Mem. Fac. Sci., Shimane Univ.,
19, pp. 75–82 Dec. 20, 1985

A Note on the Connection between the $qq\bar{q}\bar{q}$ Mesons and the Chiral Symmetry

Taketoshi INO
Department of Physics, Shimane University, Matsue 690
(Received September 14, 1985)

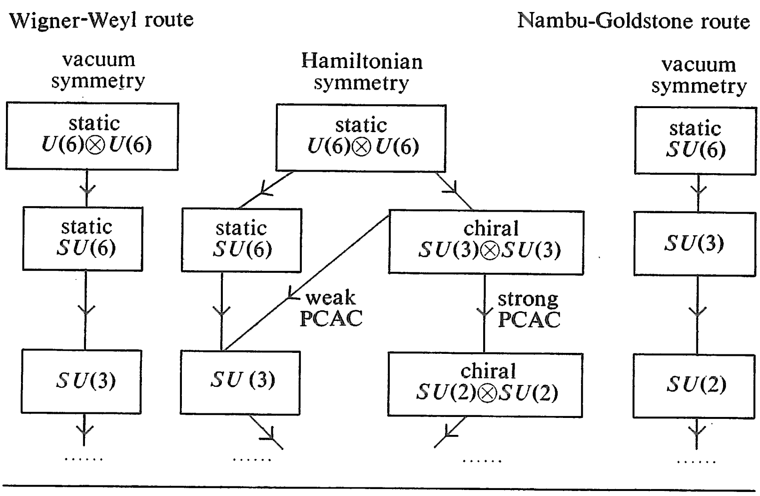
We discuss the Adler-Weisberger sum rules for the 0^--meson-0^--meson scattering in the model, which is composed of a planar dual model involving the quark-model hadron spectrum of $q\bar{q}$-mesons and a short-distance correction due to the $qq\bar{q}\bar{q}$-meson exchange.

There remains a problem in the interpretation of hadron symmetries: How can one reconcile the success of the approximate static $SU(6)$ symmetry of the quark model, in which the π and ρ are classified as members of the same 35, with the apparent role of the π and the K and η as Nambu-Goldstone states of a spontaneously broken $SU(3)\otimes SU(3)$ chiral symmetry?

An idea for a resolution of the above $\rho-\pi$ puzzle has been proposed by Caldi and Pagels.[1] They have adopted the quark model and abstracted commutation relations from QCD. Further, they assumed that in the static limit the ρ and the π and their $U(3)$ partners transform like members of a $(6, \bar{6})\oplus(\bar{6}, 6)$ representation of the Feynman-Gell-Mann-Zweig chiral $U(6)\otimes U(6)$ algebra. Their starting point is to imagine a non-relativistic world with a Hamiltonian symmetry chiral $U(6)\otimes U(6)$. The vacuum symmetry is spontaneously broken to $SU(6)$ and this is the classificatoty group for hadrons at rest. The π and ρ along with their $SU(3)$ partners are true Goldstone bosons in this non-relativistic world. In the relativistic world, in with the $SU(6)$ vacuum symmetry is necessarily broken, the ρ will be massive — however, it remembers its origin as a Goldstone state. The preudoscalars can remain strictly massless true Goldstone states in this relativistic world with a chiral $SU(3)\otimes SU(3)$ Hamiltonian symmetry. The breaking of chiral $SU(3)\otimes SU(3)$ then proceeds as in the Gell-Mann, Oakes and Renner model.[2] Their idea is shown in Table I, constrasted to the Wigner-Weyl route. And, the removal of hadron mass degeneracies in the model is shown in Fig. 1. The remaining essential points of the Caldi-Pagels model are the VMD (vector-meson dominance) as a consequence of spontaneously broken chiral symmetry (the same mechanism that conples the axial-vector current to the π couples the vector current to the ρ) and the PCTC (partial conservation of tensor current) implied by the mechanism.

However, as is well known, it is, unfortunately, impossible to construct an inter-

Table I. Group diagram for the Caldi-Pagels Nambu-Goldstone route contrasted to the Wigner-Weyl route.



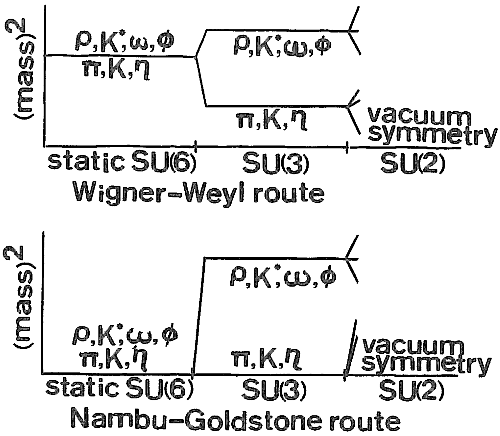


Fig. 1. Level diagrams of the ground-state pseudoscalar and vector mesons in the two routings shown in Table I.

acting relativistic field theory with the $U(6)\otimes U(6)$ symmetry.[3] In order to embody the Caldi-Pagels idea, one must find the other representation of the hadron interactions.

We are now studying a new model for hadron interactions.[4] This model is composed of two parts. One is a planar dual model involving the quark model hadron spectrum of $q\bar{q}$-mesons and qqq-baryons, the harmonic-oscillator spectrum of $SU(6)\otimes O(3)_L$ multiplets. And, the other is a correction to the planar dual model, which is about the $q\bar{q}q\bar{q}$ mesons predicted by a semi-classical approximation to the MIT bag model.[5] The $1/N_c$ expansion[6] of QCD provides the conceptual link between the colour gauge theory and the dual models.[7] It is expected that QCD gives, in the leading order of the expansion, something like the tree approximation of a planar dual model.[7] Further, the spin-spin interaction due to the one-gluon exchange, in conjunction with the confining interaction, brings about the $q\bar{q}q\bar{q}$ mesons which are analogous to the usual s-wave $q\bar{q}$-mesons and qqq-baryons and have large widths in mesonic channels, as they preferentially decay by just falling apart into two $q\bar{q}$-mesons.[5]*) As has been discussed by Jaffe,[5] the lowest nonet of $q\bar{q}q\bar{q}$ states are natural candidates for the observed 0^+ mesons

$$\varepsilon(600\sim 800),^{9,10)}S(975),^{11)}\delta(980),^{11)} \tag{1}$$

the $S(975)$ and $\delta(980)$ of which have been well established, and the $\varepsilon(600\sim 800)$ has been recently emerged from an analysis of available data on meson pair production in $\gamma\gamma$ scattering[9] and an amplitude analysis of the reaction $\pi^-\pi^+\to\pi^0\pi^0$,[10]**) after the establishment of the $\varepsilon(1300)$.[11] The present model, in the tree approximation, is shown graphically in Fig. 2 in the case of the meson-meson scattering.

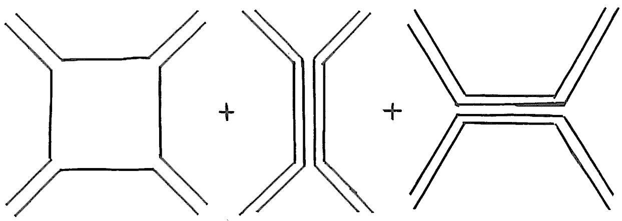


Fig. 2. The present model composed of two parts; a planar dual model involving the quark model hadron spectrum of $q\bar{q}$-mesons and a short-distance correction, that is, the $q\bar{q}q\bar{q}$-meson exchange.

The model is quite promising. It has the masses and coupling constants not as fixed parameters but as dynamical quantities.[4] The planar dual model, which involves the harmonic-oscillator spectrum of $SU(6)\otimes O(3)_L$ multiplets and is the main part of it, can uniquely determine a Born amplitude for each of the meson-meson scattering.[4,12] In fact, we have obtained a uniquely determined $\pi^-\pi^+\to\pi^-\pi^+$ Born amplitude in the planar model.[12] The amplitude contains neither negative-norm states nor tachyons in a parameter domain $(\alpha' m_\pi^2, \alpha_0)$ around the physical values of m_π, α' and α_0.[4,12] Here, α' and α_0 are the slope and zero-intercept of the exchange-degenerate $\rho-f$ trajectory. The amplitude predicts partial decay widths for low-lying resonances con-

*) These s-wave $qq\bar{q}\bar{q}$ states are not baryonium-like,[8] and they are often denoted as $q\bar{q}q\bar{q}$, which we also use in this paper.

**) The amplitude analysis has selected the so-called down-down solution as the only one making the $\pi^-\pi^+\to\pi^0\pi^0$ and $\pi^-\pi^+\to\pi^-\pi^+$ data consistent. This solution leads to a rather rapid phase variation at approximately 750 MeV and a phase shift which goes through 90° at about 800 MeV.

sistently with available experiments, when an overall multiplying factor is adjusted by $\rho\to 2\pi$ width and m_π, α' and α_0 are taken to be their physical values.[12] It provides a $\pi^-\pi^+\to\pi^-\pi^+$ Born amplitude, the uniquely determined dual Born amplitude plus a correction term due to the $\varepsilon(600\sim 800)$ exchange,*) which satisfies the soft-pion PCAC consistency condition on a segment (in the positive-norm domain $(\alpha' m_\pi^2, \alpha' m_\rho^2)=(\alpha' m_\pi^2, 1-\alpha_0)$) starting from (0, 0) and ending at the physical point of $(\alpha' m_\pi^2, \alpha' m_\rho^2)$.[4] Thus, it describes the π as a Goldstone boson in the non-relativistic limit $(\alpha' m_\pi^2, \alpha' m_\rho^2)=(0, 0)$. It describes the ρ also as a Goldstone boson in the limit, as known by constructing the $\pi^-\rho^+\to\rho^-\pi^+$ amplitude in the model.[4]

In this paper, we make a further discussion about the connection between the $q\bar{q}q\bar{q}$ mesons and the chiral symmetry. The $\pi^-\pi^+\to\pi^-\pi^+$ amplitude with the correction term due to the $\varepsilon(600\sim 800)$ exchange satisfies the Adler-Weisberger sum rule with a pion decay constant near to the experimental one from the charged-pion lifetime.[4] We apply and discuss the sum rule for the amplitudes for other 0^-0^- processes in the present model.

Amplitudes in the planar dual model

In the present planar dual model, we have the $\pi^-\pi^+\to\pi^-\pi^+$ amplitude, in the s-channel pole expansion, as[12]

$$F^{\pi\pi}(s, t)=\sum_{J=1}^{\infty}\frac{R_J^{\pi\pi}(\tilde{\alpha}_t)}{J-\alpha_s}, \tag{2}$$

where

$$\begin{aligned} R_1^{\pi\pi}(\tilde{\alpha}_t)&=-\lambda^{\pi\pi}\frac{(\tilde{\alpha}_t-\tilde{\alpha}_u)}{2},\\ R_3^{\pi\pi}(\tilde{\alpha}_t)&=-\lambda^{\pi\pi}\frac{3(\tilde{\alpha}_t-\tilde{\alpha}_u)\{(\tilde{\alpha}_t-\tilde{\alpha}_u)^2-b_3\}}{2(3+\beta)(5+\beta)},\\ R_5^{\pi\pi}(\tilde{\alpha}_t)&=-\lambda^{\pi\pi}\frac{5(\tilde{\alpha}_t-\tilde{\alpha}_u)\{(\tilde{\alpha}_t-\tilde{\alpha}_u)^2-b_5\}\{(\tilde{\alpha}_t-\tilde{\alpha}_u)^2-2^2\}}{2(3+\beta)(5+\beta)(7+\beta)(9+\beta)},\\ &\cdots\cdots\cdots\cdots, \end{aligned} \tag{3}$$

$$\begin{aligned} R_2^{\pi\pi}(\tilde{\alpha}_t)&=-\lambda^{\pi\pi}\frac{2\{(\tilde{\alpha}_t-\tilde{\alpha}_u)^2-b_2\}}{2(3+\beta)},\\ R_4^{\pi\pi}(\tilde{\alpha}_t)&=-\lambda^{\pi\pi}\frac{4\{(\tilde{\alpha}_t-\tilde{\alpha}_u)^2-b_4\}\{(\tilde{\alpha}_t-\tilde{\alpha}_u)^2-1^2\}}{2(3+\beta)(5+\beta)(7+\beta)},\\ R_6^{\pi\pi}(\tilde{\alpha}_t)&=-\lambda^{\pi\pi}\frac{6\{(\tilde{\alpha}_t-\tilde{\alpha}_u)^2-b_6\}\{(\tilde{\alpha}_t-\tilde{\alpha}_u)^2-1^2\}\{(\tilde{\alpha}_t-\tilde{\alpha}_u)^2-3^2\}}{2(3+\beta)(5+\beta)(7+\beta)(9+\beta)(11+\beta)},\\ &\cdots\cdots\cdots\cdots, \end{aligned} \tag{4}$$

Here, $\alpha_s=\alpha' s+\alpha_0$ is the exchange-degenerate $\rho-f$ Regge trajectory, and

*) According to Jaffe,[5] only the lowest of the $q\bar{q}q\bar{q}$ mesons couples strongly to two pseudoscalars. The 2^+ $q\bar{q}q\bar{q}$ mesons couple weakly to two pseudoscalars. And, the heavier 0^+ $q\bar{q}q\bar{q}$ states couple strongly to two vectors.

$$\tilde{\alpha}_t = \alpha'\tilde{t} + \alpha_0,\ \tilde{t} = -2q_J^2(1-\cos\theta),$$
$$\tilde{\alpha}_u = \alpha'\tilde{u} + \alpha_0,\ \tilde{u} = 4m_\pi^2 - m_J^2 - \tilde{t}, \tag{5}$$

$$m_J^2 = \frac{J-\alpha_0}{\alpha'},\quad q_J^2 = \frac{m_J^2 - 4m_\pi^2}{4},$$
$$b_J = \frac{(J-1)\{(J+\beta-1)^2 - J\}}{J}. \tag{6}$$

The s-channel pole expansion of the $K^-K^+ \to K^-K^+$ amplitude is

$$F^{KK}(s,t) = \sum_{J=1}^{\infty}\left(\frac{R_J^{KK}(p_J^2, \cos\theta)}{J-\alpha_s} + \frac{R_{J,1}^{KK}(p_{J,1}^2, \cos\theta)}{J-\alpha_s^1}\right), \tag{7}$$

where α_s and $\alpha_s^1 = \alpha' s + \alpha_0^1$ are the exchange-degenerate $\rho-\omega-A_2-f$ and $\phi-f'$ trajectory respectively. $R_J^{KK}(p_J^2, \cos\theta)$ is given by the substitution in Eqs. (2), (3) and (4)

$$\lambda^{\pi\pi} \longrightarrow \lambda^{KK},$$
$$\tilde{\alpha}_t - \tilde{\alpha}_u = 4\alpha' q_J^2 \cos\theta \longrightarrow 4\alpha' p_J^2 \cos\theta, \tag{8}$$
$$\beta \longrightarrow \gamma = 2 - \alpha_0 - 2\alpha_0^1 - 4\alpha' m_K^2,$$

where

$$p_J^2 = \frac{m_J^2 - 4m_K^2}{4}. \tag{9}$$

$R_J^{KK}(p_{J,1}^2, \cos\theta)$ is given by the substitution in Eqs. (2), (3) and (4)

$$\lambda^{\pi\pi} \longrightarrow \lambda_1^{KK},$$
$$\tilde{\alpha}_t - \tilde{\alpha}_u = 4\alpha' q_J^2 \cos\theta \longrightarrow 4\alpha' p_{J,1}^2 \cos\theta, \tag{10}$$
$$\beta \longrightarrow \gamma_1 = 2 - \alpha_0^1 - 2\alpha_0 - 4\alpha' m_K^2,$$

where

$$p_{J,1}^2 = \frac{m_{J,1}^2 - 4m_K^2}{4},\quad m_{J,1}^2 = \frac{J-\alpha_0^1}{\alpha'}. \tag{11}$$

In order to connect λ^{KK} and λ_1^{KK} with $\lambda^{\pi\pi}$, we assume (i) the ideal nonet scheme for mesonic resonances, (ii) the $SU(3)$ invariance for $1^- - 0^- - 0^-$ vertices, and (iii) the OZI decoupling rule. It is noted that as known from Eqs. (3), (4), (8) and the corresponding expression for $\pi^-\pi^+ \to K^-K^+$, all the coupling constants for the resonances concerned with $\rho - f$ trajectory are factorizable because of

$$\beta = \gamma \tag{12}$$

and etc. The relation (12) imply

$$m_\phi^2 - m_\rho^2 = m_{f'}^2 - m_f^2 = 2(m_K^2 - m_\pi^2), \tag{13}$$

which is consistent with the present basic assumptions.

A correction to the planar dual model

The lowest mass 0^+ $q\bar{q}q\bar{q}$ mesons, whose contributions are considered as corrections to the above planar dual model, are

$$\begin{aligned} &\varepsilon(600\sim800)=C^0(9,\,0^+)=u\bar{u}d\bar{d}, \\ &S(975)=C^s(9,\,0^+)=\frac{1}{\sqrt{2}}s\bar{s}(u\bar{u}+d\bar{d}), \\ &\delta(980)=C^s_\pi(9,\,0^+)=u\bar{d}s\bar{s},\ \text{etc.}, \\ &\kappa(?)=C_K(9,\,0^+)=u\bar{s}d\bar{d},\ \text{etc.} \end{aligned} \tag{14}$$

Here, the second notation is taken from Ref. 5). Their decay couplings are[5)]

$$\begin{array}{lll} \varepsilon(600\sim800)\longrightarrow\pi\pi & & \frac{\sqrt{3}}{2}g_0, \\ \varepsilon(600\sim800)\longrightarrow K\bar{K} & & 0, \\ S(975)\longrightarrow K\bar{K} & & \frac{1}{\sqrt{2}}g_0, \\ S(975)\longrightarrow\pi\pi & & 0, \\ \delta(980)\longrightarrow K\bar{K} & & -\frac{1}{\sqrt{2}}g_0. \end{array} \tag{15}$$

Amplitudes for the $q\bar{q}q\bar{q}$ meson exchanges are assumed tentatively in the narrow-resonance approximation.

Application of the Adler-Weisberger sum rule to the present model

We apply the Adler-Weisberger sum rule to the $\pi^-\pi^+\to\pi^-\pi^+$ amplitude in the present model

$$F^{\pi\pi}(s,\,t)=(\text{amplitude (2)})+(\varepsilon(600\sim800)\text{ exchange term}). \tag{16}$$

$\lambda^{\pi\pi}$ is adjusted and fixed by observed $\rho\to2\pi$ width,

$$-\lambda^{\pi\pi}=f^2_{\rho\pi\pi}/4\pi=2.97. \tag{17}$$

$g^2_{\varepsilon(600\sim800)\pi\pi}$ is taken to be

$$g^2_{\varepsilon(600\sim800)\pi\pi}=0.199, \tag{18}$$

which is determined Adler's soft-pion PCAC condition on the amplitude in the present model. (We note that when we demand the condition, we can not employ the s-channel expansion (2). For the first part of Eq. (16), we must use the amplitude in the planar model not expanded by the s-channel poles. The protopype dual amplitude is seen in Ref. 12).) The other input values are

$$m_\pi = 0.140 \text{ GeV},$$
$$\alpha' = 0.888 \text{ (GeV)}^{-2}, \tag{19}$$
$$\alpha_0 = 0.475.$$

We note also that $g^2_{\varepsilon(600\sim800)\pi\pi}$ in Eq. (18) predicts

$$\Gamma_{\varepsilon(600\sim800)\pi\pi} \simeq 0.6 m_{\varepsilon(600\sim800)}, \tag{20}$$

which is consistent with experimental informations.[9,10)]

The Adler-Weisberger sum rule

$$\int_{4m_\pi^2}^{\infty} \frac{ds}{(s-m_\pi^2)^2} \operatorname{Im}\left[F^{\pi^-\pi^+\to\pi^-\pi^+}(s,t) - F^{\pi^+\pi^+\to\pi^+\pi^+}(s,t)\right]\Big|_{t=0} = \frac{1}{32 f_\pi^2} \tag{21}$$

is satisfied with a value for the pion decay constant f_π

$$f_\pi = 83.6 \text{ MeV}, \tag{22}$$

which is near to the experimental value $f_\pi \simeq 93$ MeV from the charged-pion lifetime.

Our interest in this paper is to examine whether or not the Adler-Weisberger sum rule for $K\bar{K}$ scattering is also safisfied by the present model

$$F^{KK}(s,t) = (\text{amplitude (7)}) + (S(975) \text{ and } \delta(980) \text{ exchange terms}) \tag{23}$$

with input of (17), (18), (15) and (19). We note that by the assumption for the $1^- - 0^- - 0^-$ vertices, we have

$$\lambda^{KK} = \lambda_1^{KK} = \lambda^{\pi\pi}/2. \tag{24}$$

As for α_0^1, it is taken to be

$$\alpha_0^1 = 0.08. \tag{25}$$

The Adler-Weisberger sum rule

$$\int_{4m_K^2}^{\infty} \frac{ds}{(s-m_K^2)^2} \operatorname{Im}\left[F^{K^-K^+\to K^-K^+}(s,t) - F^{K^+K^+\to K^+K^+}(s,t)\right]\Big|_{t=0} = \frac{1}{32 f_K^2}$$

predicts, in conjunction with (21),

$$(f_K/f_\pi)^2 = 1.6. \tag{26}$$

The value of $(f_K/f_\pi)^2$ is consistent with experimental estimates.[13)] Therefore, the short-distance effects, that is, the $q\bar{q}q\bar{q}$ mesons are supposed to play an important role in low-energy phenomena.

In conclusion, the present model is promising to embody the Caldi-Pagels idea.

References

1) D. G. Caldi and H. Pagels, Phys. Rev. **D14** (1976), 809.
2) M. Gell-Mann, R. Oakes and B. Renner, Phys. Rev. **175** (1968), 2195.
3) For reviews, see A. Pais, Rev. Mod. Phys. **38** (1966), 215.
4) T. Ino, to be published.
See also, T. Ino, Mem. Fac. Sci. Shimane Univ. **19** (1985), 57.
5) R. L. Jaffe, Phys. Rev. **D15** (1977), 267.
6) G. 't Hooft, Nucl. Phys. **B72** (1974), 461.
7) G. Veneziano, Nucl. Phys. **B117** (1976), 519.
8) L. Montanet, G. C. Rossi and G. Veneziano, Phys. Rep. **63C** (1980), 153.
9) G. Mennessier, Z. Phys. **C16** (1983), 241.
10) N. M. Cason, P. E. Cannata, A. E. Baumbaugh, J. M. Bishop, N. N. Biswas, L. J. Dauwe, V. P. Kenney, R. C. Ruchti, W. D. Shephard, and J. M. Watson, Phys. Rev. **D28** (1983), 1586.
11) Particle Data Group, Rev. Mod. Phys. **56** (1984), Sl.
12) T. Ino, Y. Munakata and J. Sakamoto, Prog. Theor. Phys. **73** (1985), 143.
13) N. Brene, M. Roos and A. Sirlin, Nucl. Phys. **B6** (1968), 255.