Mem. Fac. Sci., Shimane Univ.,
18, pp. 21–27 Dec. 20, 1984

Effects of a Factor Suggested by a Uniquely Determined π-π Dual Amplitude

Taketoshi INO
Department of Physics, Shimane University, Matsue 690
(Received September 3, 1984)

We discuss the forward $K^-p\to\bar{K}^0n$ scattering amplitude at high energies, which is suggested by the duality scheme based on the quark-model hadron spectrum. The predicted dip at $|t|\simeq 0.9$ (GeV/c)^2 seems to have a corespondence in experiments.

§ 1. Introduction

Lately, Munakata, Sakamoto and the author[1] have obtained a uniquely determined $\pi^-\pi^+\to\pi^-\pi^+$ dual amplitude. Starting with the most general $\pi^- - \pi^+$ Veneziano-type amplitude which has just the degree of freedoms to provide an arbitrary residue for each of parents and their daughters, and restricting it by the duality scheme[2] based on the quark-model hadron spectrum and the convergence condition at $s\to\infty$, they have reached the uniquely determined amplitude. The amplitude has good properties; absence of negative-norm states, compatibility with available experimental data of 2π decay of relevant low-lying resonances, and a consistency with the PCAC hypothesis. Moreover, the amplitude well reproduces[3] the inverse Mellin transform of the imaginary part of the phenomenological scattering amplitude[4]. The transform is what proposed by Froggatt, Nielsen and Petersen[5] to examine that can the π-π scattering amplitude be represented by any Veneziano model.

Another characteristic feature of the uniquely determined amplitude is a new factor in the amplitude. The new factor has been found first in the comparison[1] of the asymptotic form of the amplitude with that of the Lovelace-Shapiro-Veneziano model[6]. Such a new factor has been rediscovered in the inverse Mellin transform of the amplitude.[3] The integral of the transform covers the region of s from $(2m_\pi)^2$ to ∞, and so the new factor is considered to be a property of the amplitude at high energies (that is, at large s) as well as at $s\to\infty$. The asymptotic form of the amplitude reproduces[3] phenomenological ρ- and f-Regge contributions[4] much better than the Lovelace-Shapiro-Veneziano model and the Frampton amplitude[7]. Unfortunately, the experimental status of the π-π scattering is, however, not so good as to test the details of the new factor.

In this paper, we discuss the $K^-p\to\bar{K}^0n$ scattering, in order to make a test of the new factor.

As for the studies of the $K^-p\to\bar{K}^0n$ scattering by the Veneziano-type model, some authors[8,9] have presented some amplitudes with a few Veneziano-like terms.

In §2, taking the t-channel helicity amplitudes (A', B) for the description of the $K^-p\to\bar{K}^0n$ scattering, we discuss the A' amplitude which dominates in small $|t|$ regions of the high-energy scattering. In §3, the results are compared with experiments and some discussions are made.

§2. The t-channel helicity nonflip $\bar{K}$-N amplitude based on the duality scheme with the quark-model hadron spectrum

Let us describe the $\bar{K}-N$ scattering by the t-channel helicity amplitudes (A', B), where A' is defined by

$$A'=A+\frac{v}{1-\dfrac{t}{4m^2}}B \tag{2.1}$$

with

$$v=\frac{s-u}{4m}\,. \tag{2.2}$$

Here, A and B are the invariant amplitudes in the standard notation, and m is the nucleon mass. The asymptotic form of the amplitudes should be

$$A'\xrightarrow[t:\text{fixed}]{s\to\infty}\gamma_{A'}(t)\left(\frac{s}{s_0}\right)^{\alpha(t)}, \tag{2.3a}$$

$$B\xrightarrow[t:\text{fixed}]{s\to\infty}\gamma_B(t)\left(\frac{s}{s_0}\right)^{\alpha(t)-1}. \tag{2.3b}$$

Here, $\alpha(t)=\alpha' t+\alpha_0$ is the exchange-degenerate $\rho-\omega-A_2-f$ Regge trajectory. While, the differential cross section in terms of the amplitudes is

$$\frac{d\sigma}{dt}=\frac{1}{\pi s}\left(\frac{m}{4q}\right)^2\left(1-\frac{t}{4m^2}\right)\left\{|A'|^2+\frac{4sq^4\sin^2\theta}{(4m^2-t)^2}|B|^2\right\}, \tag{2.4}$$

where q and θ are the c. m. momentum and scattering angle in the s-channel respectively. Therefore, the differential cross section is dominated by A' in small $|t|$ region at high energies. Hereafter, we discuss anly A', as we are interested in the $|t|<1$ (GeV/c)^2 region of the high-energy $K^-p\to\bar{K}^0n$ scattering.

It is not so easy to construct Veneziano-type amplitudes for the meson-baryon scattering at a satisfactory level, as well known. But, as for the A' amplitude, the costruction will be able to be done by the use of the duality scheme with the quark-model hadron spectrum, because the t-channel pole structure of the amplitude is very similar to that of the $0^- - 0^-$ scattering. Moreover, it is more easy to obtain the asymptotic form of the amplitude.

In the subsection 2.1, we state the outline of Ref. 1) where the $\pi^- - \pi^+$ amplitude is constructed by the use of the duality scheme and the convergence condition at $s \to \infty$. In the subsection 2.2, we shall see that the asymptotic form of the A' amplitude on the same constraints is easily suggested from that of the $\pi^- - \pi^+$ amplitude in the subsection 2.1.

2.1 *The outline of the construction of the $\pi^- - \pi^+$ amplitude*

At the starting point, the most general $\pi^- - \pi^+$ Veneziano-type amplitude is taken

$$F(s, t) = \sum_{n=1}^{\infty} \sum_{k=n}^{2n} \lambda_k^{n,n} \frac{\Gamma(n-\alpha(s))\, \Gamma(n-\alpha(t))}{\Gamma(k-\alpha(s)-\alpha(t))} \,. \tag{2.5}$$

Here, $\lambda_k^{n,n}$ are arbitrary constants and $\alpha(s)$ is the exchange-degenerate $\rho - f$ Regge trajectory. Because of the crossing symmetry, the amplitude (2.5) has just the degree of freedoms to provide an arbitrary residue at each of parents in the s- and t-channels and their daughters, including odd daughters.

The duality scheme with the harmonic-oscillator spectrum of $SU(6) \otimes O(3)_L$ multiplets, by which we restrict $\lambda_k^{n,n}$, is

$$\sum_{a \in N} R_a^{(s)}(t)|_{t=m_{N'}^2} = \sum_{b \in N'} R_b^{(t)}(s)|_{s=m_N^2}, \quad N, N' = 0, 1, \ldots. \tag{2.6}$$

Here $N(N')$ is the s-(t-) channel resonance family. The resonance family N is defined as the group of the resonances with a fixed total number N of quanta of oscillator exitations. $R_a^{(s)}(t)$ $(R_b^{(t)}(s))$ is the residue of the scattering amplitude at the s-(t-) channel resonance a (b) in the narrow-width approximation. Because J (spin of resonances) takes alternate values for a fixed N in the harmonic oscillator model, the duality scheme does not involve odd daughters, that is, Eq. (2.6) implies also that the residues of odd daughters have to vanish. It is noted that as well known, the residue of the resonance with spin J in the s-(t-) channel is proportional to $P_J(\cos\theta_{s(t)})$, the Legendre polynomial of J degree.

Restricting the arbitrary constants in the amplitude (2.5) by the duality scheme (2.6), we have

$$\lambda_n^{n,n} = \frac{1+\beta}{(n-1)!(2n-1+\beta)(2n-3+\beta)} \{(n-2)(1-\beta)\lambda_1^{1,1} + (n-1)(3+\beta)\lambda_2^{2,2}\}, \cdots\cdots, \tag{2.7}$$

where

$$\beta \equiv 2 - 3\alpha_0 - 4\alpha' m_\pi^2. \tag{2.8}$$

The terms with coefficients $\lambda_n^{n,n}$ determines the asmptotic form of $F(s, t)$. Hereafter, we call such terms as main terms. The coefficients of the non-main terms are omitted from Eq. (2.7).

We impose also the convergence condition at $s\to\infty$ and at $|t|<1$ (GeV/c)^2 on the amplitude $F(s, t)$. Then, we have

$$\lambda_2^{2,2}=-\frac{1-\beta}{3+\beta}\lambda_1^{1,1}. \tag{2.9}$$

And, the asymptotic form of the amplitude is found as

$$F(s,t)\xrightarrow[t:\text{fixed}]{s\to\infty}\lambda_1^{1,1}\Gamma(1-\alpha(t))e^{-i\pi\alpha(t)}(\alpha's)^{\alpha(t)}I(t), \tag{2.10}$$

where

$$I(t)=\frac{\Gamma\left(\frac{3+\beta}{2}\right)\Gamma(1+\alpha(t))}{\Gamma\left(\frac{1+\beta}{2}+\alpha(t)\right)}, \quad (3+\beta>0,\ 1+\alpha(t)>0). \tag{2.11}$$

We note that except for the $I(t)$ factor, Eq. (2.10) is just the asymptotic form of the Lovelace-Shapiro-Veneziano model

$$\lambda_1^{1,1}\frac{\Gamma(1-\alpha(s))\,\Gamma(1-\alpha(t))}{\Gamma(1-\alpha(s)-\alpha(t))} \tag{2.12}$$

and that of the Frampton amplitude which has an infinite number of terms, but has only one term as Eq. (2.12) as the main term. The $I(t)$ factor is the new one stated in §1.

2.2 *The A' $\bar{K}-N$ amplitude*

Now, we discuss the $A'K^-p\to\bar{K}^0n$ amplitude on the same conditions as in the subsection 2.1.

We start with

$$A'(s,t)=\sum_{n=1}^{\infty}\sum_{k=n}^{2n}\lambda_k^{n,n}\frac{\Gamma(n-\eta(s))\,\Gamma(n-\alpha(t))}{\Gamma(k-\eta(s)-\alpha(t))}+(\text{non-symmetric terms}). \tag{2.13}$$

Here, $\eta(s)=\alpha_Y(s)-1/2$, and $\alpha_Y(s)$ is the s-channel baryon Regge trajectory based on the quark-orbital baryon Regge trajectory[10] $\alpha_Y^L(s)$. We assume the exchange-degeneracy of the quark-orbital trajectory for the S(strangeness) $=-1$ components of '56' multiplets and that of '70' multiplets, and denote the exchange-degenerate $S=-1$ quark-orbital trajectory as $\alpha_Y^L(s)$. After all, $\eta(s)=\alpha's+\eta_0=(\alpha_Y(s)-1/2=\alpha_Y^L(s)+1)=1$ represents the poles $Y_1^*(J^P=3/2^+)$, $\Sigma(1/2^+)$ and $\Lambda(1/2^+)$, $Y_1^*(3/2^+)$ as the parent and $\Sigma(1/2^+)$ and $\Lambda(1/2^+)$ as its daughters. (We note that when the isospin is considered, we must, of course, use the words parents and daughters in a different sense.)

The duality scheme to restrict the arbitrary constants $\lambda_k^{n,n}$ of the amplitude (2.13) is for the residues of the A' amplitude and is formally the same as Eq. (2.6).

After the duality scheme is imposed on the amplitude (2.13), it is supposed that the coefficients of the main terms are obtained by substituting $\bar{\beta}$ for β in Eq. (2.7), where

$$\bar{\beta}=2-3\bar{\alpha}_0-2\alpha'(m_K^2+m_N^2),\quad \bar{\alpha}_0=\frac{\alpha_0+\eta_0}{2}\,. \tag{2.14}$$

The reason for the supposition is the following.

(A) The author[11] have discussed the $K^-K^0\to K^-K^0$ amplitude on the same conditions as in the present work. The amplitude has the exchange-degenerate $\rho-A_2$ trajectory in the s-schannel and the exchange-degenerate $\phi-f'$ trajectory in the t-channel. It is found that the coefficients of the main terms are reached by substituting $(2-3\bar{\alpha}_0-4\alpha' m_K^2$, $\bar{\alpha}_0=(\alpha_0+\alpha_0^0)/2$, α_0^0: zero-intercept of $\phi-f'$ trajector) for β in Eq. (2.7), and the terms with coefficients proportional to $(\alpha_0-\alpha_0^0)$ appear as the non-main terms.

(B) The residue of the t-channel resonance of spin J of the A' amplitude is also proportional to $P_J(\cos\theta_t)$. The s-channel resonances which determines the coefficients of the main terms are only the parents, and the residue of the s-channel parent of J is proportional to

$$P_{(J-1/2)}(\cos\theta_s)+\frac{(2J-2)(E_J+m_N)(M_J-m_N)}{(2J-1)(E_J-m_N)(M_J+m_N)}P_{(J-3/2)}(\cos\theta_s)+\cdots\cdots, \tag{2.15}$$

where M_J is the mass of the parent and E_J is the c. m. nucleon energy at $s=M_J^2$. After all, the constraints on the coefficients of the main terms from the s-channel poles through the duality scheme is the same as those from the t-channel poles.

(C) The coefficients of the non-symmetric terms in the amplitude (2.13) are proportional to $(\alpha_0-\eta_0)$ (see (A)) or (m_N-m_K), as easily understood. The terms with the coefficients proportional ot (m_N-m_K) are not the main terms, that is, the terms vanish at $s\to\infty$, because the mass difference should disapper at $s\to\infty$.

The convergence condition on A' can be imposed in the same way as in the $\pi^- -\pi^+$ case. Finally, we conjecture that

$$A'(s,t)\xrightarrow[t:\text{fixed}]{s\to\infty}\lambda_{1,A'}^{1,1}\,\Gamma(1-\alpha(t))e^{-i\pi\alpha(t)}(\alpha' s)^{\alpha(t)}I_{A'}(t), \tag{2.16}$$

where

$$I_{A'}(t)=\frac{\Gamma\left(\frac{3+\bar{\beta}}{2}\right)\Gamma(1+\alpha(t))}{\Gamma\left(\frac{1+\bar{\beta}}{2}+\alpha(t)\right)},\ (3+\bar{\beta}>0,\ 1+\alpha(t)>0)\,. \tag{2.17}$$

The expression (2.16) implies that the new factor $I_{A'}(t)$ should be multiplied to the asymptotic form of the single-term Veneziano-type model

$$\lambda_{1,A'}^{1,1}\frac{\Gamma(1-\eta(s))\,\Gamma(1-\alpha(t))}{\Gamma(1-\eta(s)-\alpha(t))}, \tag{2.18}$$

or models with only the term (2.18) as the main term.

§ 3. Comparison with experiments and some discussions

It is possible to compare the amplitude with experiments, if there exist data at very high energies. But, we do not know data above $P_L = 12.3$ GeV/c. So, we take an existing model with only the term (2.18) as the main term, and multiply it by the factor (2.17) to compare the present model with experiments. As stated in §1, the factor (2.17) is the property of the present model at large s as well as at $s\to\infty$.

We choose the model presented by Berger and Fox[9] as the ground one. The model has been constructed on the basis of the foregoing models[8] and detailed phenomenological analyses. But, in order to take it as the ground model, a translation is needed, as it is written in terms of the trajectories in the Chew-Frautschi plot. Such a translation, in a simplified way, is to take η_0 in Eq. (2.14) as

$$\eta_0 = \frac{\eta_\Sigma^0 + \eta_\Lambda^0}{2} + \frac{1}{2}, \quad \eta_\Sigma^0 = -0.90, \quad \eta_\Lambda^0 = -1.24. \tag{2.19}$$

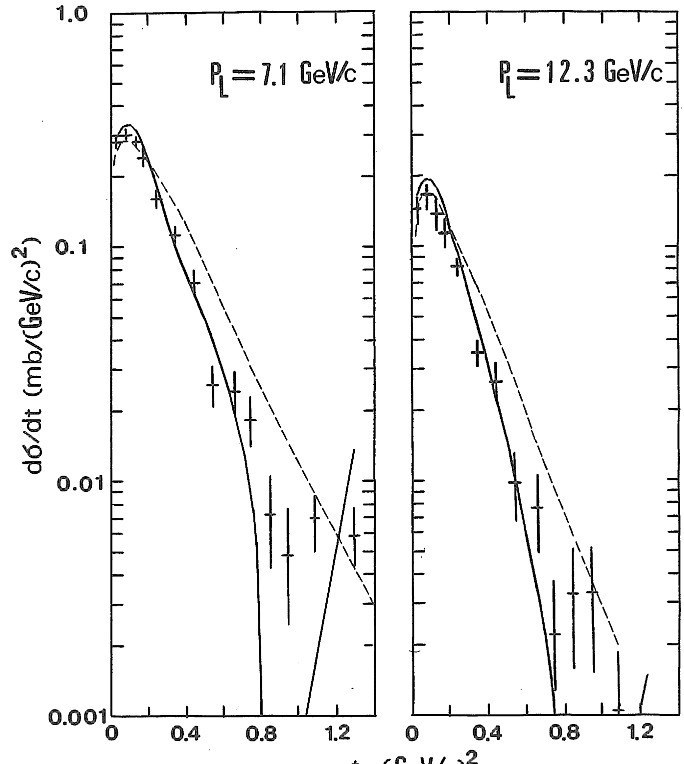

Fig. 1. Two predictions for the $d\sigma/dt(K^-p\to\bar{K}^0 n)$ and experimental data for the comparison. The dashed curve is the Berger-Fox model and the solid curve the present model.

In Fig. 1, two predictions for the $d\sigma/dt(K^- p \to \bar{K}^0 n)$, that is, the Berger-Fox model and the (Berger-Fox model) $\times |C \times I_{A'}(t)/I_{A'}(0)|^2$ model are shown together with experimental data.[12] Here, a constant C is introduced to retain the fit at the near $|t|=0$ region, and it is taken to be $|C|^2=1.40$.

As seen in Fig. 1, the dip at $|t| \simeq 0.9$ (GeV/c)^2 predicted by the new factor $I_{A'}(t)$ seems to have a correspondence in experiments.

We make some discussions about the results.

(I) When the B amplitude is also constructed on the same conditions as here, the fit to data will be much improved. We suppose that in the region of $|t| \gtrsim 1.2$ (GeV/c)^2, the $\rho - A_2$ Regge contribution calculated here will be weakened by the Regge cuts.

(II) The duality sheme used here is promising also between the baryon exchange and the baryon exchange. Thus, it will predict a dip in the backward scattering of the t-channel-exotic processes. In fact, the backward data of the $K^- p \to K^+ \Xi^-$ process[13] seems to have a dip.

References

1) T. Ino, Y. Munakata and J. Sakamoto, to be published in Prog. Theor. Phys. **73** (1985), No. 1.
2) T. Ino, Prog. Theor. Phys. **62** (1979), 1177; **61** (1979), 1863.
T. Ino, Prog. Theor. Phys. **71** (1984), 864.
3) T. Ino, Y. Munakata and J. Sakamoto, to be published.
4) C. D. Froggatt and J. L. Petersen, Nucl. Phys. **B129** (1977), 89.
5) C. D. Froggatt, H. B. Nielsen and J. L. Petersen, Phys. Rev. **D18** (1978), 4094.
6) C. Lovelace, Phys. Lett. **28B** (1968), 264; J. A. Shapiro, Phys. Rev. **179** (1969), 1345.
7) P. H. Frampton, Phys. Rev. **D7** (1973), 3077.
8) K. Igi and J. K. Storrow, Nuovo Cimento **62A** (1969), 972; T. Inami, ibid. **63A** (1969), 987; K. Pretzl and K. Igi, ibid. **63A** (1969), 609.
9) E. L. Berger and G. C. Fox, Phys. Rev. **188** (1969), 2120.
10) H. Nakkagawa, K. Yamawaki and S. Machida, Prog. Theor. Phys. **48** (1972), 939.
11) T. Ino, to be published.
12) P. Astbury, G. Brautti, G. Finocchiaro, A. Michelini, K. Terwilliger, D. Websdale, C. H. West and P. Zanella, Phys. Lett. **23** (1966), 396.
13) M. Mazzucato et. al., Nucl. Phys. ***B*178** (1981), 1.