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# A Possible Short-Distance Effect on the Meson-Meson Scattering

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It is suggested that the $q\bar{q}q\bar{q}$ mesons, which have been predicted by the MIT bag model and can be presumably identified with observed $\varepsilon(600\sim800)$ and etc., really have a short-distance effect on the $0^-$—$0^-$ scattering.

## § 1. Introduction

The $1/N_c$ expansion[1] of QCD provides the conceptual link between the colour gauge theory and the dual models.[2] Especially, it is expected that QCD leads, in the leading order of the expansion, to a confinement and gives somtehing like the tree approximation of a planar dual model.[2] In fact, for the mesons at least, experimental data show the dominance of the simplest planar topologies, for example, the exchange degeneracy, ideal mixing and the OZI rule. However, in the presence of the short-distance interaction such as the one-gluon exchange, the confinement, or, the hadron spectrum will be more complicated than what one usually imagines. In this paper, we shall suggest a possible short-distance effect on the meson-meson scattering.

Recently, an improved dual Born amplitude has been presented.[3] It explicitly involves the quark-model hadron spectrum, i.e. the harmonic-oscillator spectrum of $SU(6)\otimes O(3)_L$ multipliets. And, it is free from the ambiguity in the choice of satellite terms. The opportunity has been brought about from a recognition of a local duality. Seven years ago, Hoyer and Uschersohn[4] proposed a new-type local duality relation. Taking account of the saturation manner in Schmid's local super-convergence relation[5] and following the supposition by Bando et al.[6] and Nakkagawa et al.[7] that the dynamics of hadrons is governed essentially by the quark-orbital Regge trajectory, the author[8] has generalized and modified the Hoyer-Uschersohn relations to a local duality scheme involving the quark-model hadron spectrum. The local duality scheme, the most general representation of the global duality, i.e. $\Sigma$($s$-channel poles)$=\Sigma$($t$-channel poles) and an asymptotic convergence condition provides a uniquely determined $\pi^-\pi^+\to\pi^-\pi^+$ Born amplitude.[3] As the representation of the global duality, we take the most general Veneziano-type amplitude which provides an arbitrary residue at each of the relevant resonance poles.

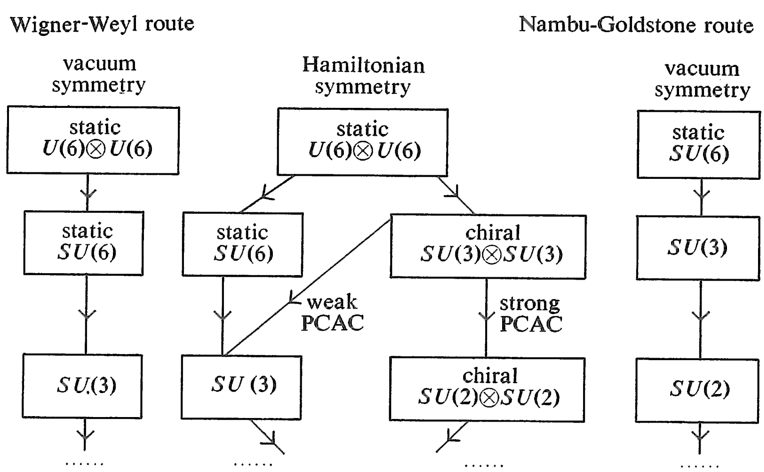
The uniquely determined $\pi^-\pi^+\to\pi^-\pi^+$ Born amplitude has good properties.[3]

It contains neither negative-norm states nor tachyons in a domain $(\alpha' m_\pi^2, \alpha_0)$ around the physical $m_\pi$, $\alpha'$ and $\alpha_0$. It predicts partial decay widths of low-lying resonances consistently with available experiments, when $m_\pi$, $\alpha'$ and $\alpha_0$ are taken to be their physical values and an overall multiplying factor is adjusted by observed $\rho \to 2\pi$ width. And, it is successful to the simultaneous description of low-lying resonances and high-energy behaviours.[9]

Further, it turns out that the uniquely determined $\pi\pi$ Born amplitude satisfies Adler's PCAC consistency condition in the limit $m_\pi = m_\rho = 0$, which is involved in the domain of no negative-norm states and no tachyons.[3] This will imply that when the short-distance spin-spin effect is also considered, the amplitude embody the Caldi-Pagels idea that the $\pi$ and $\rho$ along with their $SU(3)$ partners are Goldstone bosons in a non-relativistic world with a vacuum symmetry $SU(6)$, and in the relativistic word, in which the $SU(6)$ vacuum symmetry is necessarily broken, the $\rho$ will be massive — however, it remembers its origin as a Goldstone state.[10] Their idea is shown in Table I, contrasted to the Wigner-Weyl route. The preudoscalars can remain strictly massless ture Goldstone states in the relativistic world. The remaining essential points of the Caldi-Pagels model are the vector-meson dominance as a consequence of spontaneously broken chiral symmetry (the same mechanism that couples the axial-vector current to the $\pi$ couples the vector current to the $\rho$) and the partial conservation of tensor current (PCTC) implied by the mechanism.

According to Jaffe[12] who studied the $s$-wave $qq\bar{q}\bar{q}$ states in a semi-classical ap-

Table I. Group diagram for the Caldi-Pagels Nambu-Goldstone route contrasted to the Winger-Weyl route.




†) The breaking of chiral $SU(3) \otimes SU(3)$ proceeds as in the Gell-Mann-Oakes-Renner model.[11]

proximation to the MIT bag model, the predicted $s$-wave $qq\bar{q}\bar{q}$ states, for which the spin-spin interaction plays an important role, are analogous to the usual $s$-wave $q\bar{q}$-mesons and $qqq$-baryons. They have large widths in mesonic channels, as they preferentially decay by just falling apart into two $q\bar{q}$-mesons, as shown in Fig. 1. They are not baryonium-like states,[13] and they are often denoted as $q\bar{q}q\bar{q}$, which we also use hereafter. As also discussed by Jaffe, the lowest nonet of $q\bar{q}q\bar{q}$ states are natural candidates for the observed $O^+$ mesons

$$\varepsilon(600\sim 800),^{14,15)} S(975),^{16)} \delta(980),^{16)} \tag{1}$$

the $S(975)$ and $\delta(980)$ of which have well established, and the $\varepsilon(600\sim 800)$ has been recently emerged from an analysis of available data on meson pair production in $\gamma\gamma$ scattering[14] and an amplitude analysis of the reaction $\pi^-\pi^+\to\pi^0\pi^0$,[15] after the establishment of the $\varepsilon(1300)$.[16] (The $\varepsilon(600\to 800)$ may have a connection with the old $\varepsilon$ or $\sigma$.)

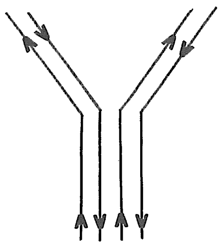


Fig. 1. Falling apart of $q\bar{q}q\bar{q}$ mesons in to two $q\bar{q}$-mesons.

Here, we discuss possible contributions of the $q\bar{q}q\bar{q}$ mesons to the meson-meson scattering. In §2, we state the present model. In §3, we compare its predictions with experiments for $\pi\pi$ scattering.

## §2. The model

In order to discuss the meson-meson scattering, we consider a model at the tree level. It is composed of a planar dual model provosed previously[3] and a short-distance correction to the model.[17]

The planar dual model involves the harmonic-oscillator spectrum of $SU(6)\otimes O(3)_L$ multiplets and determines the dynamics of them by the following: (i) The local duality scheme[8] for a process with one exotic channel (the exotic $u$-channel)

$$\sum_{a\in N} R_a^{(s)}(t)|_{t=m_{N'}^2} = \sum_{b\in N'} R_b^{(t)}(s)|_{s=m_N^2}, \quad N, N'=0, 1, 2,\dots . \tag{2}$$

Here, $R_a^{(s)}(t)(R_b^{(t)}(s))$ is the residue of the scattering amplitude at the $s$-($t$-) channel resonance $a$ ($b$) in the narrow-resonance approximation, and $N$ ($N'$) is the $s$-($t$-) channel resonance family. The resonance family $N$ is defined to be a set of resonances with a fixed total number $N$ of quanta of harmonic-oscillator excitations. The relations are assumed for each of the invariant amplitudes for the considered process. (ii) The most general representation of the global duality. (iii) An asymptotic convergence condition.

The resonance families and their members relevant to the $\pi^-\pi^+\to\pi^-\pi^+$ scattering and the appearance pattern of them are shown in Fig. 2.

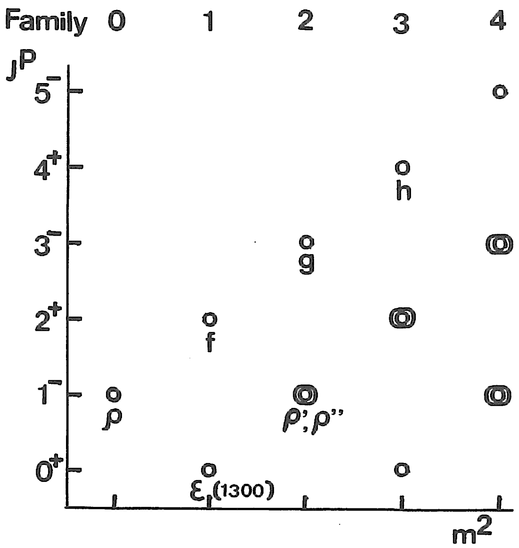



Fig. 2. The resonance families and their members in the $\pi^- - \pi^+$ sacttering. The double circles imply that there are two states with different quark-orbital angular momenta at their places.

The uniquely determined $\pi^-\pi^+\to\pi^-\pi^+$ Born amplitude in the planar dual model is

$$F(s,t) = -\lambda_{1,1}(1-\beta^2)\sum_{n=1}^{\infty}\frac{1}{(n-1)!(2n-1+\beta)(2n-3+\beta)}$$

$$\times\left(\frac{\Gamma(n-\alpha_s)\Gamma(n-\alpha_t)}{\Gamma(n-\alpha_s-\alpha_t)}+\frac{(1-\beta)}{2}\frac{\Gamma(n-\alpha_s)\Gamma(n-\alpha_t)}{\Gamma(n+1-\alpha_s-\alpha_t)}\right), \tag{3}$$

where $\alpha_s=\alpha's+\alpha_0$ is the exchange-degenerate $\rho-f$ Regge trajectory, $\lambda_{1,1}$ is an overall multiplying factor and $\beta$ is

$$\beta=2-3\alpha_0-4\alpha' m_\pi^2. \tag{4}$$

All the squared resonance coupling constants in (3) are positive in a domain in the $(\alpha' m_\pi^2, \alpha' m_\rho^2)$ plane shown in Fig. 3.[17] In the region $\alpha' m_\pi^2, \alpha' m_\rho^2>0$ of Fig. 3, all the

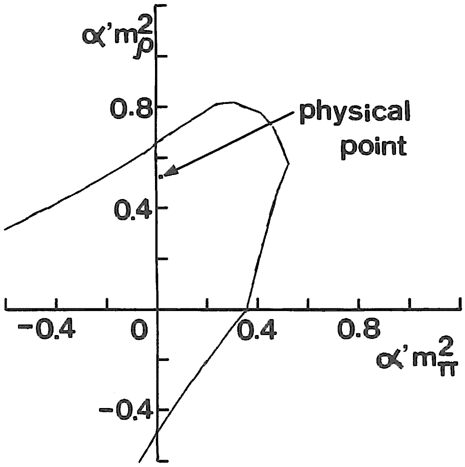



Fig. 3. The domain where all the squared coupling constants for the relevant resonances are positive. In the region $\alpha' m_\pi^2$, $\alpha' m_\rho^2 > 0$, all the squared masses of the scattering particle and resonances are also positive.

squared masses of the scattering particle and resonances are positive. (When $m_\rho^2$ is positive, then the other resonances have positive squared masses.) Thus, the hexagon-like domain in Fig. 3 is theoretically advisable. In Fig. 3, we show also the point for the physical values of $m_\pi$, $m_\rho$ and $\alpha'$.

The short-distance correction to the above planar dual model, which we take account of, is shown in Fig. 4. According to Jaffe,[12] the heavier $0^+$ $q\bar{q}q\bar{q}$ multiplets and the higher-spin $q\bar{q}q\bar{q}$ states do not contribute to the $0^- - 0^-$ scattering.

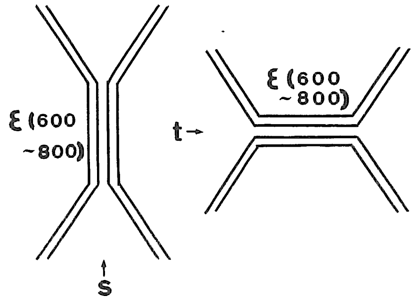



Fig. 4. The $s$- and $t$-channel exchange of the $\varepsilon(600\sim800)$ in the $0^- - 0^-$ scattering. There can not be any duality between the two diagrams.

It is noted that there can not be any duality between the $s$- and $t$-channel exchanges of the $q\bar{q}q\bar{q}$ states in Fig. 4.

For the $\pi^-\pi^+\to\pi^-\pi^+$ process, we consider a correction term due to the $\varepsilon(600\sim 800)$ exchange in Fig. 4,

$$\frac{g^2_{\varepsilon\pi\pi}m^2_\varepsilon}{m^2_\varepsilon - s} + \frac{g^2_{\varepsilon\pi\pi}m^2_\varepsilon}{m^2_\varepsilon - t}. \tag{5}$$

Here and hereafter the $\varepsilon(600\sim 800)$ is denoted as $\varepsilon$. The $\varepsilon$-exchange amplitudes are tentatively assumed in the narrow-resonance approximation.

## §3. Comparison with experiments

Now we discuss the $\pi^-\pi^+\to\pi^-\pi^+$ amplitude

$$\text{amplitude (3)} + \text{amplitude (5)}. \tag{6}$$

The comparison of the Born amplitude such as (6) with experiments is, in general, very difficult, because we must consider the unitarization. However, a method for testing the essences of the dual resonance models has been presented by Froggatt, Nielsen and Petersen,[18] using only the imaginary part of the amplitude. It is based on the observation that the general $(s, t)$ Veneziano forms have two-dimensional inverse Laplace transforms with a very characteristic support property. Because of its use of only the imaginary part of the amplitude, to consider explicit unitarity corrections is not necessary at high energies. Indeed, the support property of a certain kind of general Veneziano-type models, including the Lovelace-Shapiro-Veneziano[19] and Frampton[20] ones, is upheld by phenomenological $\pi^-\pi^+$ and isospin-$I_t=1$-exchange amplitudes in the forward directions of $0\geqq t\geqq -1.0$ (GeV/$c$)$^2$, which are essentially uniquely determined by data, analyticity and phase-shift analysis (unitarity) from threshold up to a dipion mass of 1.8 GeV. And, in details, the method is able to indicate a superiority of the Frampton model, which may imply that the spectrum of the Frampton model is more realistic than that of the Lovelace-Shapiro-Veneziano model.

Here, we use the method and the phenomenological amplitudes presented by Froggatt, Nielsen and Petersen.

As has been discussed in Ref. 9), the dual Born amplitude (3) reproduces high-energy data fairly well. This is due to an extra factor in contrast to the Frampton and Lovelace-Shapiro-Veneziano amplitudes. In Fig. 5, the extra factor $I(\alpha_t)$ in the asymptotic form is shown together with the corresponding factor of the Frampton model, $I(\alpha_t)=1$. (The Frampton amplitude possesses the same asymptoci form as the Lovelace-Shapiro-Veneziano one possesses, because its satellite terms involve only daughters.) The comparison of the prdiction from amplitude (3) with experiments at high energy is shown in Fig. 6.

We examine whether or not the correction term (5) improve fits to data at lower energies. In the threshold region, unitarity corrections to the imaginary part of the

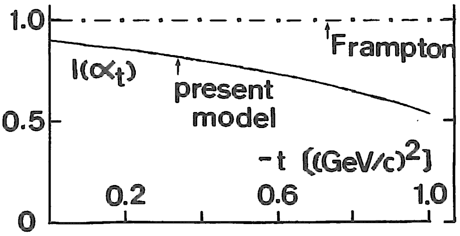



Fig. 5. The effects of the factor $I(\alpha_t)$ of the present planar dual model. A domping is brought about by the factor.

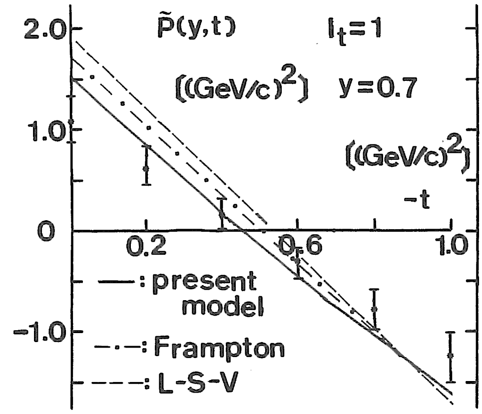



Fig. 6. The comparison of the present planar dual model with experiments at a high $y$. The predictions from the Frampton and Lovelace-Shapiro-Veneziano models are also shown.

dual Born term are important differently from at high energies. We suppose that the correction (5) will work as the dominant part of the unitarity corrections. The interest of this work is put on this supposition.

As in Ref. 9), the overall multiplying factor $\lambda_{1,1}$ is adjusted by $\rho\to 2\pi$ width. When the amplitude (3) is defined as in Ref. 18),

$$\lambda_{1,1}=-0.743. \tag{7}$$

The other parameters $m_\pi$, $\alpha'$ and $\alpha_0$ are taken to be their physical values

$$m_\pi=0.140\ \text{GeV},\quad \alpha'=0.888\ (\text{GeV})^{-2},\quad \alpha_0=0.475. \tag{8}$$

Imposing Adler's soft-poin PCAC consistency condition on the amplitude (6), we obtain[17])

$$g^2_{\varepsilon\pi\pi}=-\frac{(m_\varepsilon^2-m_\pi^2)}{2m_\varepsilon^2}F(\text{Adler point}), \tag{9}$$

where

$$F(\text{Adler point})=\lambda_{1,1}\cdot\frac{1}{2}\{1-\alpha(m_\pi^2)\}\Big(\frac{\{13-3\alpha(m_\pi^2)\}\{\Gamma(2-\alpha(m_\pi^2))\}^2}{\{5-3\alpha(m_\pi^2)\}\Gamma(3-2\alpha(m_\pi^2))}$$
$$+3\{1-3\alpha(m_\pi^2)\}\sum_{n=3}^{\infty}\frac{\{2n-1-\alpha(m_\pi^2)\}\{\Gamma(n-\alpha(m_\pi^2))\}^2}{(n-1)!\{2n-1-3\alpha(m_\pi^2)\}\{2n+1-3\alpha(m_\pi^2)\}\Gamma(n+1-2\alpha(m_\pi^2))}\Big). \tag{10}$$

$$\alpha(m_\pi^2)=\alpha' m_\pi^2+\alpha_0. \tag{11}$$

We emphasize that $g_{\varepsilon\pi\pi}^2$ of (9) provides a prediction for $\Gamma_{\varepsilon\to\pi\pi}$[17]

$$\Gamma_{\varepsilon\to\pi\pi}\simeq 0.6 m_\varepsilon, \tag{12}$$

which is consistent with experimental informations.[15,14] As for the predictions for $s$-wave scattering lengths from amplitude (6) with $g_{\varepsilon\pi\pi}^2$ of (9) and the kinematics in $\pi-\pi$ scattering, they are[17]

$$\begin{aligned} &a^{I=0}=0.205 m_\pi^{-1},\ a^{I=2}=-0.066 m_\pi^{-1},\ (2a^{I=0}-5a^{I=2}=0.740 m_\pi^{-1}),\ (a^{I=0}=\\ &\quad (0.26\pm 0.05) m_\pi^{-1},\ a^{I=2}=-(0.028\pm 0.012) m_\pi^{-1},\ 2a^{I=0}-5a^{I=2}=(0.657\pm 0.052)\\ &\quad m_\pi^{-1} \text{ from Roy equation fits constrained by } K_{e4} \text{ data.}^{21)}), \end{aligned} \tag{13}$$

when $m_\varepsilon$ is taken to be 800 MeV. Moreover, applying the Adler-Weisberger sum rule, we obtain a prediction for the poion decay constant $f_\pi$,[17]

$$f_\pi=83.6 \text{ MeV}, \tag{14}$$

which is near to the experimental value $f_\pi\simeq 93$ MeV from the charged-pion lifetime.

It is noted that if the $\varepsilon$ decouples from $\pi\pi$ in the absence of the spin-spin interaction due to the one-gluon exchange, the amplitude (6) satisfies the PCAC condition on the non-relativistic world with the $\pi$ and $\rho$ as Goldstone bosons, and the end of which does to the relativistic world where the $\rho$ is massive, but the $\pi$ remains a Goldstone state.[17]*)

The support property of the two-dimensional inverse Laplace transform of the general $(s, t)$ Veneziano-type amplitude is equivalent to the behaviour of the single inverse Laplace or Mellin transform of the fixed-t general Veneziano-type amplitude.[18] And, one can test the amplitude $F(s, t)$ of (6), considered at forward directions, by calculating

$$\tilde{G}(y, t)=\frac{\alpha'}{\pi}\int_{\text{cut}} ds\, y^{\alpha' s}\,\text{Im}\, F(s, t), \tag{15}$$

and comparing it with corresponding phenomenological amplitudes. It is noted that the calculation requires only the value of the imaginary part of the scattering amplitude along the branch cuts.

Writing amplitude (6) as

$$F(s, t)=\sum_{J=1}^{\infty}\frac{R_J(\alpha_t)}{J-\alpha_s}+\left(\frac{g_{\varepsilon\pi\pi}^2 m_\varepsilon^2}{m_\varepsilon^2-s}+\frac{g_{\varepsilon\pi\pi}^2 m_\varepsilon^2}{m_\varepsilon^2-t}\right), \tag{16}$$

*) As has been discussed in Ref. 17), it is known from the $\pi^-\rho^+\to\rho^-\pi^+$ amplitude constructed in the planar dual model stated in §2 that the limit $m_\pi=m_\rho\simeq 0$ corresponds to the non-relativistic world where the $\pi$ and $\rho$ are Goldstone bosons.

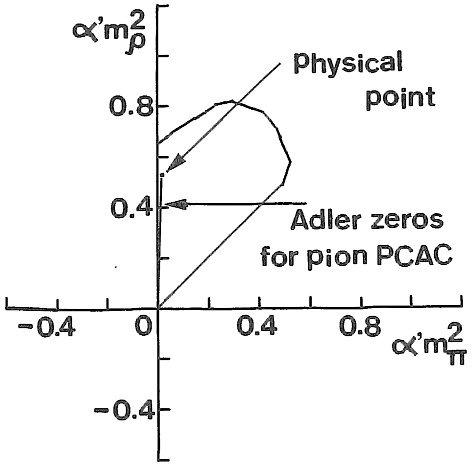



Fig. 7. The Adler zeros for pion PCAC in the present model. They are in the domain where the squared coupling constants for the $q\bar{q}$ resonances and the $\varepsilon(600\sim800)$ and the squared masses of the scattering particle and resonances are positive.

where

$$R_J(\alpha_t) = -\lambda_{1,1}\frac{\Gamma\left(\frac{\beta+3}{2}\right)}{\Gamma\left(\alpha_t+\frac{\beta+1}{2}\right)}\left[\left(\alpha_t+\frac{\beta-1}{2}\right)r(J,\,\alpha_t,\,\beta)-\frac{(\beta-1)}{2}r(J-1,\,\alpha_t,\,\beta)\right], \tag{17}$$

$$r(J,\,\alpha_t,\,\beta) = J\frac{\Gamma\left(J+\alpha_t+\frac{\beta-1}{2}\right)}{\Gamma\left(J+\frac{\beta+1}{2}\right)}, \tag{18}$$

we have

$$\tilde{G}(y,\,t) = \alpha'\tilde{P}(y,\,t)e^{-\alpha' t\cdot\ln(1-y)} + \alpha' g_{\varepsilon\pi\pi}^2 m_\varepsilon^2 y^{\alpha' m_\varepsilon^2}, \tag{19}$$

where

$$\tilde{P}(y,\,t) = -\lambda_{1,1}\frac{\{y(1-y)\}^{1-\alpha_0}}{(1-y)^2}(t-t_0)F\left(1-\alpha_t,\,\frac{\beta-1}{2},\,\frac{\beta+3}{2};\,y\right), \tag{20}$$

$$-\alpha' t_0 = \alpha_0 + \frac{(\beta-1)}{2}(1-y). \tag{21}$$

In (20), $F(a,\,b,\,c;\,y)$ is Gauss's hypergeometric function. In order to eliminate the contribution of the diffraction scattering to $\tilde{G}(y,\,t)$ explicitly, it is better to compare the model with experiments for the $I_t=1$ amplitude defined by

$$\tilde{G}_{I_t=1}(y,\ t)=\tilde{G}_{\pi^-\pi^+\to\pi^-\pi^+}(y,\ t)-\tilde{G}_{\pi^+\pi^+\to\pi^+\pi^+}(y,\ t). \tag{22}$$

The phenomenological amplitudes ($\tilde{G}(y,\ t)$) for the $\pi^+\pi^+$ scattering are nonzero and give a measure of the contribution from diffraction scattering. In the dipion mass region of $M_{\pi\pi}\leqq 1.8$ GeV, the amplitudes for the $\pi^+\pi^+$ scattering are typically of the order of 20% of those for the $\pi^-\pi^+$ scattering, except near $t=-0.5$ (GeV/$c$)$^2$ where the $\pi^-\pi^+$ amplitudes have a zero.[18] So, it seems reasonable to hope that the dominant part of the $\pi^-\pi^+$ amplitude is described by a Veneziano-type Born term. However, the property of the zero is also important in the comparison of the models with experiments. (It is noted that as for the predictions for $\tilde{G}_{\pi^+\pi^+\to\pi^+\pi^+}(y,\ t)$ from the Veneziano-type models, they are zero.)

The last factor in the first term of (19) is a property of the general dual resonance models. And, it is well upheld by the corresponding factor of phenomenological amplitudes at high energies.[18]

We compare the model with experiments about

$$\tilde{P}(y,\ t)+g^2_{\varepsilon\pi\pi}m^2_\varepsilon y^{\alpha' m^2_\varepsilon}e^{\alpha' t\cdot\ln(1-y)}. \tag{23}$$

The predictions and phenomenological amplitudes are shown in Figs. 8∼13. There, we show also the predictions from the planar dual model, i.e. the first term in (23).

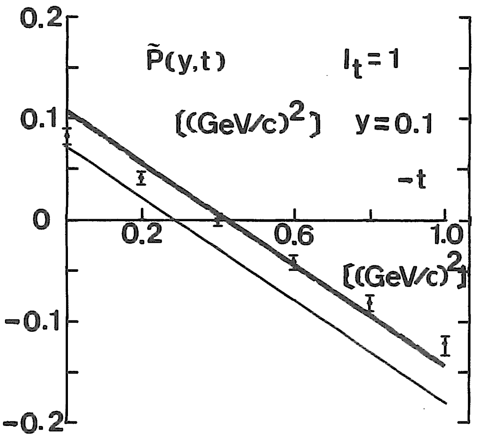



Fig. 8. The comparison of the present model with experiments at $y$=0.1. The prediction is shown by the broad solid curve. We show also the prediction from the (present) planar dual model by the slim curve.

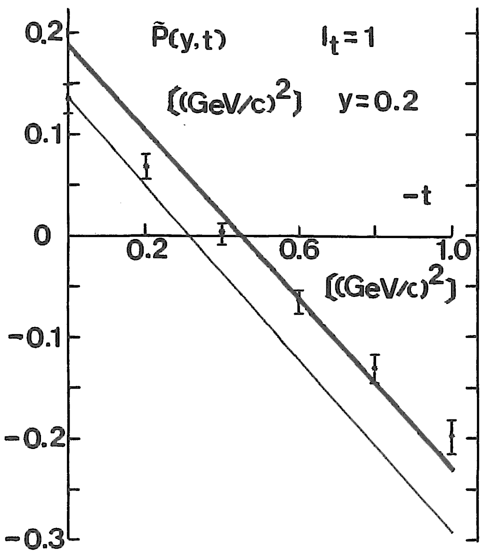



Fig. 9. The comparison of the present model with experiments at $y$=0.2. The curves are defined in Fig. 8.

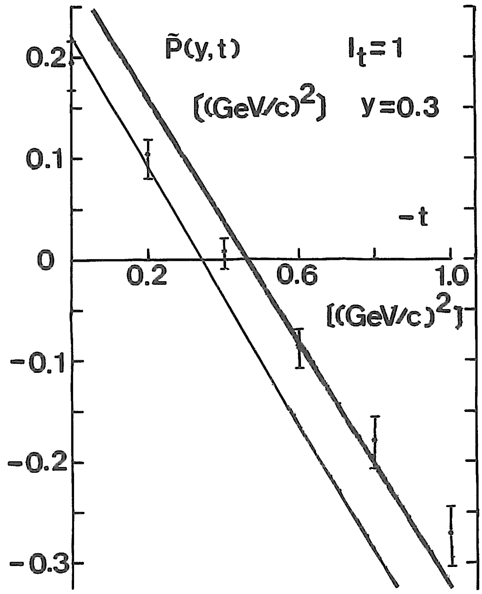



Fig. 10. The comparison of the present model with experiments at $y=0.3$. The curves are defined in Fig. 8.

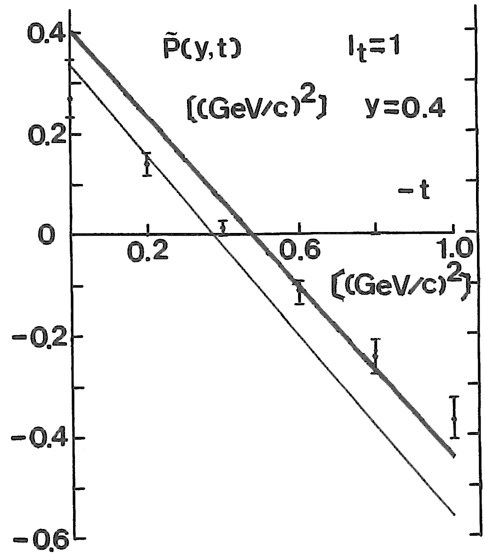



Fig. 11. The comparison of the present model with experiments at $y=0.4$. The curves are defined in Fig. 8.

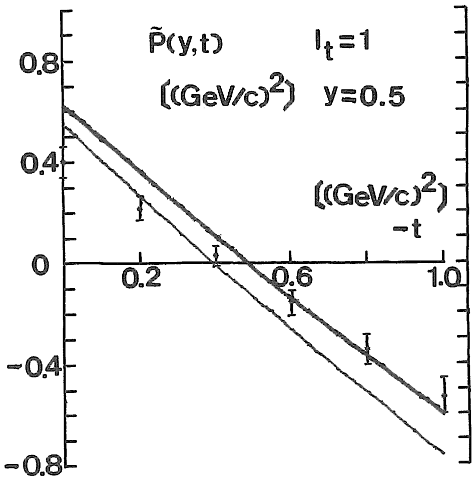



Fig. 12. The comparison of the present model with experiments at $y=0.5$. The curves are defined in Fig. 8.

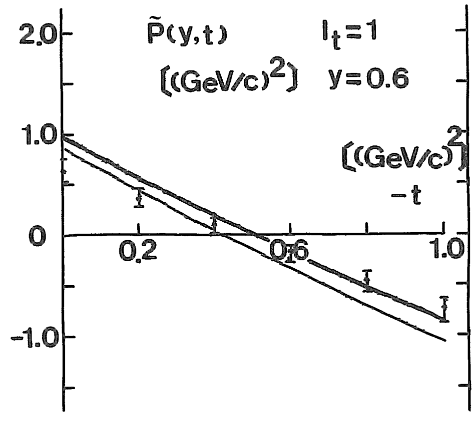



Fig. 13. The comparison of the present model with experiments at $y=0.6$. The curves are defined in Fig. 8.

In Fig. 14, we show the zeros of the theoretical and phenomenological amplitudes for various $y$'s, together with those of the Frampton and Lovelace-Shapiro-Veneziano models.

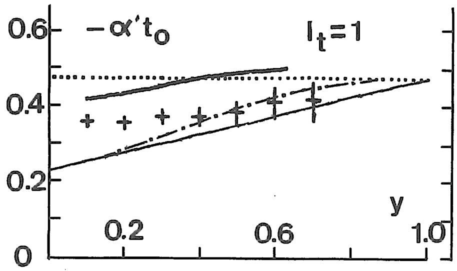



Fig. 14. The zeros of the theoretical and phenomenological amplitudes for various $y$'s, together with those of the Frampton and Lovelace-Shapiro-Veneziano models. The curves are defined in Figs. 6 and 8.

As seen in Figs. 8~14, the $\varepsilon(600\sim800)$ improve fits to experimental data. However, the description of the $\varepsilon(600\sim800)$ by the narrow resonance approximation is not suitable.

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