On the Deflections of Uniformly Loaded Clamped Rhombic Flat Plates.*

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長谷川節・岡 正 巳： 一様分布横荷重による固定菱形板の撓みについて

The problem to be treated here are those of rhombic, uniformly laterally loaded, clamped thin plates. The deflection of the plates is small compared to the thickness. Let the angle between edges be α. The displacement of points in the middle surface of the plate will be calculated by the Ritz method for the cases $\alpha=45°$, 55°, 60°, 70°, 75°, and 90°.

1. Introduction

The problems of the bending of elastic square and rectangular plate have been treated in detail many researchers. Recently, as an airplane's velocity approched at the sonic velocity, the swept wing begun to be used and then the problem of the bending of the parallelogram elastic plate became one of the most important and interest problems. The problem to be treated here is that of a rhombic laterally loaded, clamped plate.

2. The mathematical analysis

Let the plate be situated with respect to the coordinates axes as shown in Fig. 1, z axis perpendicular to the plate.

The following notation will be used:

x, y, z=coordinates of a point,

$2a$=length of side of the plate;

h=thickness of the plate;

w=z-component of displacements of points in the middle surface;

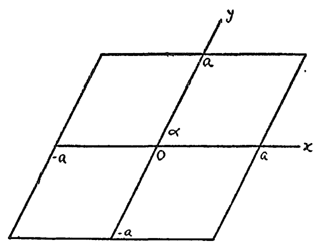

Fig. 1.

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E, μ=Young's modulus and Poisson's ratio;

$D=\dfrac{Eh^3}{12(1-\mu^2)}$, the flexural rigidity;

p=lateral load per unit of area;

U=total energy of the plate and load;

α=angle between x and y axes.

The differential equation for w is given by

$$\frac{D}{\sin^3\alpha}\left[\frac{\partial^4 w}{\partial x^4}-4\cos\alpha\frac{\partial^4 w}{\partial x^3\partial y}+2(1+2\cos^2\alpha)\frac{\partial^4 w}{\partial x^2\partial y^2}-4\cos\alpha\frac{\partial^4 w}{\partial x\partial y^3}+\frac{\partial^4 w}{\partial y^4}\right]=p. \tag{1}$$

When the four edges of the rhombic plate are clamped, the boundary conditions at the edges are

$$w=0, \quad \text{and} \quad \frac{\partial w}{\partial n}=0. \tag{2}$$

where n being normal to the boundary.

The problem will be treated by the Ritz method instead of solving Eq. (1). The total energy U of the plate is given by the following equation

$$U=\frac{D}{2}\int_{-a}^{a}\int_{-a}^{a}\frac{1}{\sin^3\alpha}\left\{(\Delta w)^2-4\cos\alpha\frac{\partial^2 w}{\partial x\partial y}\Delta w\right.$$
$$\left.+4\cos^2\alpha\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\right\}dx\,dy-\int_{-a}^{a}\int_{-a}^{a}pw\sin\alpha\,dx\,dy \tag{3}$$

where

$$\Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}.$$

Let us assume that

$$w(x,y)=(a^2-x^2)^2(a^2-y^2)^2\{a_{00}+a_{11}xy+a_{20}(x^2+y^2)+a_{22}x^2y^2$$
$$+a_{40}(x^4+y^4)+a_{31}(x^3y+xy^3)\}. \tag{4}$$

This function satisfies the boundary conditions (2) and the following symmetry conditions

$$w(x,y)=w(-x,-y)=w(y,x)=w(-y,-x).$$

If the constants a_{00}, a_{11}, a_{20}, a_{22}, a_{40}, and a_{31} are determined by the Ritz method from the equations

$$\frac{\partial U}{\partial a_{00}}=0,\ \frac{\partial U}{\partial a_{11}}=0,\ \frac{\partial U}{\partial a_{20}}=0,\ \frac{\partial U}{\partial a_{22}}=0,\ \frac{\partial U}{\partial a_{40}},\ \text{and}\ \frac{\partial U}{\partial a_{31}}=0, \tag{5}$$

which expresses the condition that the total energy has a minimum value, then w will express the approximate solution.

By the above mentioned, if we substitute Eq. (4) in w of Eq. (3) and then by the Eq. (5), we have

$$\left(\frac{1}{1225}+\frac{4m^2}{11025}\right)A_{00}+\frac{2m}{11025}A_{11}+\frac{2}{13475}A_{20}+\frac{1}{121275}A_{22}+\left(\frac{22}{675675}-\frac{8m^2}{363825}\right)A_{40}$$
$$+\frac{40}{363825}A_{31}=\frac{(1-m^2)^2}{57600}P,$$

$$\frac{2m}{11025}A_{00}+\left(\frac{67}{1091475}+\frac{4m^2}{99225}\right)A_{11}-\frac{4m}{51975}A_{20}+\frac{2m}{363825}A_{22}-\frac{236m}{4729725}A_{40}$$
$$+\left(\frac{434}{14189175}+\frac{16m^2}{1091475}\right)A_{31}=0,$$

$$\frac{2}{13475}A_{00}-\frac{4m}{51975}A_{11}+\left(\frac{528}{1576575}+\frac{8m^2}{121275}\right)A_{20}+\frac{2}{75075}A_{22}+\left(\frac{788}{4729725}+\frac{16m^2}{675675}\right)A_{40}$$
$$-\frac{8m}{4729725}A_{31}=\frac{(1-m^2)^2P}{201600},$$

$$\frac{1}{121275}A_{00}+\frac{2m}{363825}A_{11}+\frac{2}{75075}A_{20}+\left(\frac{5}{693693}+\frac{4m^2}{1334025}\right)A_{22}+\frac{1}{52026975}A_{40}$$
$$-\frac{16m}{5780775}A_{31}=\frac{(1-m^2)^2P}{2822400},$$

$$\left(\frac{22}{675675}-\frac{8m^2}{363825}\right)A_{00}-\frac{236m}{4729725}A_{11}+\left(\frac{788}{4729725}+\frac{16m^2}{675675}\right)A_{20}+\frac{1}{52026975}A_{22}$$
$$+\left(\frac{299192}{2653375725}+\frac{496m^2}{31216185}\right)A_{40}-\frac{1184m}{52026975}A_{31}=\frac{(1-m^2)^2P}{604800},$$

$$\frac{40m}{363825}A_{00}+\left(\frac{434}{14189175}+\frac{16m^2}{1091475}\right)A_{11}-\frac{8m}{4729725}A_{20}-\frac{16m}{5780775}A_{22}$$
$$-\frac{1184m}{52026975}A_{40}+\left(\frac{4976}{156080925}+\frac{1912m^2}{156080925}\right)A_{31}=0, \tag{6}$$

where $A_{00}=a^8a_{00}$, $A_{11}=a^{10}a_{11}$, $A_{20}=a^{10}a_{20}$, $A_{22}=a^{12}a_{22}$, $A_{40}=a^{12}a_{40}$, $A_{31}=a^{12}a_{31}$, $P=\frac{pa^4}{D}$ and $\cos\alpha=m$.

From these equations, we solve A_{11}, A_{20}, A_{22}, A_{40}, A_{31}, P in terms of A_{00} and α. The calculated values of these constants for various values of α are given in Table 1.

Table 1

A/A_{00} \ α	45°	55°	60°	70°	70°	90°
A_{20}/A_{00}	−1.8083	−1.3327	−1.1031	−0.6858	−0.4987	0
A_{11}/A_{00}	−0.2594	−0.0619	0.0240	0.1562	0.2005	0.2571
A_{22}/A_{00}	2.3928	1.6382	1.2959	0.7480	0.5490	0.3102
A_{40}/A_{00}	−0.0287	−0.0431	−0.0406	−0.0209	−0.0040	0.0128
A_{31}/A_{00}	−0.1386	−0.3868	−0.4532	−0.4447	−0.3747	0
P/A_{00}	169.48	99.252	81.429	61.18	55.63	49.39

In Table 1, the values of $A_{11}, \dots, A_{31}$ and P for $\alpha=90°$, that is for the square plate, are also given for reference. As this solution for the square plate shows good agreement with that of Hencky, we may consider that the above solutions would be of fairly good accuracy.

A_{00}, the deflection of the plate at its center, is the maximum deflection for various loads. Fig. 2 shows the curves for $\frac{W_{max}}{h}$, corresponding to various values of α, as functions of P/h.

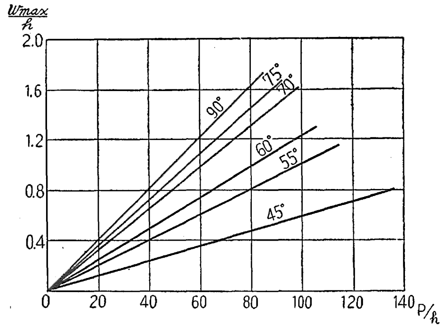

Fig. 2. Deflection load diagrams for 45°, 55°, 60°, 70°, 75° and 90°.

3. Summary

The problem treated here are those of rhombic, laterally loaded, clamped plate. The Ritz energy method is appled. The deflections of the plates are calculated for $\alpha=45^{\circ}$, 55°, 60°, 70°, 75° and 90° We shall treat the problems of parallelogram, laterally loaded, clamped plate.

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