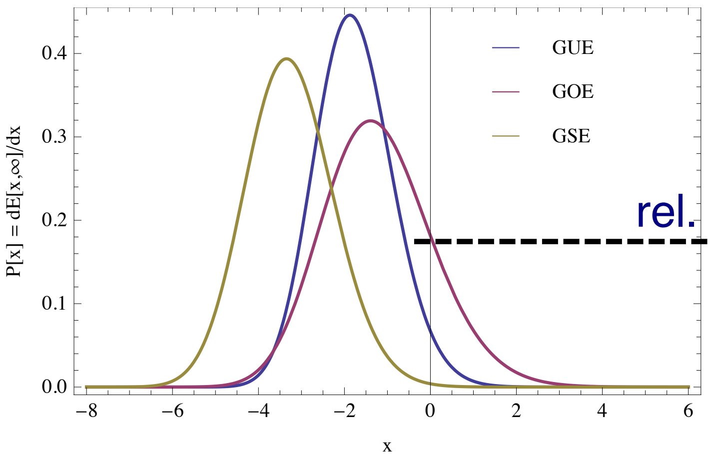
$$\int_I f(x)\,dx \ \cong\ \sum_{i=1}^{M} f(x_i)\Delta x_i \ \text{, exact for } f(x)=x^M+\text{lower}$$

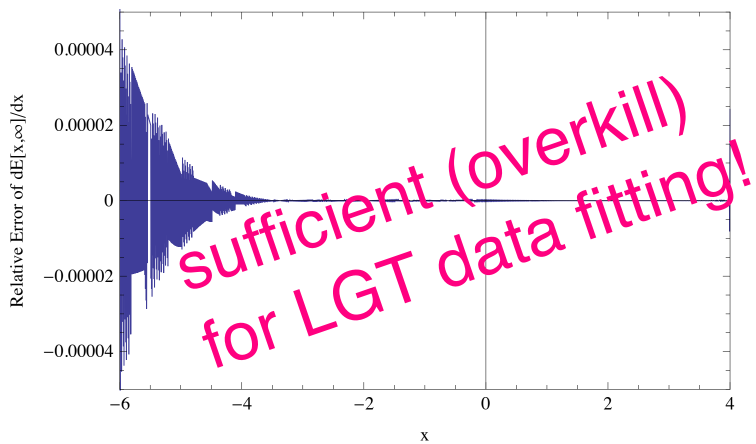
$$\boxed{\mathrm{Det}(1-K_I)\ \cong\ \det\left[\delta_{ij}-K(x_i,x_j)\sqrt{\Delta x_i\Delta x_j}\right]_{i,j=1}^{M}\ +\text{relative error } O(\mathrm{e}^{-\mathrm{const}.M})}$$

[Bornemann '10]

ex. Largest EV distribution

Nystrom approx (M=30) for K_{Airy} vs Tracy-Widom's analytic formula





4. Dirac spectra of 2-color QCD

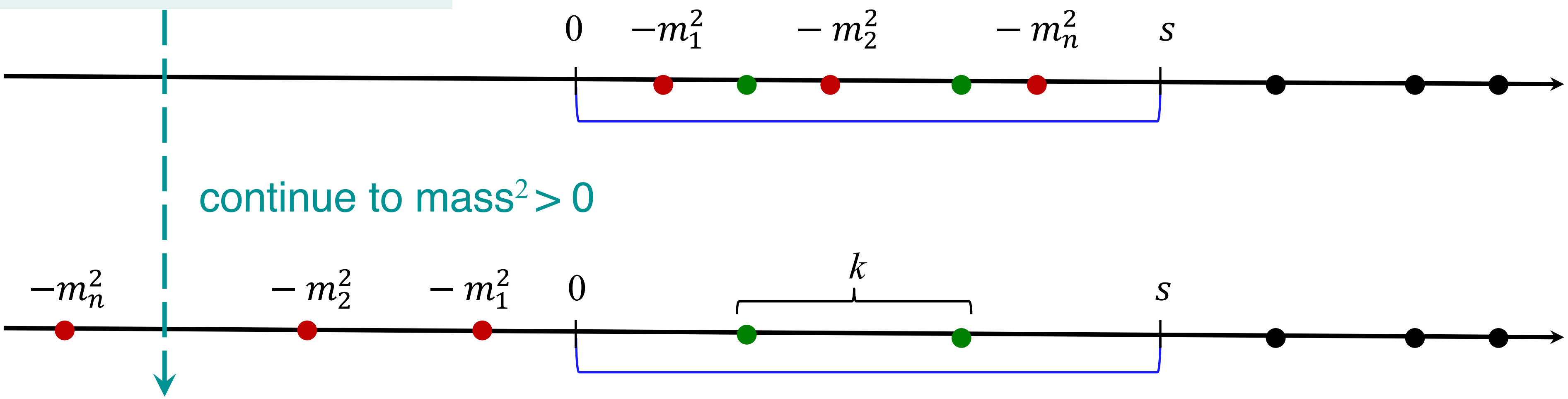
$n_F = \beta\, n$ fermions as Janossy density

$$\int dH\, \mathrm{e}^{-\mathrm{tr}\, H^2} \prod_{f=1}^{n} (H + i m_f)^{\beta} = \int_0^{\infty} \prod_{i=1}^{N} \left(d\lambda_i\, \lambda_i^{\frac{\beta(\nu+1)}{2} - 1} \mathrm{e}^{-\lambda_i} \prod_{f=1}^{n} \left| \lambda_i + m_f^2 \right|^{\beta} \right) \prod_{i>j}^{N} |\lambda_i - \lambda_j|^{\beta}$$

$$\propto \int_0^{\infty} \prod_{i=1}^{N+n} \left(d\lambda_i\, \lambda_i^{\frac{\beta(\nu+1)}{2} - 1} \mathrm{e}^{-\lambda_i} \right) \prod_{i>j}^{N+n} |\lambda_i - \lambda_j|^{\beta} \cdot \prod_{k=N+1}^{N+n} \delta(\lambda_k - (-m_f^2))$$

designated pts ● = −mass^2 > 0

$J_{k,n,[0,s]}(-m_1^2, \dots, -m_n^2)$



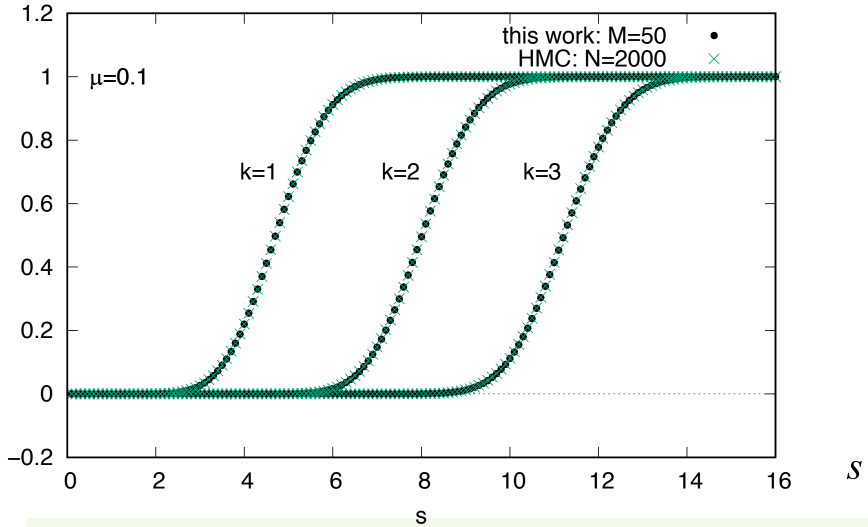
continue to mass^2 > 0

$E_k(s; m_1^2, \dots, m_n^2)$

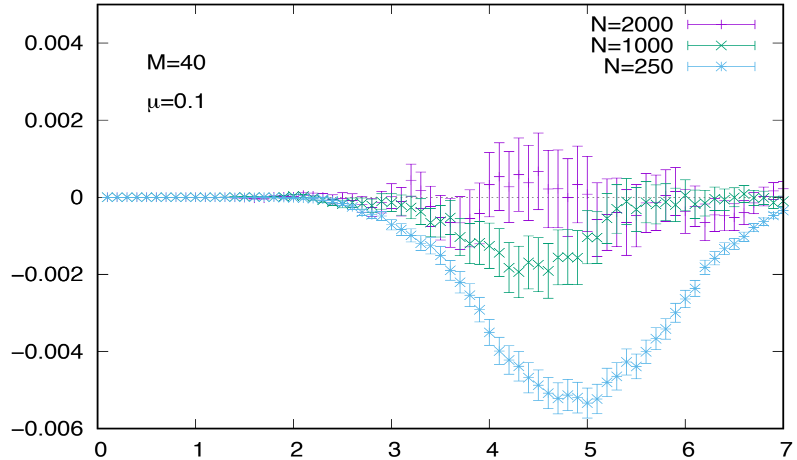
$\Longrightarrow$ evaluate Fredholm Det by quadrature approx.

$\beta = 4$ (chGSE), $n_F = 4$, $k = 0, 1, 2$, $\mu = 0.1$ vs chGSE (N= 250~2000) by HMC

1-$E_k(s\,;0.1)$



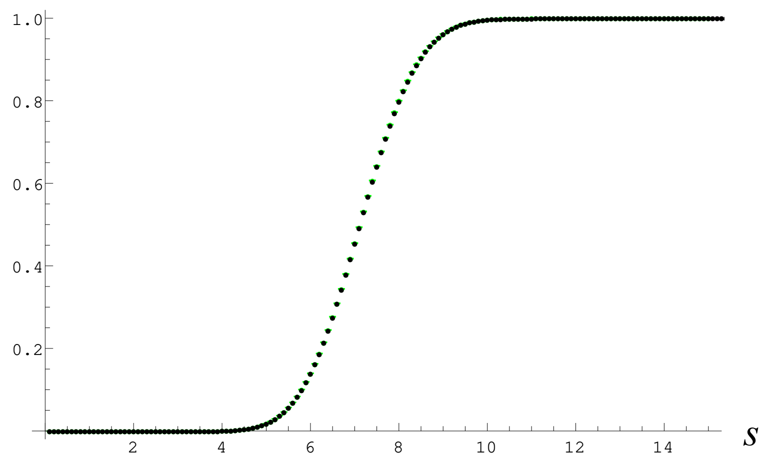
deviation



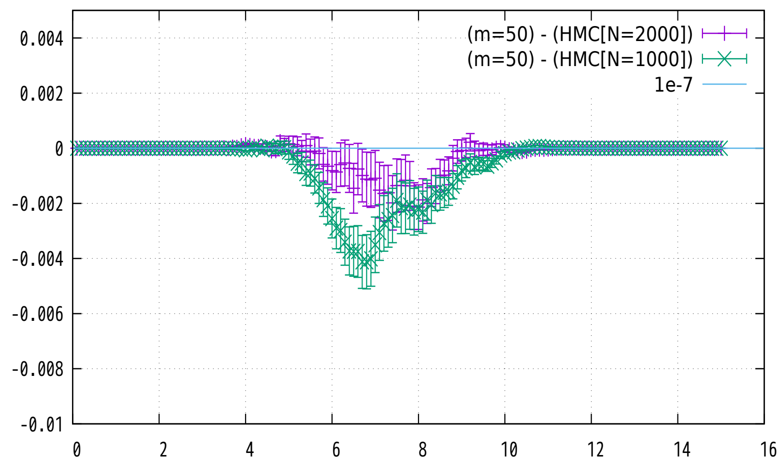
Quadrature Approx of Det : systematic error too small for bare eyes!

$\beta = 4$ (chGSE), $n_F = 8$, $k = 0$, $\mu = 0.1$ vs chGSE (N= 1000~2000) by HMC

1-$E_0(s\,;0.1)$



deviation

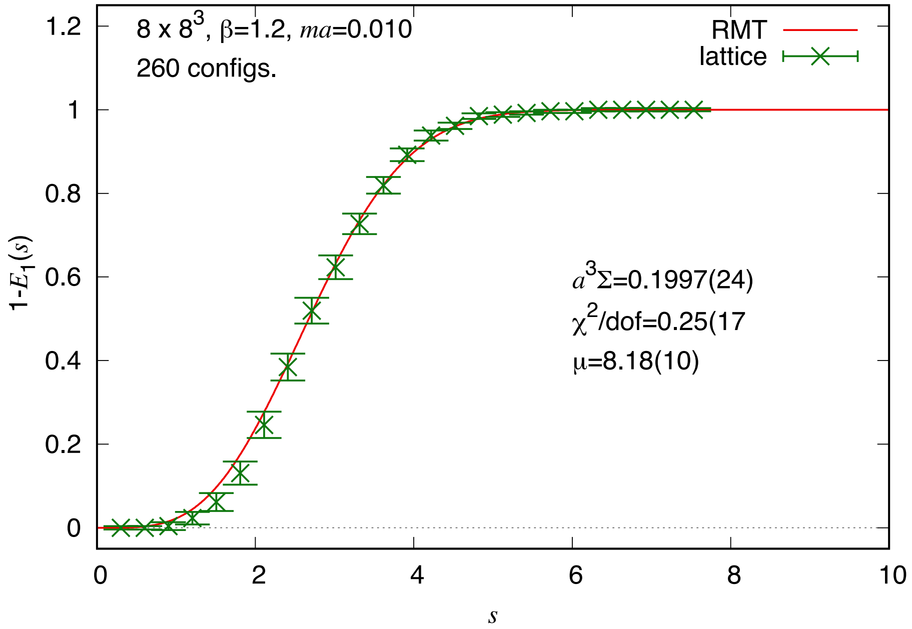


chSB check for WTC candidate

flavor taste

$\beta = 4$ (chGSE), $n_F = 4$, $k = 0$ vs 2c QCD, $n_F = 2 \times 4 = 2+(2+2+2)$

$\mu = 8.18$ $ma = 0.010$



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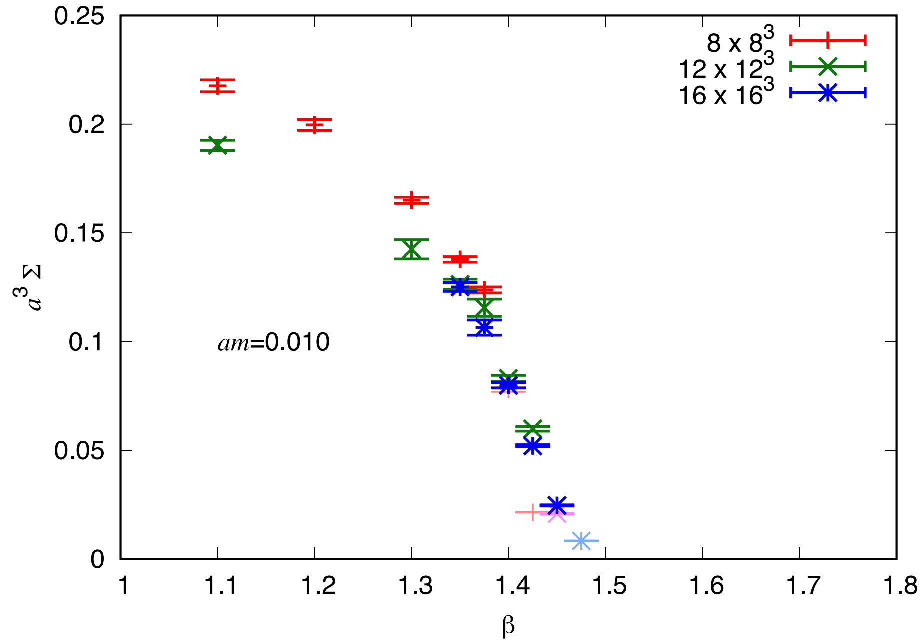
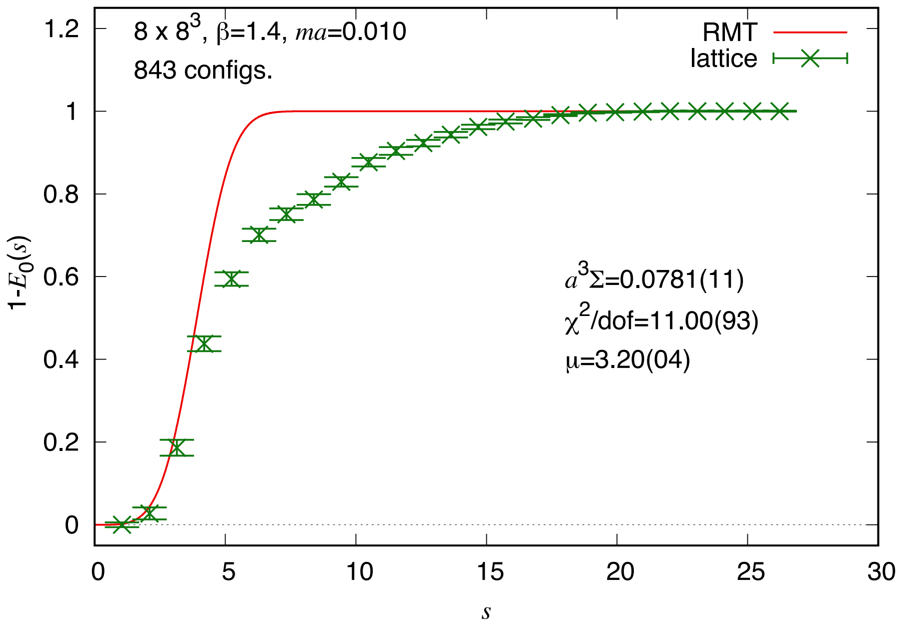


N=2, $n_F = 2+(2+2+2)$, $\beta \leq 1.4\ldots$: Chiral Symm Broken

chSB check for WTC candidate

$\beta = 4$ (chGSE), $n_F = 4$, $k = 0$

$\mu = 3.20$

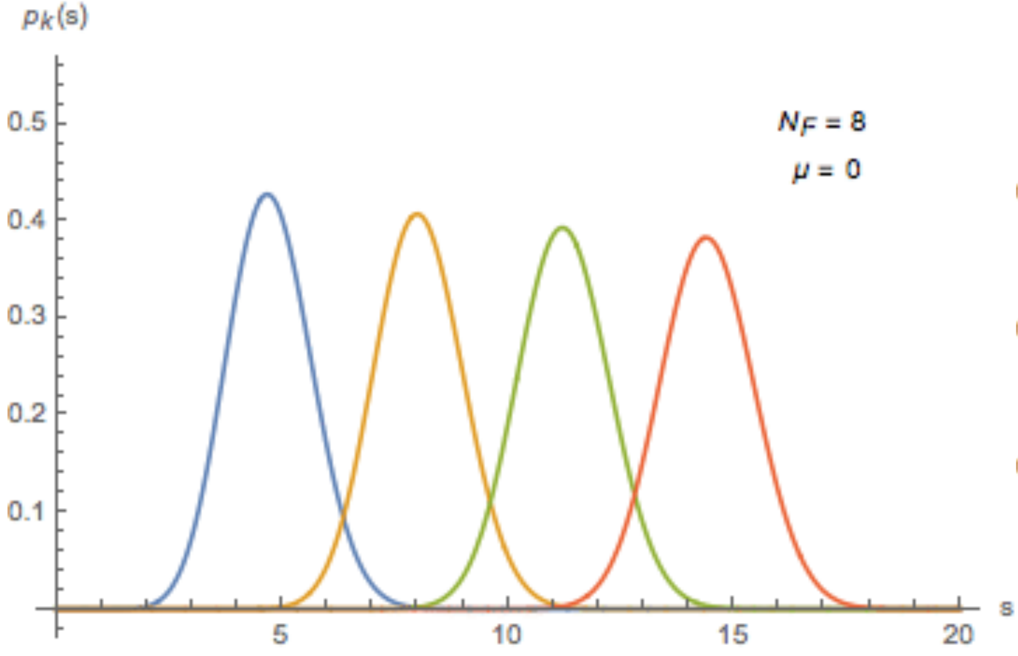


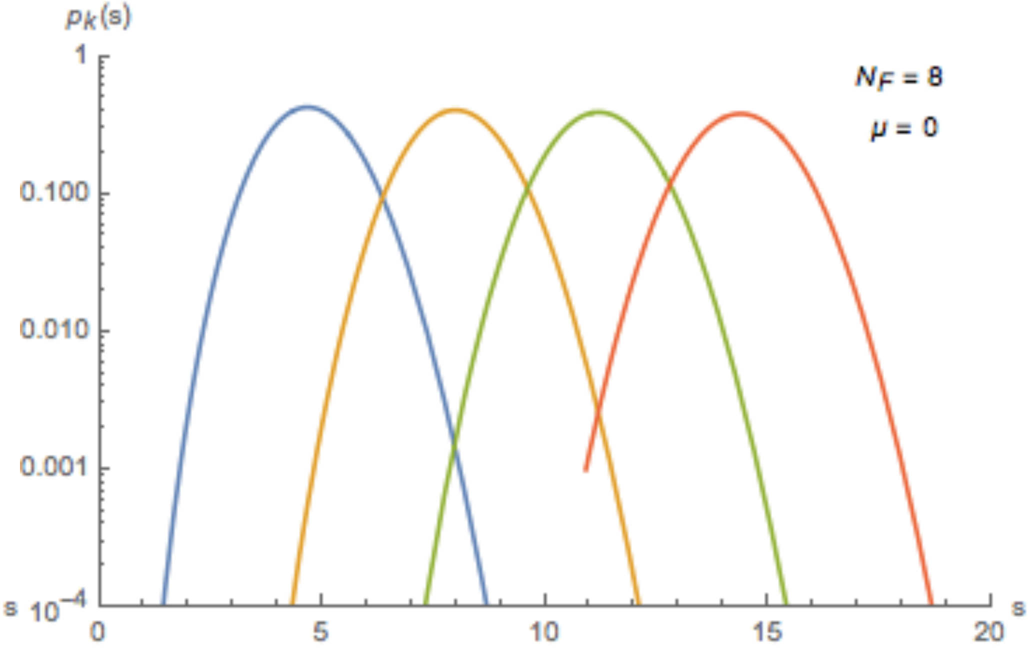
… bad χ^2 for chSym phase

N=2, n_F= 2+(2+2+2), $\beta \leq 1.4$… : Chiral Symm Broken

chSB check for WTC candidate

$\beta = 4$ (chGSE), $n_F = 8$, $k = 0,1,2,3$, $\mu = 0,1,2,\ldots,\infty$





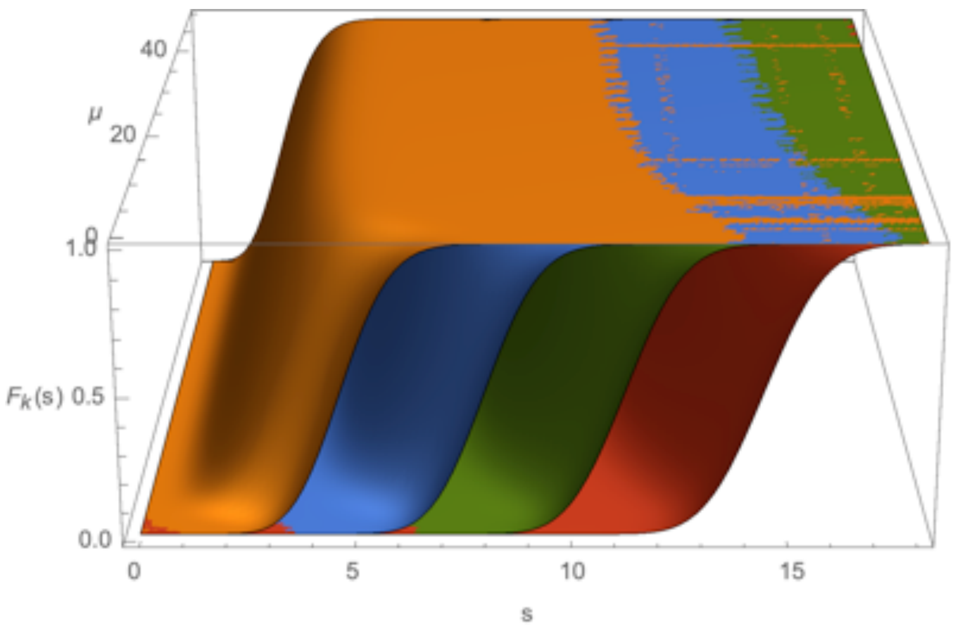
to be fitted with

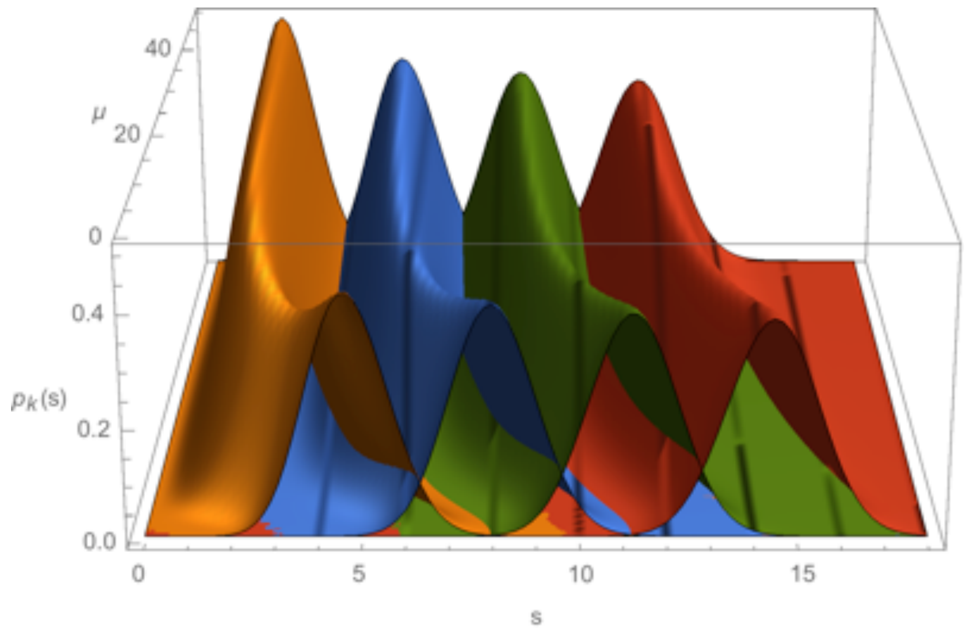
flavor taste

$N = 2$, $n_F = 8 \times 1$ lattice QCD to judge whether chSB or conformal

chSB check for WTC candidate

$\beta = 4$ (chGSE), $n_F = 8$, $k = 0,1,2,3$, $\mu = 0,1,2,\ldots,\infty$





to be fitted with

flavor taste

$N = 2$, $n_F = 8 \times 1$ lattice QCD to judge whether chSB or conformal

§5 Summary

- 2 technical difficulties in evaluating Individual EVDs of massive chRMMs are overcome by Janossy Density formula + Quadrature method

- Individual Dirac EVDs of 2C QCD with $n_F = 4n$ staggered quarks, if the theory in χSB phase, are predicted from massive chGSE

- Chiral cond Σ of 2C QCD with $n_F = 2+(2+2+2)$ is determined by fitting $\mathrm{Spec}(\not{D})$ judged to be in the chSB phase (caution: $\beta = 4/g^2 \leq 1.3$)

- feasible plan : determine whether the WTC candidates in sympl. class 2C QCD with $n_F = 8$ (stag.) , $n_{Ad} = 2$ (overlap) is chSB, conformal, or else