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多元有色信号源FIR伝送路に対するブラインド収縮法

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A Blind Deflation Approach for MIMO-FIR Channels Driven by White but Higher-order Colored Source Signals

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Abstract

The present paper deals with the blind deconvolution problem of multiple-input multiple output finite impulse response (MIMO-FIR) channels driven by source signals which are white in the sense of second-order statistics and spatially higher-order uncorrelated but temporally higher-order colored signals. Our goal is to solve the blind deconvolution problem using a deflation approach. To this goal, we propose an algorithm obtained by modifying the super-exponential deflation algorithm (SEDA) proposed by Inouye and Tanebe [8] to the case of the blind deconvolution problem of MIMO-FIR channels driven by the source signals which possess higher-order auto-correlations, which is referred to as a modified super-exponential deflation algorithm (MSEDA). Using the MSEDA, the colored source signals which are the inputs of the MIMO-FIR channels are extracted from the outputs of the MIMO-FIR channels one by one. To show the validity of the proposed approach, some simulation results are presented.

1 Introduction

The blind deconvolution problem consists of extracting source signals from their convolutive mixtures observed by sensors without knowledge about the source signals and the transfer functions (transmission channels) between the sources and the sensors. Almost all of the proposed methods for solving the blind deconvolution problem have been developed under the assumption that the source signals are temporally independent and identically distributed (i.i.d.) and spatially independent (e.g., [2, 4, 8, 13, 17, 20]. However, in some applications, the i.i.d. assumption for the source signals becomes very strong. As an example, in digital communications, the information bearing sequences are coded in order to reduce noise corruptions and channel distortions. These codes implicitly are not mutually independent among sequences and hence it is unlikely that they are i.i.d. signals. On the other hand, these coded sequences are interleaved to avoid burst errors when the codes are transmitted [15]. These interleaved sequences are usually considered to be uncorrelated. To solve the blind deconvolution problem for such an application, therefore, one must assume that the source signals have a weaker condition than the i.i.d. condition, for example, the source signals are temporally second-order uncorrelated but higher-order correlated [11, 18].

The present paper deals with the blind deconvolution problem of MIMO-FIR channels driven by source signals which are temporally higher-order colored signals (but temporally second-order white and spatially second- and higher-order uncorrelated signals). This condition for the source signals is weaker than the i.i.d. condition. To solve the blind deconvolution problem, we consider a deflation approach. The conventional algorithms based on deflation approaches have been used to achieve the blind deconvolution under the assumption that source signals are i.i.d. and spatially independent signals (e.g., [8, 20]). Therefore, it cannot be seen whether deflation approaches can be applied to the MIMO-FIR channels in the case that the sources are higher-order colored signals. It has been shown by Simon et al. [18] that a deflation approach can be applied to multiple-input multiple-output infinite impulse response (MIMO-IIR) channels in the case that source signals are colored signals (but white signals in the sense of second-order statistics). Their proposed algorithm, however, does not work so as to extract an original colored signal but works so as to extract an i.i.d. signal from the outputs of the MIMO-IIR channels. This means that if the colored source signals are generated by linearly filtering i.i.d. signals (see Section 2), the original i.i.d. signal can be recovered by the algorithm. Then if the linear filters can be estimated, the colored source signals can be acquired. However, if the colored source signals are generated by non-linearly filtering i.i.d. signals (see Section 2), it may be impossible by the algorithm to recover the original i.i.d. signal. This means that it is very difficult to obtain the colored source signals from the signals extracted by the algorithm. Such an example is shown in Section IV. Our proposed deflation algorithm can be also applied to the above two types of colored source signals. This means that our proposed deflation approach can be used to recover the colored source signals which are generated by linearly or non-linearly filtering i.i.d. signals. Such an example is shown in Section IV.

The proposed algorithm is a modification of the super-exponential deflation algorithm (SEDA) proposed by Inouye and Tanebe [8], which is referred to as a *modified super-exponential deflation algorithm* (MSEDA). The MSEDA is used to extract one by one the colored source signals which are the inputs of the MIMO-FIR channels. It consists of n repeats of deflations, where n is the number of the sources. In each repeat of deflations in the algorithm, at first, a *cascaded integrator-comb* (CIC)-filtered source signal is extracted from the outputs of the MIMO-FIR channels. Next, the colored source signal corresponding to the extracted filtered source signal is recovered by making the extracted filtered source signal white in the sense of second-order statistics. Finally, the contributions of the extracted colored source signal to the outputs are removed from the outputs of the MIMO-FIR channels. Simulation examples are presented to illustrate the performance of the proposed approach.

2 Problem formulation

Throughout the present paper, let us consider the following MIMO-FIR system:

$$\boldsymbol{x}(t) = \sum_{k=0}^{K} \boldsymbol{H}^{(k)} \boldsymbol{s}(t-k), \tag{1}$$

where $\boldsymbol{x}(t)$ represents an m-column output vector called the *observed signal*, $\boldsymbol{s}(t)$ represents an n-column input vector called the *source signal*, $\{\boldsymbol{H}^{(k)}\}$ is an $m \times n$ matrix sequence representing the impulse response of the transmission channel, and the number K denotes its order. It can be written in scalar form as

$$x_i(t) = \sum_{j=1}^{m} \sum_{k=0}^{K} h_{ij}^{(k)} s_j(t-k), \quad j = 1, \cdots, m. \tag{2}$$

Equation (1) can be written as

$$\boldsymbol{x}(t) = \boldsymbol{H}(z)\boldsymbol{s}(t), \tag{3}$$

where $\boldsymbol{H}(z) = [h_{ij}(z)]$ is the z-transform of the impulse response $\{\boldsymbol{H}^{(k)}\}$, that is, $\boldsymbol{H}(z) = \sum_{k=0}^{K} \boldsymbol{H}^{(k)} z^k$. Note that the notation z is used instead of the commonly used z^{-1} in the z-transform.

Our objective is to extract the source signals $s_i(t)$ $(i = 1, \cdots, n)$ from the observed signals one by one, using a deflation approach. To this end, the following multi-input single-output (MISO)-FIR system called a *filter* which is driven by the observed signals is used:

$$y_l(t) = \sum_{i=1}^{m} \sum_{k=0}^{L} w_{li}^{(k)} x_i(t-k) = \sum_{k=0}^{L} \boldsymbol{w}_l^{(k)T} \boldsymbol{x}(t-k), \tag{4}$$

where $y_l(t)$ is the output signal of the filter, $\{\boldsymbol{w}_l^{(k)}\} = [\{w_{l1}^{(k)}\}, \cdots, \{w_{lm}^{(k)}\}]^T$ is an m-column vector sequence. Equation (4) can be written as

$$y_l(t) = \sum_{i=1}^{m} w_{li}(z) x_i(t) = \boldsymbol{w}_l(z)^T \boldsymbol{x}(t), \tag{5}$$

where $w_{li}(z)$ is the z-transform of the impulse response $\{w_{li}^{(k)}\}$, that is, $w_{li}(z) = \sum_{k=0}^{L} w_{li}^{(k)} z^k$, and $\boldsymbol{w}_l(z)$ is an m-column vector whose elements are $w_{li}(z)$ $(i = 1, \cdots, m)$. Note that the number l corresponds to the repeat number of deflations, which means the order of extracted sources, and the number L is the order of the filter. Therefore, the filters $\boldsymbol{w}_l(z)$ $(l = 1, \cdots, n)$ are used for extracting the source signals $s_i(t)$ $(i = 1, \cdots, n)$ from the observed signals one by one, using a deflation approach.

Here, all variables can be complex-valued (this is required for such an application using quadrature amplitude modulation (QAM) signals [15]).

Using (3) and (5), the input-output relation of the composite system can be written as

$$\begin{aligned} y_l(t) &= \sum_{i=0}^{m} \sum_{j=0}^{n} w_{li}(z) h_{ij}(z) s_j(t) \\ &= \sum_{j=0}^{n} g_{lj}(z) s_j(t) = \boldsymbol{g}_l(z)^T \boldsymbol{s}(t) \end{aligned} \tag{6}$$

where

$$g_{lj}(z) := \sum_{i=0}^{m} w_{li}(z) h_{ij}(z) = \sum_{k=0}^{K+L} g_{lj}^{(k)} z^k, \tag{7}$$

and $\boldsymbol{g}_l(z)$ is an n-column vector whose elements are $g_{lj}(z)$ $(j = 1, \cdots, n)$. The composite system of the two systems is illustrated in Figure 1.

If all the $g_{lj}(z)$'s $(j = 1 \cdots n)$ except for $j = j_0$ are equal to zero, (6) becomes $y_l(t) = g_{lj_0}(z) s_{j_0}(t)$. This means that a filtered source signal is acquired. Moreover, if $g_{lj_0}(z) = z^k$, then (6) becomes $y_l(t) =$

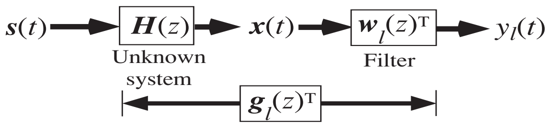

Figure 1: The composite system of an unknown system and a filter.

$s_{j_0}(t-k)$, that is, the j_0th source signal is acquired. In the present paper, for each repeat number $l = 1, \cdots, n$, at first, we consider finding a filtered version of source $s_{p_l}(t)$ as

$$y_l(t) = g_{lp_l}(z) s_{p_l}(t), \qquad l = 1, 2, \cdots, n \tag{8}$$

where $\{p_1, p_2, \cdots, p_n\}$ is a permutation of $\{1, 2, \cdots, n\}$, and then, by using the output $y_l(t)$, the source signal $s_{p_l}(t-k)$ is found. Here for $g_{lp_l}(z)$ in (8) we choose the transfer function of a CIC filter, that is,

$$g_{lp_l}(z) = \mathrm{g}_{l_0}(1 + z + \cdots + z^M), \tag{9}$$

where g_{l_0} is a non-zero complex constant and the number M is the order of $g_{lp_l}(z)$, that is, $M = K + L$. The choice of the transfer function is related to the proposed algorithm introduced in Section III. Under the following assumptions, we consider extracting n CIC-filtered source signals from the observed signals.

$A1$) The transfer function $\boldsymbol{H}(z)$ in (3) is *irreducible* [9], that is, $rank\ \boldsymbol{H}(z) = n$ for any $z \in C$ (this implies that the unknown system has less inputs than outputs, that is, $n \leq m$).

$A2$) The input sequence $\{\boldsymbol{s}(t)\}$ is a zero-mean stationary vector process whose component processes $\{s_i(t)\}$ $(i = 1, \cdots, n)$ are temporally second-order white and spatially second- and higher-order uncorrelated. At most, one component of $\{\boldsymbol{s}(\mathrm{t})\}$ can be Gaussian, and all the others should be non-Gaussian with unit variance and nonzero Γ_i $(< \infty)$, where Γ_i is the sum of all the higher-order auto-cumulants of the ith component signal (see (13)).

Under the assumption $(A1)$, we can show that there exists a filter $w_{li}(z)$ satisfying (9), because $\boldsymbol{H}(z)$ has a causal left inverse. As for $(A2)$, two examples of a component process $\{s_i(t)\}$ satisfying $A2$) are presented below: The one is a source signal generated by a linear filter and the other is a source generated by a non-linear filter.

1) A source generated by a linear filter: Let us consider a real-valued linear random process $\{x(t)\}$ generated by $x(t) = h(z)w(t)$, where $\{w(t)\}$ is a real-valued i.i.d. random process with variance $\sigma_w^2 \neq 0$ and kurtosis $\kappa_w \neq 0$, and $h(z)$ is an all-pass transfer function with real coefficients, that is, $|h(e^{-j\omega})| = 1$ for any real ω. Here, $h(z)$ may be a transfer function representing an IIR system. Then the second-order spectrum $s_x(\omega)$ is given by $s_x(\omega) = |h(e^{-j\omega})|^2 \sigma_w^2 = \sigma_w^2$, and the fourth-order spectrum $s_x(\omega_1, \omega_2, \omega_3)$ is given by $s_x(\omega_1, \omega_2, \omega_3) = h(e^{-j\omega_1}) h(e^{-j\omega_2}) h(e^{-j\omega_3}) h(e^{j(\omega_1+\omega_2+\omega_3)}) \kappa_w$, which is not constant except when $h(z)$ is monomial. This means that the random process $\{x(t)\}$ is white in the sense of second-order statistics but colored in the fourth-order sense [11].

2) A source generated by a non-linear filter: Let us consider a complex-valued process $\{x(t)\}$ generated by $x(t+1) = j\alpha(t+1)x(t)$, $t = 0, 1, 2, \cdots$, which is called a *pseudo-quaternary phase-shift keying* (PQPSK) signal [5]. Suppose that the initial value $x(0)$ of $x(t)$ is given by choosing one of values $1, -1, j, -j$ with probability 1/4, and α_t takes 1 or -1 with probability 1/2. Then the mean value $E[x(t)]$ of $x(t)$ becomes zero, and it is clear that $E[|x(t)|^2] = 1$, $E[x(t)^2] = 0$, $E[x(t)x(t-\tau)] = 0$, and $E[x(t)x(t-\tau)^*] = 0$. This means that the process $\{x(t)\}$ is white in the sense of second-order statistics. Moreover, it can be seen that the conditional expectation $E[x(t)^2 | x(t-1)]$ $(= -1$ or $1)$ is not equal to $E[x(t)^2]$ $(= 0)$. This means that the process $\{x(t)\}$ is colored in the higher-order sense.

The other examples of sources generated by non-linear filters are shown in [3]. We shall see some computer simulation results of the blind deconvolution for the both kinds of sources in Section IV. As for finding $s_{p_l}(t-k)$ using $y_l(t)$, we will show an algorithm in Section III B

3 A Blind Deflation Approach

3.1 An iterative procedure for a repeat of deflations

To generate a CIC-filtered source signal, a super-exponential deflation algorithm (SEDA) based on Inouye and Tanebe [8] is considered. In this subsection, we explain how to modify the SEDA proposed by Inouye and Tanebe to the case where the elements of the input signals $s_i(t)$ possess various higher-order auto-cumulants. Inouye and Tanebe proposed an SEDA of adjusting the elements $g_{lj}{}^{(k)}$ $(j = 1, \cdots, n)$ by the following two-step iterative procedure, in which source signals are assumed to be i.i.d. signals;

$$g_{lj}{}^{(k)[1]} = \frac{\rho_j}{\sigma_j{}^2} (g_{lj}{}^{(k)})^p (g_{lj}{}^{(k)*})^r, \tag{10}$$

$$g_{lj}{}^{(k)[2]} = \frac{g_{lj}{}^{(k)[1]}}{\sqrt{\sum_{j=1}^{n} \sum_l \sigma_j{}^2 |g_{lj}{}^{(l)[1]}|^2}}, \tag{11}$$

where $(\cdot)^{[1]}$, $(\cdot)^{[2]}$ stand for the result of the first step and the result of the second step per iteration, p and r are nonnegative integers such that $p + r \geq 2$, ρ_j denotes the $(p+r+1)$st cumulant of $s_j(t)$, that is,

$$\rho_j = \mathrm{Cum}\{\underbrace{s_j(t), s_j(t), \cdots, s_j(t)}_{p}, \underbrace{s_j(t)^*, \cdots, s_j(t)^*}_{r+1}\}, \tag{12}$$

and ${\sigma_j}^2$ denotes the variance of $s_j(t)$.

In the present paper, since the source signals $s_j(t)$ $(j = 1, \cdots, n)$ are assumed to possess non-zero various higher-order auto-cumulants, that is, the source signals are not *i.i.d.* signals, then instead of ρ_i in (12), we consider the sum of all the higher-order auto-cumulants of $s_j(t)$, which is denoted by Γ_j, that is,

$$\Gamma_j = \sum_{\underbrace{\tau_{p_1}, \cdots, \tau_{p_{p-1}}}_{p-1}, \underbrace{\tau_{r_1}, \cdots, \tau_{r_{r+1}}}_{r+1} \in Z} \mathrm{Cum}\{s_j(t), s_j(t-\tau_{p_1}), \cdots, s_j(t-\tau_{p_{p-1}}), s_j(t-\tau_{r_1})^*, \cdots, s_j(t-\tau_{r_{r+1}})^*\} \quad (13)$$

This is our original idea in which the deflation approach can be applied to the blind deconvolution in the case where the elements of the input signals $s_i(t)$ possess various higher-order auto-cumulants. Then, the equation (10) can be modified to the following equation;

$$g_{lj}{}^{(k)[1]} = \frac{\Gamma_j}{{\sigma_j}^2}\Big(\sum_{m=0}^{M} g_{lj}{}^{(m)}\Big)^p\Big(\sum_{m=0}^{M} g_{lj}{}^{(m)*}\Big)^r, \quad j = 1, \cdots, n \quad (14)$$

where $M = K + L$. Therefore, the two-step procedure (14) and (11) becomes one cycle of iterations in the modified super-exponential deflation algorithm (MSEDA) mentioned in Section III B. It should be noted in (14) that the elements $g_{lj}{}^{(k)}$'s (where $k = 0, \cdots, M$) take an identical value for fixed l and j. Hence, a CIC filter like (9) can be obtained by using (14).

Using the similar way as in [8], one can easily prove that the following two-step iterative procedure with respect to $\tilde{\boldsymbol{w}}_l$ can be derived from (14) and (11) (the proof is found in the Appendix):

$$\tilde{\boldsymbol{w}}_l{}^{[1]} = \tilde{\boldsymbol{R}}^{\dagger}\tilde{\boldsymbol{D}}_l, \qquad l = 1, \cdots, m, \quad (15)$$

$$\tilde{\boldsymbol{w}}_l{}^{[2]} = \frac{\tilde{\boldsymbol{w}}_l{}^{[1]}}{\sqrt{\tilde{\boldsymbol{w}}_l{}^{[1]*T}\tilde{\boldsymbol{R}}\tilde{\boldsymbol{w}}_l{}^{[1]}}}, \qquad l = 1, \cdots, m, \quad (16)$$

where $\tilde{\boldsymbol{w}}_l$ is an $(L+1)m$-column vector consisting of the coefficients (corresponding to the lth output) of the filter defined by

$$\tilde{\boldsymbol{w}}_l := [\tilde{\boldsymbol{w}}_{l1}^T, \tilde{\boldsymbol{w}}_{l2}^T, \cdots, \tilde{\boldsymbol{w}}_{lm}^T]^T, \quad (17)$$

$$\tilde{\boldsymbol{w}}_{lj} := [w_{lj}{}^{(0)}, w_{lj}{}^{(1)}, \cdots, w_{lj}{}^{(L)}]^T, \quad (18)$$

$\dagger$ denotes the pseudo-inverse operation of a matrix, $\tilde{\boldsymbol{R}}$ is the $m \times m$ block matrix defined by

$$\tilde{\boldsymbol{R}} := \begin{bmatrix} \tilde{\boldsymbol{R}}_{11} & \tilde{\boldsymbol{R}}_{12} & \cdots & \tilde{\boldsymbol{R}}_{1m} \\ \tilde{\boldsymbol{R}}_{21} & \tilde{\boldsymbol{R}}_{22} & \cdots & \tilde{\boldsymbol{R}}_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\boldsymbol{R}}_{m1} & \tilde{\boldsymbol{R}}_{m2} & \cdots & \tilde{\boldsymbol{R}}_{mm} \end{bmatrix} \quad (19)$$

whose (i, j)th block element $\tilde{\boldsymbol{R}}_{ij}$ is the $(L+1) \times (L+1)$ matrix with the (i_1, j_1)th element $[\tilde{\boldsymbol{R}}_{ij}]_{i_1 j_1}$ defined by

$$[\tilde{\boldsymbol{R}}_{ij}]_{i_1 j_1} = \mathrm{Cum}\{x_j(t-j_1), x_i(t-i_1)^*\}, \quad i_1, j_1 = 0, \cdots, L, \quad (20)$$

and $\tilde{\boldsymbol{D}}_l$ is the n-block vector defined by

$$\tilde{\boldsymbol{D}}_l := [\boldsymbol{d}_{l1}^T, \boldsymbol{d}_{l2}^T, \cdots, \boldsymbol{d}_{lm}^T]^T \quad (21)$$

where the $\boldsymbol{d}_{lj}$th is the $(L+1)$-column vector with the j_1th element $[\boldsymbol{d}_{lj}]_{j_1}$ given by

$$[\boldsymbol{d}_{lj}]_{j_1} = \sum_{\underbrace{\tau_{p_1}, \ldots, \tau_{p_{p-1}}}_{p-1}, \underbrace{\tau_{r_1}, \ldots, \tau_{r_{r+1}}}_{r+1} \in Z} \mathrm{Cum}\{y_l(t), y_l(t-\tau_{p_1}), \cdots, y_l(t-\tau_{p_p}), y_l(t-\tau_{r_1})^*, \cdots, y_l(t-\tau_{r_r})^*, x_l(t-j_1-\tau_{r_{r+1}})^*\}, \quad j_1 = 0, \cdots, L. \quad (22)$$

In the case of $p = 2$ and $r = 1$, (22) becomes

$$[\boldsymbol{d}_{lj}]_{j_1} = \sum_{\tau_1, \tau_2, \tau_3 \in Z} \mathrm{Cum}\{y_l(t), y_l(t-\tau_2), y_l(t-\tau_1)^*, x_j(t-j_1-\tau_3)^*\}, \quad j_1 = 0, \cdots, L. \quad (23)$$

In Section IV, we confine ourselves to the case of $p = 2$ and $r = 1$. Then (14) and (11) in the case of $p = 2$ and $r = 1$ is referred to as the *fourth-order modified super-exponential deflation algorithm* (FOMSEDA). Note that if we take into account the high-order auto-correlations of the source signals, the parameters p and r in (22) corresponding to the properties of $s_i(t)$ can be adjusted. Equations (15) and (16) are the main two steps in the proposed algorithm.

Remark 1: In the case that the source signals $s_i(t)$ of the system (3) possess non-zero various higher-order auto-cumulants, that is, the input signals are not i.i.d. signals, (15) and (16) are not obtained from (10) and (11) any longer. Therefore, equations (14) and (11) must be taken into account in the higher-order colored sources case.

Our objective of finding the filter $g_{lp_l}(z)$ in (9) from the output $y_l(t)$ in (6) using (14) and (11) is guaranteed by Theorem 1 below. Before explaining the theorem, we introduce the following notation. For each j, we define the following $(M+1)$-column vector $\tilde{\boldsymbol{g}}_{lj}$ whose elements are the impulse response parameters of the filter $g_{lj}(z)$, that is,

$$\tilde{\boldsymbol{g}}_{lj} = [g_{lj}^{(0)}, g_{lj}^{(1)}, \cdots, g_{lj}^{(M)}]^T. \quad (24)$$

Moreover, let us denote the $(M+1)$-column vector whose elements all equal a complex constant g_{l_0} whose absolute value is $1/\sqrt{M+1}$ by $\hat{\boldsymbol{g}}$, that is,

$$\hat{\boldsymbol{g}} = [g_{l_0}, g_{l_0}, \cdots, g_{l_0}]^T. \quad (25)$$

Theorem: Let us consider the MISO system defined by (6). Infinite iterations of two steps (14) and (11) can yield an SISO cascade system such that the impulse response vectors $\tilde{\boldsymbol{g}}_{lj}$'s of (6) satisfy

$$\begin{array}{rclr} \tilde{\boldsymbol{g}}_{lj} & = & \hat{\boldsymbol{g}} \neq \boldsymbol{0} & \text{for some } j = j_0, \\ \tilde{\boldsymbol{g}}_{lj} & = & \boldsymbol{0} & \text{for all } j \neq j_0, \end{array} \quad (26)$$

where $j_0 = \arg\max_{j\in\{1,\cdots,n\}} |\Gamma_j||\sum_{k=0}^{M} g_{lj}{}^{(k)}(0)|$, and the impulse response sequence $\{g_{lj}^{(k)}(0)\}$ means the sequence of the initial values of $\{g_{lj}^{(k)}\}$, [where the argument j_0 is assumed to be unique by choosing appropriate values of $\{g_{lj}^{(k)}(0)\}$]. *Proof* From (14), choosing j_0 so that $\sum_{k=0}^{M} g_{lj_0}{}^{(k)}(i) \neq 0$, we obtain

$$\frac{|\sum_{k=0}^{M} g_{lj}{}^{(k)[1]}(i)|}{|\sum_{k=0}^{M} g_{lj_0}{}^{(k)[1]}(i)|} = \frac{|\Gamma_j|}{|\Gamma_{j_0}|}\left(\frac{|\sum_{k=0}^{M} g_{lj}{}^{(k)[1]}(i-1)|}{|\sum_{k=0}^{M} g_{lj_0}{}^{(k)[1]}(i-1)|}\right)^{p+r}, \quad (27)$$

where an integer i denotes the iteration time. Note that $|\sum_{k=0}^{M} g_{lj}{}^{(k)}(i)|/|\sum_{k=0}^{M} g_{lj_0}{}^{(k)}(i)|$ is not modified by the normalization of the second step. Therefore, it is possible to solve $|\sum_{k=0}^{M} g_{lj}{}^{(k)}(i)|/|\sum_{k=0}^{M} g_{lj_0}{}^{(k)}(i)|$ from the recursive formula (27), which yields

$$\frac{|\sum_{k=0}^{M} g_{lj}{}^{(k)[2]}(i)|}{|\sum_{k=0}^{M} g_{lj_0}{}^{(k)[2]}(i)|} = \frac{|\Gamma_{j_0}|^{\frac{1}{p+r-1}}}{|\Gamma_j|^{\frac{1}{p+q-1}}}\left(\frac{|\Gamma_j|^{\frac{1}{p+r-1}}}{|\Gamma_{j_0}|^{\frac{1}{p+q-1}}}\frac{|\sum_{k=0}^{M} g_{lj}{}^{(k)[2]}(0)|}{|\sum_{k=0}^{M} g_{lj_0}{}^{(k)[2]}(0)|}\right)^{(p+q)^i} \quad (28)$$

$\forall i > 0$. For $j_0 = \max_j |\Gamma_j||\sum_{k=0}^{M} g_{lj}{}^{(k)[2]}(0)|$ (where j_0 is assumed to be unique), we see that all the other values $|\sum_{k=0}^{M} g_{lj}^{(k)[2]}(i)|$, $j \neq j_0$, quickly become small compared to $|\sum_{k=0}^{M} g_{lj_0}{}^{(k)[2]}(i)|$. Taking into account $\sum_{j=1}^{n}\sum_{k=0}^{M} |g_{lj}{}^{(k)[2]}(i)|^2 = 1$, this means that $|\sum_{k=0}^{M} g_{lj_0}{}^{(k)[2]}(i)| \neq 0$ and $|\sum_{k=0}^{M} g_{lj}{}^{(k)[2]}(i)| \to 0$ for all $j \neq j_0$. This implies that the algorithms (14) and (11) give the expected solution (26), taking into account that $|\sum_{k=0}^{M} g_{lj}{}^{(k)[2]}(i)| \to 0$ becomes $g_{lj}^{(k)}(i) = 0$ $(k = 0,\cdots,M)$ for all $j \neq j_0$ and $|g_{lj_0}^{(k)}(i)|$ $(k = 0,\cdots,M)$ become $1/\sqrt{M+1}$ from (11).

Remark 2: It can be seen from the proof of Theorem 1 that if $|\sum_{k=0}^{M} g_{lj}{}^{(k)}(0)|$ for all j are equal to zero, then $\{g_{lj}^{(k)}(1)\}$ which is the sequence of the values of $\{g_{lj}^{(k)}\}$ obtained by calculating (14) and (11) at the first cycle of the iterations becomes a sequence of zeros. This corresponds to a pathological case. In this case, we consider that by resetting the initial values of $g_{lj}{}^{(k)}(0)$ to be appropriate values, one of $|\sum_{k=0}^{M} g_{lj}{}^{(k)}(0)|$ $(j = 1,\cdots,n)$ becomes at least nonzero.

3.2 The modified super-exponential deflation algorithm (MSEDA)

The modified super-exponential deflation algorithm (MSEDA) is used to recover the n colored source signals one by one from the observed signals $\{x_i(t)\}$ $(i = 1,\cdots,m)$. The MSEDA can be summarized in the following steps:

Step 1: Set $l = 1$ (where l denotes the repeat number of deflations and corresponds to the order of a CIC-filtered input extracted).

Step 2: Choose random initial values $w_{lj}{}^{(k)}(0)$ of $w_{lj}{}^{(k)}$ $(k = 0,\cdots,M)$, and then calculate $\tilde{\boldsymbol{w}}_l(0)/\sqrt{\tilde{\boldsymbol{w}}_l(0)^{*T}\tilde{\boldsymbol{R}}\tilde{\boldsymbol{w}}_l(0)}$, where $\tilde{\boldsymbol{w}}_l(0)$ is the initial value of $\tilde{\boldsymbol{w}}_l$. Set i in $\tilde{\boldsymbol{w}}_l(i)$ to 0 (i denotes the iteration number).

Step 3: Calculate $\tilde{\boldsymbol{w}}_l(i)$ using (15) and (16). The expectation can be estimated using large samples of $y_l(t)$ and $x_j(t)$ $(j = 1,\cdots,m)$ (in our experimental studies, we used about 10,000 points for discrete time t)

Step 4: If $|\tilde{\boldsymbol{w}}_l{}^{*T}(i)\tilde{\boldsymbol{R}}\,\tilde{\boldsymbol{w}}_l(i-1)|$ is not close enough to 1, set $i = i + 1$ and go back to Step 3. Otherwise go to the next step.

Step 5: Consider an AR model

$$y_l(t) = -\sum_{k=1}^{M'} v_l{}^{(k)} y_l(t-k) + \beta u_l(t). \quad (29)$$

with sufficiently large order M' ($M' \gg$ M) and find the parameters β and $v_i{}^{(k)}$ by using the Yule-Walker equations and the Levinson algorithm [14].

Step 6: We compute a scaled and time-shifted version $u_l(t)$ of the input $s_{p_l}(t)$ by using the following equations:

$$y_l(t) = \sum_{j=1}^{m}\sum_{k} w_{lj}{}^{(k)} x_j(t-k),$$

$$u_l(t) = \beta^{-1}(y_l(t) + \sum_{k=1}^{M'} v_l{}^{(k)} y_l(t-k)) = v_l(z) y_l(t),$$

where $w_{lj}{}^{(k)}$ and $v_l{}^{(k)}$ are obtained by the above five steps, and $v_l(z) = \beta^{-1}(1 + \sum_{k=1}^{M'} v_l{}^{(k)} z^k)$.

Step 7: Estimate the scale and the time-shift of $h_{jp_l}{}^{(\tau)}$ by using the cross-correlation of the observed signals $x_j(t)$ and $u_l(t)$ as

$$\hat{h}_{jp_l}{}^{(\tau)} = E[x_j(t) u_l(t-\tau)^*],\ j = 1,2,\cdots,m. \quad (30)$$

Step 8: Estimate the contribution of $s_{p_l}(t)$ to the observed signals $x_j(t)$ $(j = 1,2,\cdots,m)$, that is, $\sum_\tau h_{jp_l}(\tau)\ s_{p_l}(t-\tau)$, using

$$\hat{x}_{jp_l}(t) = \sum_{\tau} \hat{h}_{jp_l}(\tau) u_l(t-\tau), \quad (31)$$

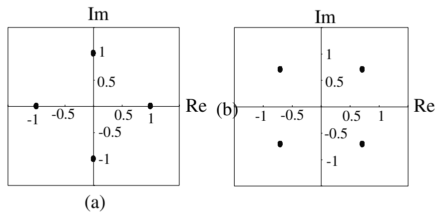

Figure 2: Signal constellations of the source signals: (a) source signal $s_1(t)$; (b) source signal $s_2(t)$.

Step 9: Remove the above contribution using the following equation:

$$x_j^{(l)}(t) = x_j(t) - \hat{x}_{jp_l}(t), \tag{32}$$

where $x_j^{(l)}(t)$ $(j = 1, \cdots, m)$ are the outputs of a linear unknown multichannel system with m outputs and $n-1$ inputs.
Step 10: If the superscript (l) of $x_j^{(l)}(t)$ is less than n, then set $x_j(t) = x_j^{(l)}(t)$ $(j = 1, \cdots, m)$, $l = l + 1$, and the procedures mentioned above (Step 2 to Step 9) are continued until $l = n$.

In Step 4, $|\tilde{\boldsymbol{w}}_l^{*T}(i)\tilde{\boldsymbol{R}}\ \tilde{\boldsymbol{w}}_l(i-1)| \approx 1$ means $\tilde{\boldsymbol{w}}_l(i) \approx \tilde{\boldsymbol{w}}_l(i-1)$ [7]. Step 5 is a procedure of recovering approximately a source signal $s_{p_l}(t)$ from the output $y_l(t)$ in (8) by making $y_l(t)$ white in the sense of second-order statistics, because the source signal $s_{p_l}(t)$ is white in the sense of second-order statistics but the obtained output $y_l(t)$ based on Theorem 1 is a colored signal. Therefore, in Step 5, one can acquire the source signal $s_{p_l}(t)$. The procedure from Step 7 to Step 10 are implemented for making it possible to extract the other source signals from the observed signals and the extracted source signals. On a justification of this extraction, for example, see [20].

4 Computer Simulations

To demonstrate the validity of the proposed algorithm, many computer simulations were conducted. Some results are shown in this section. We considered the following two-input and three-output FIR system.

$$\boldsymbol{H}(z) = \begin{bmatrix} 1.0 + 0.6z + 0.3z^2 & 0.6 + 0.5z - 0.2z^2 \\ 0.5 - 0.1z + 0.2z^2 & 0.3 + 0.4z + 0.5z^2 \\ 0.7 + 0.1z + 0.4z^2 & 0.1 + 0.2z + 0.1z^2 \end{bmatrix}. \tag{33}$$

It should be noted that $\boldsymbol{H}(z)$ satisfies $(A1)$. The observed signals $\boldsymbol{x}(t) = [x_1(t), x_2(t), x_3(t)]^T$ $(t = 0, 1, 2, \cdots)$ were calculated by (2). We used (23) (p = 2 and r = 1 in (22)). The values of τ_1, τ_2, and τ_3 in (23) were set to belong to the intervals [0,10], [0,10], and [0,10], respectively. The order L of the filter was overestimated by calculating $L = nK - 1$ proposed by Inouye and Liu [9]. We considered the case where the order K of $\boldsymbol{H}(z)$ given in (33) is unknown. Therefore, we assumed $K = 3$. The order of the filter was set to $L = 5$ calculating $L = 2 \times 3 - 1$. The order of the AR model was set to $M' = 30$. The expectations in (15) were estimated using 3000 data samples of $y_i(t)$ and $x_j(t)$.

Example 1: In this example, the source signals $\{s_i(t)\}$ $(i = 1, 2)$ were generated by using the following system:

$$\begin{bmatrix} s_1(t) \\ s_2(t) \end{bmatrix} = \begin{bmatrix} \frac{0.3+z}{1+0.3z} & 0 \\ 0 & \frac{0.7+z}{1+0.7z} \end{bmatrix} \begin{bmatrix} \nu_1(t) \\ \nu_2(t) \end{bmatrix}. \tag{34}$$

where $\{\nu_1(t)\}$ and $\{\nu_2(t)\}$ were non-Gaussian i.i.d. signals with zero mean and unit variance, but were independent with each other. Since the filters $(0.3 + z)/(1 + 0.3z)$ and $(0.7 + z)/(1 + 0.7z)$ in (34) are all-pass filters, the source signals $\{s_1(t)\}$ and $\{s_2(t)\}$ become temporally second-order white but temporally higher-order colored signals.

We examined the FOMSEDA using randomly chosen values as the initialization of $\tilde{\boldsymbol{w}}_i$. The algorithm was tested in 20 independent Monte Carlo runs. The algorithm converged to a desired solution and succeeded in recovering the first and the second sources. We confirmed the first and the second sources were recovered by Theorem 1 and the comments mentioned below the algorithm. As a measure of performance, we used the multichannel intersymbol interference (MISI) [8] defined by

$$\begin{aligned} \text{MISI} \;=\; & \sum_{l=1}^{n} \frac{|\sum_{j=1}^{n}\sum_{k=0}^{M+M'} |{g'_{lj}}^{(k)}|^2 - |{g'_{l\cdot}}^{(\cdot)}|^2_{max}|}{|{g'_{l\cdot}}^{(\cdot)}|^2_{max}} \\ & + \sum_{j=1}^{n} \frac{|\sum_{l=1}^{n}\sum_{k=0}^{M+M'} |{g'_{lj}}^{(k)}|^2 - |{g'_{\cdot j}}^{(\cdot)}|^2_{max}|}{|{g'_{\cdot j}}^{(\cdot)}|^2_{max}} \end{aligned} \tag{35}$$

where ${g'_{lj}}^{(k)}$ denote the parameters of the filter between $u_l(t)$ and $s_i(t)$ and $|{g'_{l\cdot}}^{(\cdot)}|^2_{max}$ and $|{g'_{\cdot j}}^{(\cdot)}|^2_{max}$are defined by

$$\begin{aligned} |{g'_{l\cdot}}^{(\cdot)}|^2_{max} &:= \max_{j,k} |{g'_{lj}}^{(k)}|^2, \\ |{g'_{\cdot j}}^{(\cdot)}|^2_{max} &:= \max_{l,k} |{g'_{lj}}^{(k)}|^2. \end{aligned}$$

In this experiment, we found that the average MISI is -9.2515 [dB].

Table 1: Average MISI values when Gaussian noise was added to the outputs of channels at various SNR's.

	MISI [dB]	SNR [dB]
Result 1	−8.1471	30
Result 2	−7.7281	25
Result 3	−5.8975	20
Result 4	−0.4192	15

We examined the effects of noise presented at channels. Two independent white Gaussian noises (with

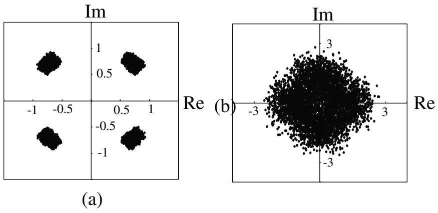

Figure 3: Signal constellations of equalizer outputs and filter outputs: (a) outputs obtained using the Simon et al. algorithm; (b) outputs obtained by applying outputs $\eta(t)$ to Step 5.

identical variance σ_w^2) were added to the three output channels at various SNR levels. The SNR is, for convenience, defined as SNR $:= 10log_{10}(\sigma_i^2/\sigma_w^2)$, where σ_i^2's are the variances of $s_i(t)$'s and are equal to 1. Table 1 shows the results (Result 1 through Result 4) of performance for the algorithm when the SNR level was taken to be 30[dB], ,25[dB], 20[dB], and 15[dB], respectively, for the algorithm.

Example 2: In this example, we show the results obtained by using the Simon et al. algorithm and our proposed algorithm (FOMSEDA) in the case that the source signals $\{s_i(t)\}$ $(i = 1, 2)$ were generated by

$$\begin{bmatrix} s_1(t+1) \\ s_2(t+1) \end{bmatrix} = \begin{bmatrix} j\alpha_1(t+1) & 0 \\ 0 & j\alpha_2(t+1) \end{bmatrix} \times \begin{bmatrix} s_1(t) \\ s_2(t) \end{bmatrix}, \quad t = 0, 1, \cdots, \quad (36)$$

where the initial values $s_1(0)$ and $s_2(0)$ were set to, respectively, with probability 1/4, one of $\{1, -1, j, -j\}$ and one of $\{\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}}, -\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}}, \frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}, -\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\}$, and $\alpha_i(t)$ $(i = 1, 2)$ took 1 or -1 with probability 1/2. The source signals are shown in Figure 2.

Figure 3 (a) shows the result obtained by applying the Simon et al. algorithm to the observed signals $x_i(t)$ $(i = 1, 2, 3)$ in Example 1. From Figure 2 and Figure 3(a), one may consider that the output $\eta(t)$ obtained by their algorithm is similar to the source signal $s_2(t)$. However, the statistical property of the output is different from those of the source signals $s_1(t)$ and $s_2(t)$, because the auto-correlation $E[\eta(t)\eta(t+1)^*]$ is not equal to zero (it is equal to $0.5172+j0.0091$), while $E[s_i(t)s_i(t+1)^*]$ $(i = 1, 2)$ are equal to zero. Figure 3(b) shows the result obtained by applying the output $\eta(t)$ to Step 5 and Step 6 in Section III B. One can see that one of the source signals is not acquired, even if the whitening is applied to the output $\eta(t)$. On the other hand, Figure 4(a) and Figure 4(b) show $y_1(t)$ (l = 1, that is, the first repeat of deflations) obtained using the FOMSEDA and $u_1(t)$ obtained applying $y_1(t)$ to Step 5 in Section III B. It can be shown that the output $u_1(t)$ is close to the source signal $s_1(t)$, because $v_1(z)g_{11}(z) \approx 0$ and $v_1(z)g_{12}(z) \approx 1$.

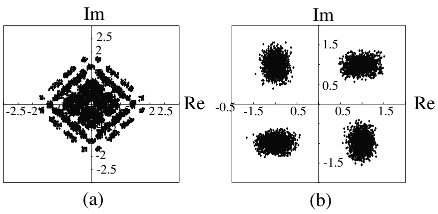

Figure 4: Signal constellations of equalizer outputs and filter outputs: (a) outputs $y_1(t)$; (b) outputs $u_1(t)$.

5 Conclusions

The present paper proposed an iterative algorithm for the blind deconvolution problem in the case of temporally second-order white and spatially second- and higher-order uncorrelated signals. The proposed algorithm, referred to as the modified super-exponential deflation algorithm (MSEDA), was a modification of the super-exponential deflation algorithm (SEDA) proposed by Inouye and Tanebe [8] to the case of the blind deconvolution problem of MIMO-FIR channels driven by higher-order colored source signals. The MSEDA was used to generate the colored source signals from their mixtures one by one.

Computer simulations were carried out to demonstrate the proposed approach using FOMSEDA. The results without noise have shown that the proposed algorithm can be used successfully to achieve the blind deconvolution. Moreover, from the computer simulations, it can also be confirmed that the proposed deflation algorithm will be globally convergent almost always (this result is similar to [8]). The results with noise have shown that the performance of the algorithm gradually becomes worse as the SNR level is low.

The authors have not yet carried out any theoretical analysis of the effects of measurement noise. This will be treated in our future work.

Appendix

Derivations of (15) and (16) from (14) and (11): Since the second step (11) is the same one as in [8], the proof of the derivation of (16) from (11) is omitted. To derive (15) from (14), the same strategy shown in [8] is used. Namely, we consider that $\tilde{\boldsymbol{w}}_l^{[1]}$'s are given by

$$\tilde{\boldsymbol{w}}_l^{[1]} := (\tilde{\boldsymbol{H}}^{T*}\tilde{\boldsymbol{\Lambda}}\tilde{\boldsymbol{H}})^{\dagger}\tilde{\boldsymbol{H}}^{T*}\tilde{\boldsymbol{\Lambda}}\tilde{\boldsymbol{f}}_l, \quad l = 1, 2, \cdots, m, \quad (37)$$

which are the solutions of the weighted squares problem

$$\min_{\tilde{\boldsymbol{w}}_l}(\tilde{\boldsymbol{H}}\tilde{\boldsymbol{w}}_l - \tilde{\boldsymbol{f}}_l)^{T*}\tilde{\boldsymbol{\Lambda}}(\tilde{\boldsymbol{H}}\tilde{\boldsymbol{w}}_l - \tilde{\boldsymbol{f}}_l), \quad (38)$$

where $\tilde{\boldsymbol{H}}$ is the $n \times m$ block matrix defined by

$$\tilde{\boldsymbol{H}} := \begin{bmatrix} \boldsymbol{H}_{11} & \boldsymbol{H}_{12} & \cdots & \boldsymbol{H}_{1m} \\ \boldsymbol{H}_{21} & \boldsymbol{H}_{22} & \cdots & \boldsymbol{H}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{H}_{n1} & \boldsymbol{H}_{n2} & \cdots & \boldsymbol{H}_{nm} \end{bmatrix} \quad (39)$$

whose (i,j)th block element $\boldsymbol{H}_{ij}$ is a $(K+L+1) \times (L+1)$ matrix with the (i_1, j_1)th element $[\boldsymbol{H}_{ij}]_{i_1 j_1}$ defined by $[\boldsymbol{H}_{ij}]_{i_1 j_1} := h_{ji}(i_1 - j_1)$, $i_1 = 0, \cdots, K+L$; $j_1 = 0, \cdots, L$. $\tilde{\boldsymbol{\Lambda}}$ is a diagonal matrix whose elements are positive, and $\tilde{\boldsymbol{f}}_l$ is an $(K+L+1)n$-column vector consisting of the coefficients defined by

$$\tilde{\boldsymbol{f}}_l := [\tilde{\boldsymbol{f}}_{l1}^T, \tilde{\boldsymbol{f}}_{l2}^T, \cdots, \tilde{\boldsymbol{f}}_{ln}^T]^T, \quad (40)$$

$$\tilde{\boldsymbol{f}}_{lj} := [f_{lj}{}^{(0)}, f_{lj}{}^{(1)}, \cdots, f_{lj}{}^{(K+L)}]^T, \quad (41)$$

$$f_{lj}{}^{(k)} := \frac{\gamma_j}{\sigma_j{}^2} (\sum_{m=0}^{M} g_{lj}{}^{(m)})^2 (\sum_{m=0}^{M} g_{lj}{}^{(m)*}). \quad (42)$$

Here, as for (42), we consider the case of (14) with $p = 2$ and $r = 1$, and γ_j is given by

$$\gamma_j = \sum_{\tau_1, \tau_2, \tau_3 \in Z} \mathrm{Cum}\{s_j(t), s_j(t - \tau_2), s_j(t - \tau_1)^*, s_j(t - \tau_3)^*\}. \quad (43)$$

Suppose we put $\tilde{\boldsymbol{\Lambda}} = \tilde{\boldsymbol{\Sigma}}$, where $\tilde{\boldsymbol{\Sigma}}$ is the $n \times n$ block diagonal matrix defined by

$$\tilde{\boldsymbol{\Sigma}} := \begin{bmatrix} \boldsymbol{\Sigma}_{11} & 0 & \cdots & 0 \\ 0 & \boldsymbol{\Sigma}_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \boldsymbol{\Sigma}_{nn} \end{bmatrix} \quad (44)$$

$$\boldsymbol{\Sigma}_{ii} := \sigma_i \boldsymbol{I} \qquad i = 1, 2, \cdots, n,$$

Here, $\boldsymbol{I}$ denotes the $(K+L+1) \times (K+L+1)$ identity matrix. Since it has been shown in [8] that the term $\tilde{\boldsymbol{H}}^{T*} \tilde{\boldsymbol{\Sigma}} \tilde{\boldsymbol{H}}$ can be obtained from (20), let us show that the term $\tilde{\boldsymbol{H}}^{T*} \tilde{\boldsymbol{\Sigma}} \tilde{\boldsymbol{f}}_l$ can be obtained from (23). From (2), (4), $A2$), and the properties of cumulants, the right-hand side of (23) can be modified to the following equation:

$$\sum_{\tau_1, \tau_2, \tau_3 \in Z} \mathrm{Cum}\{y_l(t), y_l(t - \tau_2), y_l(t - \tau_1)^*, x_{i_3}(t - j_1 - \tau_3)^*\}$$
$$= \sum_{j=1}^{n} \sum_{k} h_{i_3 j}^{(k - j_1)*} \sum_{\tau_1, \tau_2, \tau_3 \in Z} \mathrm{Cum}\{y_l(t), y_l(t - \tau_2), y_l(t - \tau_1)^*, s_j(t - k - \tau_3)^*\}, \quad (45)$$

where

$$\sum_{\tau_1, \tau_2, \tau_3 \in Z} \mathrm{Cum}\{y_l(t), y_l(t - \tau_2), y_l(t - \tau_1)^*, s_j(t - k - \tau_3)^*\}$$
$$= \sum_{j_1}^{n} \sum_{j_2}^{n} \sum_{j_3}^{n} \sum_{m_1} \sum_{m_2} \sum_{m_3} g_{lj_1}^{(m_1)} g_{lj_2}^{(m_2)} g_{lj_3}^{(m_3)*}$$
$$\times \sum_{\tau_1, \tau_2, \tau_3 \in Z} \mathrm{Cum}\{s_{j_1}(t - m_1), s_{j_2}(t - m_2 - \tau_2), s_{j_3}(t - m_3 - \tau_1)^*, s_j(t - k - \tau_3)^*\}$$
$$= \gamma_j (\sum_{m=0}^{M} g_{lj}{}^{(m)})^2 (\sum_{m=0}^{M} g_{lj}{}^{(m)*}). \quad (46)$$

Using (45) and (46) together with (40)-(42), we find

$$\sum_{\tau_1, \tau_2, \tau_3 \in Z} \mathrm{Cum}\{y_l(t), y_l(t - \tau_2), y_l(t - \tau_1)^*, x_{i_3}(t - j_1 - \tau_3)^*\}$$
$$= \sum_{j=1}^{n} \sum_{k} h_{i_3 j}^{(k - j_1)*} \sigma_j^2 f_{lj}^{(k)}$$
$$= \sum_{j=1}^{n} (\boldsymbol{H}_{j i_3}^{T*} \boldsymbol{\Sigma}_{jj} \tilde{\boldsymbol{f}}_{lj})_{j_1} \quad (47)$$

from which we obtain $\tilde{\boldsymbol{H}}^{T*} \tilde{\boldsymbol{\Sigma}} \tilde{\boldsymbol{f}}_l$. Therefore, the two-step iterative procedure (15) and (16) can be derived from (14) and (11).

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