Detectable dimension-6 proton decay in SUSY SO(10) GUT at Hyper-Kamiokande

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ABSTRACT: In the minimal SUSY SU(5) GUT with $O(1)$ TeV SUSY particles and $O(1)$ or below self-coupling for the GUT-breaking Higgs field, the width of the dimension-6 proton decay is suppressed below the reach of Hyper-Kamiokande. In this paper, we point out that a SUSY SO(10) GUT which adopts only $\mathbf{45}_H + \mathbf{16}_H + \overline{\mathbf{16}}_H$ GUT-breaking Higgs fields leads to an enhanced dimension-6 proton decay width detectable at Hyper-Kamiokande. The enhancement is because the SU(5)-breaking VEV of $\mathbf{45}_H$ arises due to Planck-suppressed terms, $W \propto (\mathbf{45}_H^2)^2/M_* + \mathbf{45}_H^4/M_*$, and is therefore substantially larger than the other VEVs that conserve SU(5). As a result, the $(\mathbf{3}, \mathbf{2}, 1/6)$ GUT gauge boson mass is about $1/5$ smaller than the $(\mathbf{3}, \mathbf{2}, -5/6)$ GUT gauge boson mass and can induce a fast dimension-6 proton decay. Through a numerical analysis on threshold corrections of the GUT gauge bosons and the physical components of the GUT-breaking Higgs fields, we confirm that the dimension-6 proton decay can be within the reach of Hyper-Kamiokande.

KEYWORDS: GUT, Supersymmetric Standard Model

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Contents

1 Introduction

The dimension-6 proton decay is an important prediction of the grand unified theory (GUT) [1]. The Super-Kamiokande experiment currently gives the bound of the partial proton lifetime $\tau(p \to \pi^0 e^+) > 1.6 \times 10^{34}$ years (90% confidence level) [2], and it will be searched up to 6.3×10^{34} years at 3σ level by a 10 year exposure of one 187 kton fiducial volume detector at Hyper-Kamiokande (HK) [3]. Now that the HK experiment is scheduled to start in 2026, it is time to survey GUT models which predict the dimension-6 proton decay within the discovery reach.

In the minimal supersymmetric (SUSY) SU(5) GUT [4–6] with $O(1)$ TeV SUSY particles, the partial lifetime for the dimension-6 proton decay via GUT gauge boson exchange is predicted to be more than a few times 10^{35} years naively. The gauge coupling unification condition does not directly give the mass of the GUT gauge boson, because the mass of the physical components of the SU(5)-breaking Higgs field $\mathbf{24}_{\rm H}$ cannot be determined theoretically. The GUT gauge boson mass becomes heavier (the proton lifetime is longer) if the self-coupling of $\mathbf{24}_{\rm H}$ is smaller.[1] The SUSY GUTs also predict the dimension-5 proton decay via colored Higgs exchange [7, 8], such as $p \to K^+ \bar{\nu}$, whose current bound reads $\tau(p \to K^+ \bar{\nu}) > 5.9 \times 10^{33}$ years [9] and which often gives a severe constraint on the model construction. There are several ways to suppress the dimension-5 decay to a harmless level, e.g., by enhancing the colored Higgs mass with SUSY particle threshold with large

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[1]In non-SUSY SU(5) GUTs, some choices of GUT Higgses yield models that survive the SK bound and will be explored at HK [17].

wino/gluino mass ratio [10], with non-renormalizable superpotential of adjoint representations [11], or with GUT particle thresholds in non-minimal models for the gauge coupling evolutions [12]. Other ways include assuming heavy squarks, or utilizing a cancellation among multiple Higgs couplings. Compared to the dimension-5 decay, the dimension-6 proton decay involves less parameters and its naive prediction is above the current experimental bound. Therefore, it is worth pursuing the possibility that $p \to \pi^0 e^+$ will be observed at HK. In fact, as the LHC results imply that the SUSY particles have mass above multi-TeV scale, some people revisit the unification conditions in the context of the high-scale SUSY scenario [13–16]. As the wino and gluino are heavier, the unification scale becomes lower, and it can reach the discovery range of HK for ∼10-100 TeV wino and gluino masses.

What about the dimension-6 proton decay in SUSY SO(10) GUTs? The SO(10) GUT is attractive since all the observed fermions (including right-handed neutrinos) can be unified in one spinor representation in each generation. The breaking pattern of the SO(10) symmetry, however, has room for the existence of intermediate scales, and thus the prediction of the dimension-6 proton decay varies in a wide range. Among various choices of the Higgs representations to break SO(10) to the SM gauge symmetry, the simplest choice is $\mathbf{45}_H + \mathbf{16}_H + \overline{\mathbf{16}}_H$, which is also the most economical in view of the total contribution to the beta coefficient for gauge couplings. The above choice of the Higgs representations gives characteristic vacua where the GUT gauge boson with SM charge $(\mathbf{3}, \mathbf{2}, 1/6)$, which is absent in SU(5) GUT, is about 1/5 lighter than the GUT gauge boson with SM charge $(\mathbf{3}, \mathbf{2}, -5/6)$, which is also present in SU(5) GUT. In the vacua, therefore, the dimension-6 proton decay width is enlarged compared to the minimal SU(5) model due to the exchange of the light $(\mathbf{3}, \mathbf{2}, 1/6)$ gauge boson. So, it is worth scrutinizing the prediction of the dimension-6 proton decay in the above model, since the predicted proton lifetime can be in the range of HK. To our best knowledge, this simple SO(10) model has not been investigated in light of experimental accessibility of the dimension-6 proton decay. In this paper, we will show a numerical calculation of the dimension-6 proton decay $p \to \pi^0 e^+$ in the SO(10) model with $\mathbf{45}_H + \mathbf{16}_H + \overline{\mathbf{16}}_H$ GUT-breaking Higgs fields.

We also find that in the characteristic vacua of the above model, the colored Higgs mass is enhanced by about 576 compared to the minimal SU(5) model due to threshold corrections of GUT gauge bosons and physical components of GUT-breaking Higgs fields.[2] So, this SO(10) model exhibits an interesting tendency that the dimension-6 decay width is enhanced and the dimension-5 decay width is suppressed.

This paper is organized as follows: in section 2, we present the spectrum of the SO(10) gauge bosons that gain mass via symmetry breaking with $\mathbf{45}_H + \mathbf{16}_H + \overline{\mathbf{16}}_H$, and show that the dimension-6 proton decay width can be enlarged. In section 3, we study how the characteristic SO(10) breaking vacua of the model are obtained. In section 4, GUT-scale threshold corrections are evaluated for the calculation of the dimension-6 proton decay width. In section 5, a detailed numerical result for the proton lifetime is presented. Section 6 is for the conclusion. In appendix A, we show the mass spectrum of the multiplets which

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[2]Considering this enhancement and uncertainty of the Yukawa coupling unification, we omit an analysis of the dimension-5 decay in this paper.

come from $\mathbf{16}_H + \overline{\mathbf{16}}_H$. In appendix B, an alternative, renormalizable superpotential with $\mathbf{54}_H$ representation GUT Higgs field is shown.

2 Spectrum of the SO(10) gauge bosons and proton decay

There are many ways to break the SO(10) gauge symmetry to the SM gauge symmetry. The most economical choice of the Higgs representations to break SO(10) is $\mathbf{45}_H + \mathbf{16}_H$. We also introduce $\overline{\mathbf{16}}_H$ for D-flatness. $\mathbf{45}_H$ contains two SM singlets: a SU(5) singlet (a_1) and a SM singlet in SU(5) adjoint (a_{24}). By general vacuum expectation values (VEVs) of a_1 and a_{24} (without particular relations between them), SO(10) is broken down to $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)'$. The SM singlet in $\mathbf{16}_H$ (v_R), which is a SU(5) singlet, develops a VEV to break U(1)$'$ (the singlet in $\overline{\mathbf{16}}_H$ ($\bar{v}_R$) also gains a VEV with $|v_R| = |\bar{v}_R|$ to keep D-flatness). Due to the absence of cubic term of $\mathbf{45}_H$, a_{24} cannot acquire a VEV from the renormalizable superpotential. However, by introducing non-renormalizable, quartic terms of $\mathbf{45}_H$, a_{24} can acquire a VEV to break SU(5).

The GUT gauge boson masses generated by the VEVs of $\mathbf{45}_H$, $\mathbf{16}_H$ and $\overline{\mathbf{16}}_H$ are

$$M_X^2/g_5^2 = \frac{5}{6}|a_{24}|^2, \tag{2.1}$$

$$M_Q^2/g_5^2 = \frac{4}{5}\left|a_1 + \frac{1}{2\sqrt{6}}a_{24}\right|^2 + |v_R|^2, \tag{2.2}$$

$$M_U^2/g_5^2 = \frac{4}{5}\left|a_1 - \sqrt{\frac{2}{3}}a_{24}\right|^2 + |v_R|^2, \tag{2.3}$$

$$M_E^2/g_5^2 = \frac{4}{5}\left|a_1 + \sqrt{\frac{3}{2}}a_{24}\right|^2 + |v_R|^2, \tag{2.4}$$

where X, Q, U and E denote SO(10) gauge bosons whose SM charges are $X : (\mathbf{3}, \mathbf{2}, -5/6)$, $Q : (\mathbf{3}, \mathbf{2}, 1/6)$, $U : (\mathbf{3}, \mathbf{1}, 2/3)$ and $E : (\mathbf{1}, \mathbf{1}, 1)$, and a_1, a_{24} and v_R are the VEVs of canonically-normalized SM singlets. The extra U(1) gauge boson mass is $g_5\sqrt{5/2}v_R$.

When $a_{24} \gg a_1, v_R$, we obtain

$$M_X : M_Q : M_U : M_E \simeq 5 : 1 : 4 : 6. \tag{2.5}$$

This ratio is easily obtained by the rule $(3+2) : (3-2) : (2+2) : (3+3)$.

The dimension-6 proton decay operators are generated not only by the X gauge boson exchange but also by the Q gauge boson exchange. The partial width of the dimension-6 proton decay is given by

$$\Gamma \propto A_R^2\left(\frac{1}{M_X^2} + \frac{1}{M_Q^2}\right)^2 + \frac{4A_L^2}{M_X^4}, \tag{2.6}$$

where $A_{L,R}$ are the renormalization factors for $q\ell(u^c)^\dagger(d^c)^\dagger$ and $qq(e^c)^\dagger(u^c)^\dagger$ operators. One finds that the Q gauge boson exchange gives much larger contribution when $M_X : M_Q \simeq$

5 : 1. The ratio of the decay width in SU(5) GUT ($M_Q \to \infty$) and in the SO(10) GUT with $a_{24} \gg a_1, v_R$ (now) is

$$\Gamma^{\mathrm{SU}(5)} : \Gamma^{\mathrm{now}} \simeq 1 : 136, \tag{2.7}$$

for $A_L \simeq A_R$, if the X gauge boson masses are the same. Since the naive prediction of $p \to \pi^0 e^+$ partial lifetime in SU(5) GUT is $\tau_p \sim 10^{36}$ years, the prediction in the SO(10) with $a_{24} \gg a_1, v_R$ is 10^{34} years, which is on the current experimental bound at SK.

3 SO(10) breaking vacua in the model

We consider a superpotential for the GUT breaking Higgs fields $\mathbf{45}_H$ (A), $\mathbf{16}_H$ (χ) and $\overline{\mathbf{16}}_H$ ($\bar{\chi}$),

$$W = \frac{1}{2} m_A A^2 + \frac{\lambda_1}{M_*}(A^2)^2 + \frac{\lambda_2}{M_*} A^4 + m_\chi \chi \bar{\chi} + \kappa A \chi \bar{\chi}, \tag{3.1}$$

where we define $A^2 \equiv A_{ab} A_{ab}/2$, and $A^4 \equiv A_{ac} A_{ad} A_{bc} A_{bd}/2$ so that the multiplication of contraction of 2-anti-symmetric indices is removed by dividing by 2. The superpotential in terms of canonically-normalized SM singlets a_1, a_{24} in A and v_R in χ (Clebsch-Gordan coefficients for $\mathbf{16}$ representation can be found in [18]) is given by

$$W|_{\mathrm{SM\ singlets}} = m_\chi v_R \bar{v}_R + \frac{1}{2} m_A (a_1^2 + a_{24}^2) + \sqrt{5} \kappa\, a_1 v_R \bar{v}_R$$
$$+ \frac{\lambda_1}{M_*}(a_1^2 + a_{24}^2)^2 + \frac{\lambda_2}{5 M_*} \left(a_1^4 + 6 a_1^2 a_{24}^2 + 2\sqrt{\frac{2}{3}} a_1 a_{24}^3 + \frac{7}{6} a_{24}^4 \right). \tag{3.2}$$

The F-flat conditions read

$$a_1 = -\frac{m_\chi}{\sqrt{5}\kappa}, \tag{3.3}$$

$$a_{24} = \frac{m_\chi}{\sqrt{5}\kappa} a, \tag{3.4}$$

$$v_R \bar{v}_R = \frac{4 m_\chi^3}{25 \kappa^4 M_*} \left(\lambda_1 (1 + a^2) + \frac{\lambda_2}{5} \left(1 + 3a^2 - \frac{1}{\sqrt{6}} a^3 \right) \right) + \frac{m_\chi m_A}{5 \kappa^2}, \tag{3.5}$$

where a is a solution to

$$\lambda_1 + \frac{3}{5} \lambda_2 + \frac{5 \kappa^2 m_A M_*}{4 m_\chi^2} - \sqrt{\frac{3}{2}} \frac{\lambda_2}{5} a + \left(\lambda_1 + \frac{7}{30} \lambda_2 \right) a^2 = 0. \tag{3.6}$$

In eq. (3.3), the condition $\partial W / \partial v_R = (m_\chi + \sqrt{5}\kappa a_1)\bar{v}_R$ fixes the VEV of a_1 to be around m_χ. In eq. (3.6), on the other hand, the VEV of a_{24} (proportional to a) is fixed by a balance between the quadratic mass term and the quartic non-renormalizable term, and $|a|$ is large if $|m_A M_*/m_\chi^2| \gg 1$, in which case we obtain

$$a_{24}^2 \sim -\frac{m_A M_*}{4(\lambda_1 + \frac{7}{30}\lambda_2)}, \qquad v_R \bar{v}_R \sim \frac{\lambda_2}{5\sqrt{30}\kappa} \frac{m_A a_{24}}{\lambda_1 + \frac{7}{30}\lambda_2}. \tag{3.7}$$

Thus, vacua with $|v_R|, |a_1| \ll |a_{24}|$ are obtained[3] with a feasible assumption $m_\chi, m_A \ll M_*$.

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[3]If one adds a non-renormalizable term $(\chi\bar{\chi})^2/M_*$ to the superpotential, new vacua with $v_R \sim \sqrt{m_\chi M_*}$ appear. However, the vacua we obtain in the main text remain stable with a correction of $O(v_R^2/M_*^2)$.

4 GUT-scale threshold corrections for the gauge coupling unification

The gauge coupling unification conditions [25] in SUSY SO(10) GUT are written as[4]

$$-\frac{2}{\alpha_3(m_Z)} + \frac{3}{\alpha_2(m_Z)} - \frac{1}{\alpha_1(m_Z)} = \frac{1}{2\pi}\left(\frac{12}{5}\ln\frac{M_H}{m_Z} - 2\ln\frac{m_{\text{SUSY1}}}{m_Z}\right), \qquad (4.3)$$

$$-\frac{2}{\alpha_3(m_Z)} - \frac{3}{\alpha_2(m_Z)} + \frac{5}{\alpha_1(m_Z)} = \frac{1}{2\pi}\left(36\ln\frac{M_G}{m_Z} + 8\ln\frac{m_{\text{SUSY2}}}{m_Z}\right), \qquad (4.4)$$

$$M_H = \frac{M_Q^4}{M_E M_U^3}M_{H_C}\prod_i M_i^{l_A^i}, \qquad (4.5)$$

$$M_G^6 = \frac{M_X^4 M_E^2 M_U^2}{M_Q^4}\prod_i M_i^{l_B^i}, \qquad (4.6)$$

where $M_{X,Q,U,E}$ are the SO(10) gauge boson masses which we have already defined, M_{H_C} is the colored Higgs mass, and i stands for the degree of physical modes under the SM decompositions. We define $l_A = \frac{5}{12}(2l_3 - 3l_2 + l_1)$ and $l_B = \frac{1}{6}(2l_3 + 3l_2 - 5l_1)$ where l_i gives the beta coefficient contribution of the respective multiplet, $l_i = \Delta b_i^{\text{SUSY}}$. Because the would-be-Goldstone modes which are eaten by the gauge bosons lack from the multiplets, we obtain

$$\sum_i l_A^i = 0, \qquad \sum_i l_B^i = 2. \qquad (4.7)$$

The RGEs give

$$M_H \sim 10^{15} - 10^{16}\text{ GeV}, \qquad M_G \sim 2\times 10^{16}\text{ GeV}, \qquad (4.8)$$

and the GUT gauge boson and colored Higgs masses depend on threshold corrections of GUT-scale particles.

The $\mathbf{45}_H$ and $\mathbf{16}_H$ representations are decomposed under SU(5) as $\mathbf{45}_H = \mathbf{24} + \mathbf{10} + \overline{\mathbf{10}} + \mathbf{1}$. and $\mathbf{16}_H = \mathbf{10} + \overline{\mathbf{5}} + \mathbf{1}$. One linear combination of the $\mathbf{10}$'s (and $\overline{\mathbf{10}}$'s) from $\mathbf{45}_H$ and $\mathbf{16}_H$ ($\overline{\mathbf{16}}_H$) is absorbed by the gauge bosons Q, U, E. For $|v_R| \ll |a_{24}|$, the linear combination to be absorbed mainly comes from $\mathbf{45}_H$. The other linear combination is a physical mode and we denote its components by χ_Q, χ_U, χ_E (which respectively have the same SM charge as Q, U, E). For $|m_A|, |m_\chi| \ll M_*$, their masses satisfy the ratio (see appendix A for the derivation)

$$M_{\chi Q} : M_{\chi U} : M_{\chi E} \simeq 1 : 4 : 6. \qquad (4.9)$$

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[4]The SUSY particle threshold contributions in the respective equations are more precisely written as

$$-2\ln\frac{m_{\text{SUSY1}}}{m_Z} = -\frac{8}{5}\ln\frac{\mu_H}{m_Z} - \frac{2}{5}\ln\frac{m_H}{m_Z} + 4\ln\frac{M_{\tilde{g}}}{M_{\tilde{w}}} + \frac{3}{5}\ln\frac{m_{\tilde{q}^c}^3 m_{\tilde{d}^c}^2 m_{\tilde{e}^c}}{m_{\tilde{q}}^4 m_{\tilde{\ell}}^2}, \qquad (4.1)$$

$$8\ln\frac{m_{\text{SUSY2}}}{m_Z} = 4\ln\frac{M_{\tilde{g}}M_{\tilde{w}}}{m_Z^2} + 3\ln\frac{m_{\tilde{q}}^2}{m_{\tilde{u}}m_{\tilde{e}^c}}, \qquad (4.2)$$

where μ_H, m_H are higgsino and heavier Higgs masses, $M_{\tilde{g}}$ and $M_{\tilde{w}}$ are gluino and wino masses. From these equations, one finds that the colored Higgs mass is larger for a smaller ratio of $M_{\tilde{g}}/M_{\tilde{w}}$, and the unification scale M_G becomes smaller for heavier wino and gluino masses.

The **24** representation in $\mathbf{45}_H$ contains a SU(3)$_c$ adjoint $(\mathbf{8},\mathbf{1},0)$ and a SU(2)$_L$ adjoint $(\mathbf{1},\mathbf{3},0)$ as physical modes. Their masses can be calculated (using the minimization conditions) as

$$M_8 = \frac{2}{3}\frac{\lambda_2}{M_*}a_{24}(-3\sqrt{6}a_1 + a_{24}), \tag{4.10}$$

$$M_3 = \frac{2}{3}\frac{\lambda_2}{M_*}a_{24}(3\sqrt{6}a_1 + 4a_{24}), \tag{4.11}$$

and when $|a_1| \ll |a_{24}|$, we find

$$M_8 : M_3 \simeq 1 : 4. \tag{4.12}$$

The other physical Higgs modes are the $\mathbf{5} + \bar{\mathbf{5}}$ in $\mathbf{16}_H + \overline{\mathbf{16}}_H$. We additionally introduce two $\mathbf{10}_H$ representations to generate renormalizable Yukawa couplings that give fermion masses after electroweak symmetry breaking. Then, there are three heavy colored Higgs fields H_C, $H_{T_{1,2}}$, and two heavy Higgs doublets $H_{D_{1,2}}$ around the GUT scale.

Now the gauge coupling unification conditions are specified as

$$M_H = \frac{M_Q^4}{M_E M_U^3}\frac{(M_{\chi_E}M_{\chi_U}^3)^{1/2}}{M_{\chi_Q}^2}\left(\frac{M_8}{M_3}\right)^{5/2}\frac{M_{T_1}M_{T_2}}{M_{D_1}M_{D_2}}M_{H_C}, \tag{4.13}$$

$$M_G^6 = \frac{M_E^2 M_U^2}{M_Q^4}\frac{M_{\chi_Q}^2}{M_{\chi_E}M_{\chi_U}}M_X^4 M_8 M_3. \tag{4.14}$$

In the vacua with $|v_R|, |a_1| \ll |a_{24}|$, we obtain from eqs. (2.5), (4.9), (4.12),

$$\frac{M_Q^4}{M_E M_U^3}\frac{(M_{\chi_E}M_{\chi_U}^3)^{1/2}}{M_{\chi_Q}^2}\left(\frac{M_8}{M_3}\right)^{5/2} \simeq \frac{1}{576}, \tag{4.15}$$

$$\frac{M_E^2 M_U^2}{M_Q^4}\frac{M_{\chi_Q}^2}{M_{\chi_E}M_{\chi_U}} \simeq 24. \tag{4.16}$$

Due to the factor 1/576, the colored Higgs mass can be much larger than in the minimal SU(5) model. As for the gauge boson mass, in the minimal SU(5) model, one has $M_G^6 = M_X^4 M_8 M_3$ and $M_8 = M_3 = \lambda M_X$ where λ is proportional to the self-coupling of the SU(5) adjoint representation. λ is arbitrary unless it far exceeds $O(1)$, and people often assume $\lambda \sim 1$, which gives $M_X \sim M_G$. In the current SO(10)-breaking vacua $|v_R|, |a_1| \ll |a_{24}|$, if we write $4M_8 \simeq M_3 = \rho M_X$, ρ is always much smaller than 1 because the masses of the SU(3)$_c$ adjoint and SU(2)$_L$ adjoint particles are roughly m_A, while the X gauge boson mass is roughly $(m_A M_*)^{1/2}$. To be specific, we get from eqs. (4.14), (4.16),

$$M_X \simeq M_G\frac{1}{(6\rho^2)^{1/6}}, \tag{4.17}$$

and from eqs. (2.1), (4.11),

$$\rho = \frac{M_3}{M_X} \simeq \frac{16}{5}\lambda_2\frac{1}{g_5^2}\frac{M_X}{M_*}. \tag{4.18}$$

Therefore we find

$$\rho \simeq 2\left(\frac{\lambda_2}{g_5^2}\right)^{3/4}\left(\frac{M_G}{M_*}\right)^{3/4}, \tag{4.19}$$

which equals 0.1 for $\lambda_2 = 1$ and $M_* = 2 \times 10^{18}$ GeV. It follows that M_X is a little larger than M_G. Nevertheless, the Q gauge boson satisfying $M_X : M_Q \simeq 5 : 1$ enhances the dimension-6 proton decay width compared to the minimal SU(5) model.

To summarize, in the SO(10)-breaking vacua with $|a_{24}| \gg |a_1|, |v_R|$, the colored Higgs is made heavier by the GUT-scale threshold corrections, and the dimension-5 proton decay is suppressed compared to the minimal SU(5) model. On the other hand, the dimension-6 proton decay width is roughly 100 times enlarged and we have $\tau_p \sim 10^{34}$ years, which is in the scope of HK.

Suppression of the dimension-5 proton decay is also achieved by making the ratio of gluino and wino masses $M_{\tilde{g}}/M_{\tilde{w}}$ smaller, and enhancement of the dimension-6 proton decay is achieved by increasing their product $M_{\tilde{g}} M_{\tilde{w}}$, as seen from the SUSY particle threshold correction formulas. Hence, in the high scale SUSY scenario, the dimension-6 proton decay is detectable at HK even in the minimal SUSY SU(5) model. In contrast, in our SO(10)-breaking vacua $|a_{24}| \gg |a_1|, |v_R|$, the GUT-scale threshold corrections enhance the dimension-6 proton decay width to a detectable level, even if SUSY particle masses are a few TeV.

We comment on the case when the $\mathbf{16}_H$ is replaced by $\mathbf{126}_H$ representation. Choice of $\mathbf{16}_H$ needs additional R-parity symmetry to avoid rapid dimension-4 proton decays, and adopting $\mathbf{126}_H$ has a merit that R-parity in MSSM is induced due to the fact that the vevs of $\mathbf{126}_H, \overline{\mathbf{126}}_H$ have $B-L = \pm 2$ (while the vevs of $\mathbf{16}_H, \overline{\mathbf{16}}_H$ have $B-L = \pm 1$). However, when a vacuum with $|a_1| \ll |a_{24}|$ is chosen, $(\mathbf{6}, \mathbf{3}, 1/3)$ multiplet in $\mathbf{126}_H$ is about $1/3$ lighter than the other components in the representation. Since this multiplet has $l_A = -33/2$, it gives a large threshold correction and renders the colored Higgs too light. To avoid this unwanted large thresholds, we adopt the choice of $\mathbf{16}_H$ (with R-parity) in this paper.

5 Numerical result

In the previous section, we have used 1-loop relations to describe qualitative behaviors. In this section, we will show a numerical result using 2-loop RGE evolutions [19, 20]. In the result, we use the central value of the 5-flavor strong coupling, $\alpha_s^{(5)}(M_Z)^{\overline{\mathrm{MS}}} = 0.1181 \pm 0.0011$ [21]. The colored Higgs mass is sensitive to the value of the strong coupling, while the GUT gauge boson masses are less sensitive. The proton lifetime is about 50% larger if we use the value $+3\sigma$. We assume all the SUSY particle masses to be 2 TeV except for the wino mass, which is taken to be 500 GeV.

The decay width of $p \to \pi^0 e^+$ is [22]

$$\Gamma = \frac{\pi}{4} \frac{m_p \alpha_H^2}{f_\pi^2} (1 + F + D)^2 \alpha_U^2 \left[A_R^2 \left(\frac{1}{M_X^2} + \frac{1}{M_Q^2} \right)^2 + \frac{4 A_L^2}{M_X^4} \right], \tag{5.1}$$

where we use proton mass $m_p = 0.9383$ GeV, chiral Lagrangian parameters $F = 0.46$, $D = 0.80$, hadron matrix element for proton decay $\alpha_H = -0.014$ GeV^3 at 2 GeV [23], decay constant $f_\pi = 0.1307$ GeV, renormalization factors down to 2 GeV, $A_L = 2.91$, $A_R = 2.78$ (The two-loop renormalization factors are calculated in [24]). From the SO(10) gauge coupling unification, we obtain $1/\alpha_U(M_X) = 4\pi/g_5^2 \simeq 25.2$.

Before presenting the main result, we show an estimate on the partial proton lifetime. Under the approximations with

$$M_X : M_3 : M_8 = 1 : \rho : \rho/4, \tag{5.2}$$

$$M_{\chi Q} : M_{\chi U} : M_{\chi E} = 1 : 4 : 6, \tag{5.3}$$

and

$$M_X : M_Q : M_U : M_E = 5 : 1 : 4 : 6, \tag{5.4}$$

the partial proton lifetime is found to be

$$\tau_p \simeq \left(\frac{0.1}{\rho}\right)^{\frac{4}{3}} \times 2.6 \times 10^{34} \text{ years.} \tag{5.5}$$

As discussed in the previous section, $\rho \ll 1$ in the current SO(10)-breaking vacua because the VEV of a_{24} is roughly the geometrical average of m_A and M_* while M_3, M_8 are roughly m_A, and we get $\rho \simeq 0.1$ for $\lambda_2 = 1$ and $M_* = 2 \times 10^{18} \text{ GeV}$.

It is interesting to compare the above estimate with the prediction of the minimal SU(5) model. In the minimal SU(5), we define $M_8 = M_3 = \lambda M_X$ where λ is proportional to the self-coupling of the adjoint field that breaks SU(5). Then, the partial proton lifetime is found to be[5]

$$\tau_p^{\text{SU(5)}} \simeq \lambda^{-4/3} \times 5.5 \times 10^{35} \text{ years.} \tag{5.6}$$

We observe that the partial lifetime decreases by $1/20$ in our SO(10)-breaking vacua compared to the minimal SU(5) model, for natural values of $\rho = 0.1$ and $\lambda = 1$.

The estimate for our SO(10)-breaking vacua, eq. (5.5), receives corrections from the small VEVs of a_1, v_R that perturb the mass ratios. In table 1, we show precise numerical values. Here, we fix $M_* = 2 \times 10^{18} \text{ GeV}$, and take benchmark values for $\lambda_1, \lambda_2, \kappa$ and m_χ. We solve the F-flat conditions eqs. (3.3)–(3.6) and the unification conditions by varying m_A and the colored Higgs mass. Since eq. (3.6) is a quadratic equation, there are two solutions. If $m_A < 0$, a_{24} is real and the mass spectrum splits into two, both of which are tabulated. If $m_A > 0$, a_{24} is complex and the two solutions yield the same mass spectrum in terms of the absolute values. From (iii) and (iii″) of table 1, we find that the mass spectrum is not sensitive to λ_1. This is because the relation $|a_1|, |v_R| \ll |a_{24}|$ gives $M_3/a_{24} \propto \lambda_2 a_{24}/M_*$. Although a_{24} depends on λ_1, the ratio M_3/M_X does not depend on λ_1 for $|a_1|, |v_R| \ll |a_{24}|$. As a result, once M_X is chosen to realize the gauge coupling unification, the mass spectrum is almost independent of λ_1. On the other hand, when λ_2 is smaller, the SU(3)$_c$ adjoint and SU(2)$_L$ adjoint particles become lighter ($\rho = M_3/M_X$ is smaller), and the proton lifetime becomes longer, as seen from (iii) and (iii′) of table 1. Consequently, the proton lifetime cannot be bounded from above theoretically. Still, it is interesting that for $\lambda_2 \sim 1$, the dimension-6 proton decay is detectable at HK.

In the benchmarks of table 1, the effective colored Higgs mass, $M_{H_T} = \frac{M_{T_1} M_{T_2} M_{H_C}}{M_{D_1} M_{D_2}}$, is $2 \times 10^{17} \text{ GeV}$. The relation $M_{H_T} > M_X$ is realized with a large coupling of $AH_1 H_2$ (see

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[5]We remark that the LHC bound of gluino mass reduces the dimension-6 partial lifetime due to eqs. (4.2) and (4.4), though the prediction of the minimal SUSY SU(5) model is often referred to be 10^{36} years.

	$(\lambda_1, \lambda_2, \kappa)$	m_χ	m_A	M_X	M_Q	M_3	M_8	v_R	$\tau_p(10^{34}$ years$)$
(i-A)	$(1,1,1)$	0.1	-0.47	2.8	0.55	0.25	0.068	0.24	3.4
(i-B)	$(1,1,1)$	0.1	-0.46	2.8	0.61	0.25	0.057	0.25	4.7
(ii-A)	$(1,1,1)$	0.01	-0.48	2.8	0.59	0.26	0.069	0.25	4.3
(ii-B)	$(1,1,1)$	0.01	-0.44	2.7	0.57	0.24	0.059	0.24	3.7
(iii)	$(1,1,1)$	0.1	0.46	2.8	0.58	0.25	0.063	0.24	4.0
(iii$'$)	$(1,0.5,1)$	0.1	0.58	3.2	0.67	0.17	0.043	0.22	7.2
(iii$''$)	$(0.1,1,1)$	0.1	0.12	2.8	0.58	0.25	0.063	0.24	4.0
(iii$'''$)	$(1,1,0.5)$	0.1	0.48	2.8	0.62	0.26	0.066	0.36	5.1

Table 1. The $p \to \pi^0 e^+$ partial lifetime and the mass spectrum for various input values of $\lambda_1, \lambda_2, \kappa$ and m_χ. The masses and VEV are in units of 10^{16} GeV. From (i-A) to (ii-B), $m_A < 0$ and thus a_{24} is real and the mass spectrum splits into two, for which (i-A) and (ii-A) correspond to the cases with $a_{24} > 0$, and (i-B) and (ii-B) to the cases with $a_{24} < 0$. In (iii), $m_A > 0$ and thus a_{24} is complex. We change one of $\lambda_1, \lambda_2, \kappa$ in (iii$'$), (iii$''$), (iii$'''$).

appendix A). Since the dimension-5 proton decay amplitudes also depend on details of the Yukawa coupling unification, we do not discuss the dimension-5 decay in this paper.

Before concluding, we comment on the realization of fermion masses and mixings in the model. All the quarks and leptons including the right-handed neutrinos are unified in spinor representation ψ_i (i stands for a generation index). The right-handed neutrino masses are generated by $\psi_i \psi_j \bar\chi \bar\chi$ (or $S \psi_i \bar\chi + M_S S^2$ with singlet field S). The Yukawa interaction (including Dirac neutrino Yukawa interaction) can come from $Y_1^{ij} \psi_i \psi_j H_1 + Y_2^{ij} \psi_i \psi_j H_2$, where H_1 and H_2 are **10**-dimensional reps, as well as non-renormalizable terms such as $\psi_i \psi_j \chi \chi$. The MSSM Higgs doublets are contained in H_1, H_2, and χ and $\bar\chi$, which are mixed and one linear combinations are light (see appendix A). Because of the non-trivial mixings of Higgs doublets, CKM mixings are generated. The Higgs mixings can also induce the freedom of $\tan\beta$, which is the ratio of the Higgs doublet vevs. Suppose that Y_1^{33} gives the top Yukawa coupling and up-type Higgs doublet is contained in H_1 dominantly. In order to break the up-down symmetry and generate CKM mixings, down-type Higgs doublet should contain H_2 component. In order to realize the muon/strange and electron/down-quark mass ratios, one needs $Y_2'^{22} \psi_2 \psi_2 A H_2 / M_*$ term. The muon Yukawa coupling is about 0.025 for $\tan\beta \simeq 50$ and one can find that $Y_2^{22} \sim Y_2'^{22} 3/\sqrt{30} a_{24}/M_* \sim 0.01$ can realize the muon/strange masses. In the current fit, a_{24} can be calculated from eq. (2.1) and one can check that $Y_2'^{22} \simeq 1$ (with $M_* = 2 \times 10^{18}$ GeV) can realize the proper masses. Because of the freedom of Higgs mixing and $\tan\beta$, the coefficient can be even smaller. After all, the fermion masses and mixings (including the neutrino mixings) can be realized in the current setup of SO(10) model. Compared to the models with $\mathbf{126}_H$, relations of fermion masses and mixings are less predictive, and we do not touch more detail of the Yukawa structure in this paper.

6 Conclusion

We have studied the dimension-6 proton decay in a SUSY SO(10) GUT with only $\mathbf{45}_\mathrm{H} + \mathbf{16}_\mathrm{H} + \overline{\mathbf{16}}_\mathrm{H}$ GUT-breaking Higgs fields. Since the SU(5)-breaking VEV of $\mathbf{45}_\mathrm{H}$ is induced by the Planck-suppressed, quartic superpotential for $\mathbf{45}_\mathrm{H}$, this VEV is larger than the SU(5)-conserving VEVs. This results in a 1/5 suppression of the $Q(\mathbf{3}, \mathbf{2}, 1/6)$ gauge boson mass compared to the $X(\mathbf{3}, \mathbf{2}, -5/6)$ gauge boson mass. On the other hand, the masses of the $SU(3)_c$ adjoint and $SU(2)_L$ adjoint particles from $\mathbf{45}_\mathrm{H}$ are much smaller than the X gauge boson mass and this enhances the latter when the unification condition is fulfilled. Still, the mass of the Q gauge boson can be below 0.6×10^{16} GeV (for $\lambda_2 = 1$ and $M_* = 2 \times 10^{18}$ GeV) and can thus give rise to a fast dimension-6 proton decay detectable at Hyper-Kamiokande.

Acknowledgments

A Mass spectrum

$\mathbf{45}_\mathrm{H}(A)$ and $\mathbf{16}_\mathrm{H}(\chi)$ representations are decomposed under SU(5) as $\mathbf{45}_\mathrm{H} = \mathbf{24} + \mathbf{10} + \overline{\mathbf{10}} + \mathbf{1}$ and $\mathbf{16}_\mathrm{H} = \mathbf{10} + \overline{\mathbf{5}} + \mathbf{1}$. One linear combination of the $\mathbf{10}$'s (and $\overline{\mathbf{10}}$'s) from $\mathbf{45}_\mathrm{H}$ and $\mathbf{16}_\mathrm{H}$ ($\overline{\mathbf{16}}_\mathrm{H}$) is absorbed by GUT gauge bosons, $Q : (\mathbf{3}, \mathbf{2}, 1/6)$, $U : (\bar{\mathbf{3}}, \mathbf{1}, -2/3)$, and $E : (\mathbf{1}, \mathbf{1}, 1)$. The other linear combination yields physical modes $\chi_Q : (\mathbf{3}, \mathbf{2}, 1/6)$, $\chi_U : (\bar{\mathbf{3}}, \mathbf{1}, -2/3)$, and $\chi_E : (\mathbf{1}, \mathbf{1}, 1)$. The mass matrix of each component of the $\mathbf{10} + \overline{\mathbf{10}}$'s can be written as

$$\begin{pmatrix} M_{AA} & M_{A\bar{\chi}} \\ M_{\chi A} & M_{\chi\bar{\chi}} \end{pmatrix}, \tag{A.1}$$

where

$$M_{AA} = m_A + 4a_1^2 \frac{\lambda_1 + \frac{1}{5}\lambda_2}{M_*} + 4a_{24}^2 \frac{\lambda_1}{M_*} + \frac{4}{5}\frac{\lambda_2}{M_*}\left(\frac{C_{Q,U,E}}{\sqrt{6}} a_1 a_{24} + \frac{D_{Q,U,E}}{6} a_{24}^2\right), \tag{A.2}$$

$$M_{A\bar{\chi}} = 2\kappa v_R, \tag{A.3}$$

$$M_{\chi A} = 2\kappa \bar{v}_R, \tag{A.4}$$

$$M_{\chi\bar{\chi}} = m_\chi + \frac{1}{\sqrt{5}}\kappa a_1 - \sqrt{\frac{2}{15}}\kappa C_{Q,U,E} a_{24} \tag{A.5}$$

where $(C_Q, C_U, C_E) = (1, -4, 6)$ and $(D_Q, D_U, D_E) = (19, 4, 9)$. We can verify that one eigenvalue is zero when the F-flat conditions are used. The mass of the physical mode is $M_{AA} + M_{\chi\bar{\chi}}$. In the limit with $m_A, m_\chi \ll M_*$, $M_{\chi\bar{\chi}}$ dominates, but M_{AA} can be non-negligible for χ_Q due to the large factor D_Q/C_Q. Using the minimization condition, we obtain $M_{AA} \simeq m_A(7 - D_{Q,U,E})\lambda_2/(30\lambda_1 + 7\lambda_2)$ for $m_A, m_\chi \ll M_*$.

The masses of isospin doublet and color triplet Higgses are obtained from the super-potential

$$W_H = M_{ij}H_iH_j + \lambda_H A H_1 H_2 + \lambda_\chi^i H_i \chi\chi + \bar{\lambda}_\chi^i H_i \bar{\chi}\bar{\chi}, \tag{A.6}$$

and the mass term is

$$(H_1^{\mathbf{5}} \ H_2^{\mathbf{5}} \ \bar{\chi}^{\mathbf{5}}) M_{H_{D,T}} \begin{pmatrix} H_1^{\bar{\mathbf{5}}} \\ H_2^{\bar{\mathbf{5}}} \\ \chi^{\bar{\mathbf{5}}} \end{pmatrix}, \tag{A.7}$$

$$M_{H_{D,T}} = \begin{pmatrix} M_{11} & M_{12} + \lambda_H A_{D,T} & \lambda_\chi^1 v_R \\ M_{12} - \lambda_H A_{D,T} & M_{22} & \lambda_\chi^2 v_R \\ \bar{\lambda}_\chi^1 \bar{v}_R & \bar{\lambda}_\chi^2 \bar{v}_R & m_\chi + \frac{\kappa}{\sqrt{5}}\left(-3a_1 + \sqrt{\frac{2}{3}}c_{D,T}a_{24}\right) \end{pmatrix}, \tag{A.8}$$

where

$$A_{D,T} = \frac{1}{\sqrt{5}}(a_1 + \frac{1}{\sqrt{6}}c_{D,T}a_{24}) \tag{A.9}$$

and $(c_D, c_T) = (3, -2)$. The doublet-triplet splitting needs fine-tuning. Without loss of generality, λ_χ^1 is set to zero by a rotation of (H_1, H_2). In this basis, by the fine-tuning $M_{11} = M_{12} + \lambda_H A_D = 0$, we have one pair of doublets massless. H_1 in this basis should dominantly give the large top quark Yukawa coupling. The mass of the corresponding triplet is roughly $\sim 5/3 \lambda_H A_D$ for $|a_1|, |v_R| \ll |a_{24}|$.

B Renormalizable model obtained by employing 54_H

In the main text, we have considered the model with $\mathbf{45}_H + \mathbf{16}_H + \overline{\mathbf{16}}_H$ and with non-renormalizable quartic terms of $\mathbf{45}_H$. In this appendix, for readers who prefer renormalizable models, we show that a renormalizable superpotential with $\mathbf{54}_H$ (whose SM singlet component is denoted by E) can also provide the wanted vacua where $|a_{24}| \gg |a_1|, |v_R|$ (and $|a_{24}| \gg |E|$).

The superpotential for the SM singlets is

$$W|_{\text{SM singlets}} = m_\chi v_R \bar{v}_R + \frac{1}{2}m_A(a_1^2 + a_{24}^2) + \frac{1}{2}m_E E^2 + \sqrt{5}\kappa\, a_1 v_R \bar{v}_R \tag{B.1}$$

$$+ \frac{\kappa_1}{3}E^3 + \kappa_2 E\left(\sqrt{6}a_1 a_{24} + \frac{1}{2}a_{24}^2\right) \tag{B.2}$$

From the F-flat conditions, we obtain

$$a_1 = -\frac{m_\chi}{\sqrt{5}\kappa}, \quad a_{24} = \sqrt{\frac{6}{5}}\frac{m_\chi}{\kappa}a, \quad v_R \bar{v}_R = \frac{m_\chi m_A}{5\kappa^2}\frac{6a^2 + a - 1}{a - 1}, \quad E = -\frac{m_A}{\kappa_2}\frac{a}{a - 1}, \tag{B.3}$$

where a is a solution of the following equation:

$$m_E = \kappa_1 \frac{m_A}{\sqrt{5}\kappa}\frac{a}{a - 1} + \frac{3\kappa_2^2 m_\chi^2}{5\kappa^2 m_A}(a - 1)(a - 2). \tag{B.4}$$

Vacua with $|E|, |a_1|, |v_R| \ll |a_{24}|$ are obtained by assuming $m_\chi, m_A \ll m_E$, which gives

$$a_{24} \simeq \frac{\sqrt{2m_A m_E}}{\kappa_2}. \tag{B.5}$$

The $\mathbf{54}_{\rm H}$ is decomposed as $\mathbf{54}_{\rm H} = \mathbf{24} + \mathbf{15} + \overline{\mathbf{15}}$ under SU(5). The mass matrices of the adjoint representations after SU(5) breaking are

$$\begin{pmatrix} m_A - 2\kappa_2 E & -2\kappa_2 a_{24} + \sqrt{6}\kappa_2 a_1 \\ -2\kappa_2 a_{24} + \sqrt{6}\kappa_2 a_1 & m_E - 4\kappa_1 E \end{pmatrix} \tag{B.6}$$

for $(\mathbf{8}, \mathbf{1}, 0)$ and

$$\begin{pmatrix} m_A + 3\kappa_2 E & 3\kappa_2 a_{24} + \sqrt{6}\kappa_2 a_1 \\ 3\kappa_2 a_{24} + \sqrt{6}\kappa_2 a_1 & m_E + 6\kappa_1 E \end{pmatrix} \tag{B.7}$$

for $(\mathbf{1}, \mathbf{3}, 0)$. For $a \gg 1$, $\kappa_2 E \simeq -m_A$, and we obtain the masses of the lighter adjoint fields (using $2m_A m_E \simeq (\kappa_2 a_{24})^2$) as $M_8 \simeq (3 - 4 \times 2)m_A = -5m_A$ and $M_3 \simeq (-2 - 9 \times 2)m_A = -20m_A$, and hence $M_8 : M_3 \simeq 1 : 4$. We have thus verified that the mass ratio $M_8 : M_3$ is the same as the model with the non-renormalizable terms, which is obtained by integrating out $\mathbf{54}_{\rm H}$.

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