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Muon and electron $g-2$ and the origin of the fermion mass hierarchy

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We present a model that gives a natural explanation to the charged lepton mass hierarchy and study the contributions to the electron and the muon $g-2$. In the model, we introduce lepton-flavor-dependent $U(1)_F$ symmetry and three additional Higgs doublets with $U(1)_F$ charges, to realize that each generation of charged leptons couples to one of the three additional Higgs doublets. The $U(1)_F$ symmetry is softly broken by $+1$ charges, and the smallness of the soft breaking naturally gives rise to the hierarchy of the Higgs vacuum expectation values, which then accounts for the charged lepton mass hierarchy. Since electron and muon couple to different scalar particles, each scalar contributes to the electron and the muon $g-2$ differently. We survey the space of parameters of the Higgs sector and find that there are sets of parameters that explain the muon $g-2$ discrepancy. On the other hand, we cannot find parameter sets that can explain the $g-2$ discrepancy within $2\,\sigma$. Here, the $U(1)_F$ symmetry suppresses charged lepton flavor violation.

Subject Index B40, B53, B59

1. Introduction

The Standard Model (SM) explains almost all experimental data, but there are several anomalies in low-energy observables. Among the anomalies, experimental and theoretical studies on the muon $g-2$ are extensive. Comparing the experimental value to its SM prediction, the current discrepancy of the muon $g-2$ is reported as [1–4]

$$\delta a_\mu = a_\mu^{\rm obs} - a_\mu^{\rm SM} = (27.4 \pm 7.3) \times 10^{-10}. \tag{1}$$

Recently, the discrepancy of the electron $g-2$ is also reported as [5–8]

$$\delta a_e = a_e^{\rm obs} - a_e^{\rm SM} = (-8.7 \pm 3.6) \times 10^{-13}. \tag{2}$$

Although the electron $g-2$ discrepancy is less than $3\,\sigma$, both discrepancies may be signs of physics beyond the SM.

In the effective theoretical approach, the contribution to the lepton $g-2$ is described by the chirality-breaking operator, $\sim \overline{\ell_L}\sigma_{\mu\nu}\ell_R F^{\mu\nu}$. If the chirality breaking in a new physics model is proportional simply to the lepton mass, as in the minimal supersymmetric Standard Model with minimal flavor violation (MFV), the contribution to the lepton $g-2$ is proportional to its mass, and there is a simple relation between the electron and muon $g-2$:

$$\frac{\delta a_e}{\delta a_\mu} \simeq \frac{m_e^2}{m_\mu^2} = 2.3 \times 10^{-5}. \tag{3}$$

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This relation does not hold experimentally in Eqs. (1) and (2); in particular, the sign is opposite. Hence, any new physics model that explains both discrepancies must include flavor violation beyond MFV in the interactions of muon and electron. Various sources of flavor violation beyond MFV have been considered to address the discrepancies [9–17].

A stringent restriction [11] on the model building attempts comes from the absence of the $\mu \to e\gamma$ decay. In the effective theory point of view, this indicates that two operators $\overline{\mu_L}\sigma_{\mu\nu}\mu_R F^{\mu\nu}$ and $\overline{e_L}\sigma_{\mu\nu}e_R F^{\mu\nu}$ appear in a way that breaks MFV, but the operators $\overline{e_L}\sigma_{\mu\nu}\mu_R F^{\mu\nu}$ and $\overline{\mu_L}\sigma_{\mu\nu}e_R F^{\mu\nu}$ are forbidden. The above situation is realized naturally by assuming a muon-specific and/or electron-specific $U(1)$ symmetry. One possibility is that this $U(1)$ symmetry is the same as the accidental muon-number and electron-number symmetries of the SM, namely, the new physics sector respects the muon-number and electron-number symmetries so that $\overline{\mu_L}\sigma_{\mu\nu}\mu_R F^{\mu\nu}$ and $\overline{e_L}\sigma_{\mu\nu}e_R F^{\mu\nu}$ are generated with arbitrary strengths, but $\overline{e_L}\sigma_{\mu\nu}\mu_R F^{\mu\nu}$ and $\overline{\mu_L}\sigma_{\mu\nu}e_R F^{\mu\nu}$ are not generated. Another possibility is that only μ_R or e_R is charged under a new (anomalous) $U(1)$ symmetry, and there exists a new Higgs field charged under it that couples exclusively to μ_R or e_R.

In this paper we construct a model along the second possibility to explain the electron and muon $g-2$ discrepancies without invoking large charged lepton flavor violation. We go one step further and connect the new $U(1)$ symmetry to the origin of the fermion mass hierarchy.

In our model (which we name the "lepton-flavored Higgs model"), we introduce a new $U(1)_F$ symmetry under which τ_R, μ_R, e_R are charged by $-1, -2, -3$, respectively, and introduce three additional Higgs doublets with $U(1)_F$ charges $+1, +2, +3$. Due to the $U(1)_F$ symmetry, each generation of charged leptons couples to one of the three Higgs doublets. We assume that the $U(1)_F$ symmetry is softly broken by a small amount by $+1$ charges, which naturally generates a hierarchy among the vacuum expectation values (VEVs) of the Higgs doublets. We consider that this hierarchy of VEVs accounts for the charged lepton mass hierarchy, and that the charged lepton Yukawa couplings are all $O(1)$. A notable feature of the above setup is that, since the charged and heavy neutral scalars in the Higgs sector couple differently to the electron and muon, there is little correlation between the new scalar contributions to the electron and muon $g-2$.

We survey the space of parameters of the Higgs sector and find that there are sets of parameters that explain the muon $g-2$ discrepancy within the $1\,\sigma$ region. Unfortunately, we cannot find any parameter set that explains the electron $g-2$ discrepancy within the $2\,\sigma$ region.

This paper is organized as follows. In Sect. 2 we explain our lepton-flavored Higgs model. In Sect. 3, we conduct a numerical search of parameters of the Higgs sector. Section 4 summarizes the paper.

2. Lepton-flavored Higgs model

The lepton-flavored Higgs model includes four Higgs doublets (H_1, H_2, H_3, H_0) in addition to the SM leptons. The fields are charged under the SM $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group and a new anomalous $U(1)_F$ symmetry as in Table 1. Note that since only the right-handed charged leptons have $U(1)_F$ charges, the model does not restrict the Weinberg operator for the tiny neutrino mass.

The $U(1)_F$ symmetry is assumed to be softly broken by $+1$ charges. The soft breaking by $+1$ charges can be realized by introducing an SM gauge-singlet scalar S with $U(1)_F$ charge $1/2$ and demanding that renormalizable interactions preserve the $U(1)_F$. Then, one introduces a VEV of S to break the $U(1)_F$. The resulting Nambu–Goldstone boson gains mass from non-renormalizable terms that explicitly violate the $U(1)_F$.

Table 1. The fields and their charge assignments; α labels the three generations.

Field	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	Anomalous $U(1)_F$
H_1	**1**	**2**	$+1/2$	$+3$
H_2	**1**	**2**	$+1/2$	$+2$
H_3	**1**	**2**	$+1/2$	$+1$
H_0	**1**	**2**	$+1/2$	0
ℓ_L^α $(\alpha = 1,2,3)$	**1**	**2**	$-1/2$	0
e_R	**1**	**1**	-1	-3
μ_R	**1**	**1**	-1	-2
τ_R	**1**	**1**	-1	-1

The Yukawa couplings for the SM leptons, which respect the $U(1)_F$ symmetry, are given by

$$-\mathcal{L}_{\text{Yukawa}} = y_1\ \bar{\ell}_L^1 H_1 e_R + y_2\ \bar{\ell}_L^2 H_2 \mu_R + y_3\ \bar{\ell}_L^3 H_3 \tau_R + \text{H.c.} \tag{4}$$

The Higgs potential, where the $U(1)_F$ symmetry is softly broken by $+1$ charges, is given by

$$\begin{aligned}
-\mathcal{L}_{\text{Higgs}} = {} & m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 + m_3^2 H_3^\dagger H_3 + m_0^2 H_0^\dagger H_0 \\
& - \mu_{12}^2 (H_1^\dagger H_2 + H_2^\dagger H_1) - \mu_{23}^2 (H_2^\dagger H_3 + H_3^\dagger H_2) - \mu_{03}^2 (H_0^\dagger H_3 + H_3^\dagger H_0) \\
& + \lambda_1 (H_1^\dagger H_1)^2 + \lambda_2 (H_2^\dagger H_2)^2 + \lambda_3 (H_3^\dagger H_3)^2 + \lambda_0 (H_0^\dagger H_0)^2 \\
& + \sum_{i,j=0,1,2,3;i>j} \lambda_{ij} (H_i^\dagger H_i)(H_j^\dagger H_j) + \sum_{i,j=0,1,2,3;i>j} \rho_{ij} (H_i^\dagger H_j)(H_j^\dagger H_i) \\
& + \kappa_1 \big((H_0^\dagger H_2)(H_1^\dagger H_3) + (H_2^\dagger H_0)(H_3^\dagger H_1) \big) \\
& + \kappa_2 \big((H_0^\dagger H_3)(H_1^\dagger H_2) + (H_3^\dagger H_0)(H_3^\dagger H_2) \big).
\end{aligned} \tag{5}$$

For simplicity, we assume that the Higgs potential is CP invariant and all the parameters are real.

Now we make a crucial assumption on the Higgs potential. We assume

$$m_0^2 < 0, \quad m_1^2 > 0, \quad m_2^2 > 0, \quad m_3^2 > 0. \tag{6}$$

H_0 develops a VEV, $\langle H_0 \rangle = v/\sqrt{2}$, which is estimated to be

$$v \simeq \sqrt{\frac{-m_0^2}{\lambda_0}}. \tag{7}$$

The VEV of H_0 induces a VEV of H_3 through the term $\mu_{03}^2 (H_0^\dagger H_3 + H_3^\dagger H_0)$. The latter induces a VEV of H_2 through the term $\mu_{23}^2 (H_2^\dagger H_3 + H_3^\dagger H_2)$, which then induces a VEV of H_1 through the term $\mu_{12}^2 (H_1^\dagger H_2 + H_2^\dagger H_1)$. Consequently, writing the VEVs as $\langle H_3 \rangle = v_3/\sqrt{2}$, $\langle H_2 \rangle = v_2/\sqrt{2}$, and $\langle H_1 \rangle = v_1/\sqrt{2}$, we get

$$v_3 \simeq \frac{\mu_{03}^2}{m_3^2} v, \qquad v_2 \simeq \frac{\mu_{23}^2}{m_2^2} v_3, \qquad v_1 \simeq \frac{\mu_{12}^2}{m_1^2} v_2. \tag{8}$$

We arrange the masses such that (m_t denotes the top quark mass)

$$\frac{\mu_{03}^2}{m_3^2} \sim \frac{m_\tau}{m_t} \ll 1, \quad \frac{\mu_{23}^2}{m_2^2} \sim \frac{m_\mu}{m_\tau} \ll 1, \quad \frac{\mu_{12}^2}{m_1^2} \sim \frac{m_e}{m_\mu} \ll 1. \tag{9}$$

The arrangement of Eq. (9) is natural because $\mu_{03}^2, \mu_{23}^2, \mu_{12}^2$ break the $U(1)_F$ symmetry while m_3^2, m_2^2, m_1^2 preserve it. It follows from Eq. (9) that the lepton Yukawa couplings are all $O(1)$,

$$y_1 \sim y_2 \sim y_3 \sim 1. \tag{10}$$

We thus naturally explain the hierarchy of the charged lepton masses in terms of the small soft breaking of the $U(1)_F$ symmetry.

One can implement a similar structure in the quark sector, where we have five more Higgs doublets whose VEVs are on the order of $(m_b/m_t)v$, $(m_s/m_b)v$, $(m_d/m_s)v$, $(m_c/m_t)v$, and $(m_u/m_c)v$, respectively, and which couple exclusively to the right-handed bottom, strange, down, charm, and up quarks, respectively. Also, H_0 couples only to the right-handed top quark. The resulting model is basically the same as the progressive $U(1)$ model of Ref. [18].

We move to phenomenological aspects of the model. After electroweak symmetry breaking, there appear four CP-even scalar particles, three CP-odd scalar particles, and three charged scalar particles. Since $v_1 \ll v_2 \ll v_3 \ll v$, we can make the approximation that each CP-odd scalar particle comes exclusively from H_1, H_2, or H_3, each CP-even scalar particle comes exclusively from H_1, H_2, H_3, or H_0, and each charged scalar particle comes exclusively from H_1, H_2, or H_3. Under the above approximation, the Yukawa couplings in Eq. (4) are rewritten in terms of physical particles as

$$\begin{aligned}
\mathcal{L}_{\rm Yukawa} \simeq & \frac{y_1 v_1}{\sqrt{2}} \bar{e}e + \frac{y_2 v_2}{\sqrt{2}} \bar{\mu}\mu + \frac{y_3 v_3}{\sqrt{2}} \bar{\tau}\tau + \frac{y_1}{\sqrt{2}} \frac{v_1}{v} h \bar{e}e + \frac{y_2}{\sqrt{2}} \frac{v_2}{v} h \bar{\mu}\mu + \frac{y_3}{\sqrt{2}} \frac{v_3}{v} h \bar{\tau}\tau \\
& + \frac{y_1}{\sqrt{2}} H_1^0 \bar{e}e + \frac{y_2}{\sqrt{2}} H_2^0 \bar{\mu}\mu + \frac{y_3}{\sqrt{2}} H_3^0 \bar{\tau}\tau \\
& + \frac{y_1}{\sqrt{2}} A_1 \bar{e} i\gamma_5 e + \frac{y_2}{\sqrt{2}} A_2 \bar{\mu} i\gamma_5 \mu + \frac{y_3}{\sqrt{2}} A_3 \bar{\tau} i\gamma_5 \tau \\
& + y_1 H_1^+ \bar{\nu}_e e_R + y_2 H_2^+ \bar{\nu}_\mu \mu_R + y_3 H_3^+ \bar{\nu}_\tau \tau_R + \text{H.c.},
\end{aligned} \tag{11}$$

where A_1, A_2, A_3 denote CP-odd scalar particles, h, H_1^0, H_2^0, H_3^0 CP-even scalar particles, and H_1^+, H_2^+, H_3^+ charged scalar particles; h has SM-like Yukawa couplings and can be identified with the observed 125 GeV scalar particle.

We concentrate on the contribution of H_1^0, A_1, H_1^+ to the electron $g-2$ and that of H_2^0, A_2, H_2^+ to the muon $g-2$. They are given by [19]

$$\delta a_e = \frac{1}{16\pi^2} y_1^2 \left\{ \frac{m_e^2}{m_{H_1^0}^2} \left(\log \frac{m_{H_1^0}^2}{m_e^2} - \frac{7}{6} \right) - \frac{m_e^2}{m_{A_1}^2} \left(\log \frac{m_{A_1}^2}{m_e^2} - \frac{11}{6} \right) - \frac{m_e^2}{6 m_{H_1^+}^2} \right\}, \tag{12}$$

$$\delta a_\mu = \frac{1}{16\pi^2} y_2^2 \left\{ \frac{m_\mu^2}{m_{H_2^0}^2} \left(\log \frac{m_{H_2^0}^2}{m_\mu^2} - \frac{7}{6} \right) - \frac{m_\mu^2}{m_{A_2}^2} \left(\log \frac{m_{A_2}^2}{m_\mu^2} - \frac{11}{6} \right) - \frac{m_\mu^2}{6 m_{H_2^+}^2} \right\}. \tag{13}$$

It is important to note that different sets of scalar masses enter the formulas for the electron and muon $g-2$. This allows us to simultaneously explain the negative deviation of the electron $g-2$ and the positive deviation of the muon $g-2$. We comment that the two-loop Barr–Zee diagrams are suppressed by the electron mass or muon mass and hence are negligible.

We derive the masses of the scalar particles $H_1^0, A_1, H_1^+, H_2^0, A_2, H_2^+$ from the scalar potential in Eq. (5), and show that there exists a parameter region where the discrepancies of electron and muon $g-2$ can be explained, with $O(1)$ values for Yukawa couplings y_1, y_2 and without conflicting with experimental bounds on the masses of CP-even, CP-odd, and charged scalar particles. Expanding

the Higgs potential around the VEVs in Eq. (8), the tadpole parameters t_{H_i} for the CP-even scalars should vanish:

$$t_{H_1} \simeq m_1^2 v_1 - \mu_{12}^2 v_2 + \frac{\lambda_{10}+\rho_{10}}{2} v^2 v_1 = 0, \tag{14}$$

$$t_{H_2} \simeq m_2^2 v_2 - \mu_{23}^2 v_3 + \frac{\lambda_{20}+\rho_{20}}{2} v^2 v_2 = 0, \tag{15}$$

$$t_{H_3} \simeq m_3^2 v_3 - \mu_{30}^2 v_0 + \frac{\lambda_{30}+\rho_{30}}{2} v^2 v_3 = 0. \tag{16}$$

Here, the hierarchy of the VEVs is assumed to be $v_1 \ll v_2 \ll v_3 \ll v$. The physical Higgs mass spectrum is given by

$$m_{H_i^0}^2 = m_{A_i}^2 \simeq m_i^2 + \frac{\lambda_{i0}+\rho_{i0}}{2} v^2, \tag{17}$$

$$m_{H_i^+}^2 \simeq m_i^2 + \frac{\lambda_{i0}}{2} v^2. \tag{18}$$

Notice that the CP-even and the CP-odd scalars have different masses due to the quartic coupling constants ρ_{i0}.

3. Numerical results

First, let us estimate the contributions of the new scalar particles to δa_e and δa_μ. From Eqs. (12) and (13),

$$\delta a_e = 2.5 \times 10^{-13} \times \left(\frac{y_1}{3}\right)^2 \left(\frac{100\,\mathrm{GeV}}{m_{H_1^0}}\right)^2 \left(4 - \left(\frac{m_{H_1^0}}{m_{H_1^+}}\right)^2\right) \mathrm{e\,cm}, \tag{19}$$

$$\delta a_\mu = 1.2 \times 10^{-9} \times \left(\frac{y_2}{1}\right)^2 \left(\frac{100\,\mathrm{GeV}}{m_{H_2^0}}\right)^2 \left(4 - \left(\frac{m_{H_2^0}}{m_{H_2^+}}\right)^2\right) \mathrm{e\,cm}, \tag{20}$$

where $m_{A_i} = m_{H_i^0}$ is assumed. In order to explain the magnitude of the current discrepancies in Eqs. (2) and (1), the Yukawa coupling constants must be $O(1)$ and the new scalar masses must be $O(100)$ GeV. Since the current deviation of the electron $g-2$ is negative, there is a condition that

$$m_{H_1^0} > 2 m_{H_1^+}. \tag{21}$$

From Eqs. (17) and (18), this inequality can be satisfied if ρ_{10} is $O(1)$. On the other hand, the current deviation of the muon $g-2$ is positive, so that

$$m_{H_2^0} < 2 m_{H_2^+}. \tag{22}$$

This inequality can be satisfied if ρ_{20} is sufficiently small.

In the numerical analysis, we take the VEVs of the four Higgs doublets as follows:

$$v_3 = \frac{m_\tau}{m_t} v, \quad v_2 = \frac{m_\mu}{m_\tau} v_3, \quad v_1 = \frac{m_e}{m_\mu} v_2, \tag{23}$$

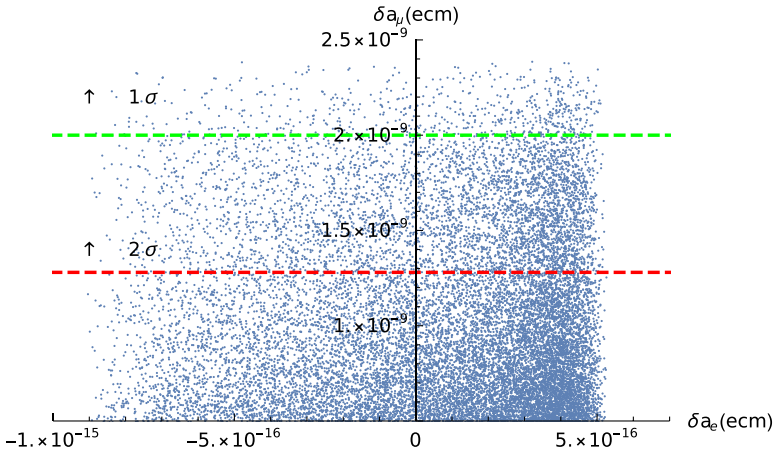

Fig. 1. Scatter plot of δa_e and δa_μ. The Yukawa coupling constants are fixed at $y_1 = 0.6$ and $y_2 = 3.3$; the other input parameters are explained in the text. The green and red lines correspond to the experimental $1\,\sigma$ and $2\,\sigma$ bounds, respectively.

with $v = 246\,\text{GeV}$. For the other Higgs sector parameters, we scan them randomly in the following regions:

$$0 < m_1, m_2, m_3 < 50\,\text{GeV}, \tag{24}$$

$$0 < \lambda_i, \lambda_{ij}, \kappa_i < 1, \tag{25}$$

$$0 < \rho_{ij} < 6. \tag{26}$$

For each parameter set, we impose the vanishing tadpole conditions in Eqs. (14)–(16) and calculate the physical Higgs spectrum. We cannot find the Large Hadron Collider (LHC) constraint on the lepton-specific Higgs. If the Yukawa couplings are O(1), the charged Higgs mainly decay into the charged lepton and neutrino. In this case, the charged Higgs search is almost the same as the left-handed slepton search. The current LHC bound on the left-handed slepton is $\sim$300 GeV for the combination of selectron and smuon [20,21]. There is no separate mass bound, and we impose the following flavor-dependent constraints:

$$m^+_{H_1} > 200\,\text{GeV}, \quad m^+_{H_{2,3}} > 400\,\text{GeV}. \tag{27}$$

If y_1 is large, the effective operator $\frac{y_1^2}{m^2_{H_1^+}} \bar{e} e \bar{\nu} \nu$ through the H_1^+-mediated diagram is enhanced and there is a severe constraint on the effective operator from the mono-photon search at the Large Electron–Positron Collider [22]:

$$m_{H_1^+}/y_1 > 340\,\text{GeV}. \tag{28}$$

Figure 1 shows the contributions of the new scalar particles to δa_e and δa_μ. Here, we fix the Yukawa coupling constants as $y_1 = 0.6$ and $y_2 = 3.3$. We observe that there are parameter sets that give δa_μ in the $1\,\sigma$ region. Although there are parameter sets where δa_e is negative and reaches $\sim -9 \times 10^{-16}$ e cm, we cannot find any parameter set that gives δa_e even in the $2\,\sigma$ region. In these parameter sets, the charged scalar masses are around the current experimental bound $\sim$200 GeV, and ρ_{10} is $O(1)$.

4. Summary

We have studied the lepton-flavored Higgs model to explain the electron and muon $g-2$ discrepancies simultaneously, and to give a natural explanation to the charged lepton mass hierarchy. In the model, we introduce the $U(1)_F$ symmetry under which e_R, μ_R, τ_R are charged with $-3, -2, -1$, respectively, and introduce three additional Higgs doublets with $U(1)_F$ charges $+3, +2, +1$, so that each generation of charged leptons couples to one of the three additional Higgs doublets due to the $U(1)_F$ symmetry. We assume that the $U(1)_F$ symmetry is softly broken by $+1$ charges, and take the soft breaking to be sufficiently small that the hierarchy of the charged lepton masses originates from the hierarchy of the Higgs VEVs while the Yukawa couplings are $O(1)$. Note that it is natural to take the soft breaking small. Since electron and muon couple to different scalar particles with $O(1)$ Yukawa couplings, the electron and the muon $g-2$ discrepancies can be explained simultaneously. Specifically, the negative deviation of a_e is explained when the almost-electron-specific charged scalar particle is much lighter than the CP-even scalar particle. On the other hand, the positive deviation of a_μ is explained when the mass of the almost-muon-specific charged scalar particle is similar to that of the CP-even scalar particle. We have searched the space of the Higgs sector parameters and found sets that give a_μ in the $1\,\sigma$ region. Although there are parameter sets that give negative δa_e, we cannot find any parameter set that gives a_e in the $2\,\sigma$ region.

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