Improvements of
the Rotary Grating Ultrasonic Velocity Meter

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石 田 美 雄：回転格子式超音波速度測定装置の改良

1. Introduction

The rotary grating ultrasonic velocity meter is a convenient apparatus by which the sound velocity of the liquid can be measured rapidly. It is based on a variation of Debye-Sears ultrasonic diffraction method. The principle, the experimental process of measurement and some data taken by means of it have been published befote.[1) 2)] In the preceding cases, the rotary grating whose proper constant is 1.369×10^{-2} cm, slightly smaller than the wavelength of the measured liquid has been used. The author reports here the case in which the proper constant of the rotary grating is 2.000×10^{-3} cm, far smaller than the wavelength of the measured liquid. By using such a rotary grating, detection of the coincidence of diffraction lines has become easier. Consequently, the accuracy of the apparatus has been promoted.

In the preceding papers, the measured values were somewhat higher than the acknowledged values and the reason could not be clarified. Here are reported the results of the analysis of the reason on the basis of the following four hypotheses.

a) The deviation from the exact parallelism of the light beam might affect the velocity value.

b) The friction between the liquid and the wall of the sample container might affect the velocity value, when the width of the ultrasonic beam is larger than that of the sample container. In order to test this hypothesis, a " Beam Forming Cylinder " is used to control the width of the beam.

c) The curving up of the free surface of the sample liquid by intense ultrasonic waves might change the proper wavelength near the surface. In order to test the effect, the apparatus is equipped with a reflector.

d) The deviation from tne parallelism of the wave front and the slit might affect the velocity.

2. Experimental Arrangement

Before stating the results of the experimental verification of the hypotheses, the author wants to say about experimental arrangement. Fig. I shows a vertical section of the main part of the experimental arrangement.

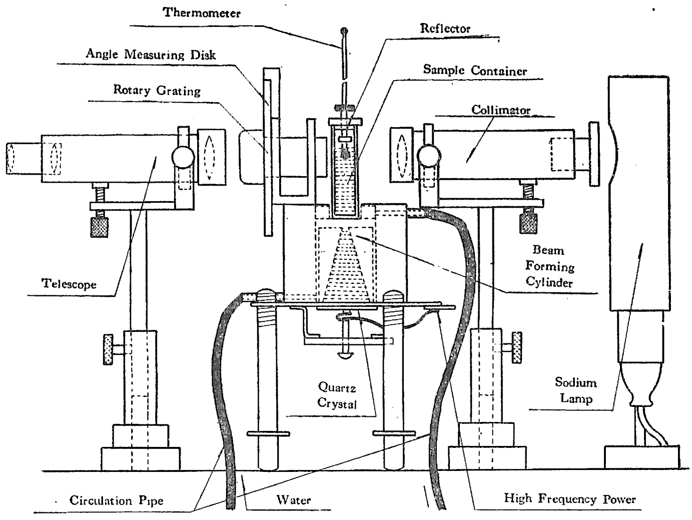

Fig. I Vertical Section of the Experimental Arrangement

The light beam emitted from a sodium lamp is made parallel by a collimator which has a horizontal slit and enters the sample container. It is diffracted doubly by the ultrasonic waves and the rotary grating. The diffraction image can be observed by the telescope.

The ultrasonic waves are generated by a 5Mc/s piezo-electric crystal of X-cut and 30mm long in diameter. The generated waves go up through the water tank, the width being reduced by the beam forming cylinder and enter the sample cotainer through the glass base of the container. The reflector assembly which is made up of a reflector driven by a screw and a thermometr, is mounted on the sample container. The beam forming cylinder is a massive metal cylinder, whose inside surface is conically shaped. The diameters of the lower and the upper mouths of the beam forming cylinder are 20 mm and 5 mm respectively. By the way, the shortest linear dimension of the sample container is 15 mm. The beam forming cylinder is removable.

The oscillator which drives the ultrasonic crystal is crystal controlled and output amplifier is composed of two 807′ s in parallel whose plate dissipation is about 80 watts.

The temperature of the sample container is controlled by circulating warm or cool water respectively through the water tank.

An angle measuring disk is attached to the cylinder in which the rotary grating is fixed. The circumference of the disk is divided into 360 equal parts. The angle can be measured to two minutes by an attached vernier.

3. Process of the Measurement by the New Grating

The proper constant of the new grating has been chosen far smaller than the wavelength of the measured liquid. The diffraction image generated by the ultrasonic waves only has several lines in the full view of the telescope. Thus, the diffraction image generated by the new rotary grating alone shows only first order lines in the view, while there appear not only first order lines but also higher lines in the case of an old grating whose proper constant is of the same order as the wavelength of the measured liquid. Therefore, the composite diffraction image which is generated both by a grating and ultrasonic waves is much simpler in the case of a new grating than in the case of an old one.

The diffraction images corresponding to the rotation of the gratng in the case of a new grating are shown in Fig. 2. Figure (a) shows the diffraction image when the lines of the grating are perpendicular to the slit and only the image of the slit is seen. Figures (b) and (c) show the appearances when the grating is rotated and there appear two rotary main lines which are first order lines generated by a grating only. Figure (d) shows the appearance when the ultrasonic waves are applied. Figures (e) and (f) show the appearances when the grating is rotated from the state (d). The rotary main lines move followed respectively by the diffraction lines which are generated by the ultrasonic waves. On the way, there occur successively the positions that the rotary main lines coincide with diffraction lines generated by the ultrasonic waves only. Figures (g) and (h) show positions of the first order coincidence at which the rotary main lines coincide with first order lines generated by the ultrasonic waves only, and those of the third order coincidence respectively.

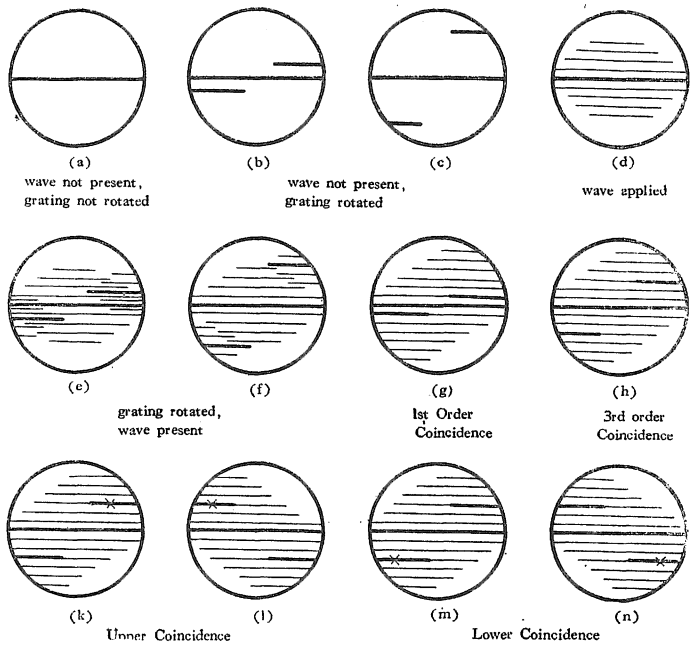

Fig. 2 Diffraction Images Seen in the View of the Telescope

In the measurements, the third order coincidence is of the highest order among the coincidences which are easily observed. First, by rotating a grating anti-clockwise the coincidence position is sought in the upper half of the view, as shown in Fig. (k) and the angle reading θ_1 is obtained. Secondly, by rotating it clockwise, the coincidence in the upper half as shown in Fig. (l) and the angle reading θ_2 are determined. Then, we get

$$\theta = \frac{1}{2}(\theta_1 - \theta_2).$$

This is called the upper coincidence angle. The angle θ is the angle which corresponds to the wavelength. The angles θ' s are all measured from the position at which the lines of grating and the slit are perpendicular. In the same way, we can get the lower coincidence angle, shown in Figures (m) and (n). The upper and the lower coincidence angles should be the same. But, in order to compensate the errors originated from the mutual inclinations of the grating, the collimator and the telescope, the upper and the lower coincidence angles are sought alternately in the measurements.

Considering that diffraction lines coincide with each other when the wavelength of the ultrasonic waves becomes integral multiple of the effective constant of the grating d, we obtain

$$\Lambda = nd,$$

where Λ is the wavelength, n is a positive integer. It has been ascertained that the formula

$$d = d_o/\sin\theta \tag{1}$$

holds good between the effective constant d and the proper constant d_0 of the rotary grating. Therefore, we get as a velocity calculating formula

$$V = \frac{nd_o f}{\sin\theta} \tag{2}$$

where f is the frequency of the wave. In the present case, substituting the numerical values, $d_o = 2\times10^{-3}$ cm, $f = 5\times10^6 c/s$, and $n = 3$ in (2), we obtain

$$V = \frac{300}{\sin\theta} \quad (\mathrm{m/sec}) \tag{3}.$$

4. The Advantage of the Method over the Ordinary Debye-Sears Method

In Debye-Sears method, the angle of light diffraction is measured directly by means of a spectrometer or photographing, and ordinarily the angle which should be measured is very small. In our method, the angle of light diffraction is measured, so to speak, indirectly by means of a rotary grating, and the angle which should be measured is much larger than the former one. Therefore, the accuracy of the angle measurement in this method should be theoretically better than that in Debye-Sears method. We discuss this point in some details.

The velocity calculating formula in Debye-Sears method is

$$V = \frac{n\lambda f}{\sin\varphi} \tag{4},$$

where, n is the order of the light diffraction, λ the wavelength of the light, f the fre- -quency of the ultrasonic wave and φ the angle by which the telescope of a spectrometer is rotated. We can calculate from the formula the error of the velocity measurement ΔV which corresponds to the error of the angle measurement $\Delta\varphi$.

Taking the differentials of the formula (4), we get

$$\Delta V = -n\lambda f \frac{\cos\varphi}{\sin^2\varphi} \frac{\pi}{180} \Delta\varphi$$

where, $\Delta\varphi$ is expressed in degree. Substituting the numerical values $n = 1$, $\varphi = 7'$ (in the case of water) and $\lambda = 6\times10^{-5}$ cm (sodium light) in it, we get

$$\frac{\Delta V}{\Delta\theta} = -13\times10^3 \,(\mathrm{m/sec/deg}) \tag{6}$$

This means that the error of the velocity measurement becomes about 200 (m/sec) when the error of the angle measurement is 1′. Even when $n = 20$ (that is the highest order we can utilize), we obtain that $\Delta V = 10$(m/sec) for $\Delta\varphi = 1'$.

The velocity calculating formula in our method shown in (2) has the same form as (4). Therefore, the error formula should be written in the same form,

$$\Delta V = -nd_o f \frac{\cos\theta}{\sin^2\theta} \frac{\pi}{180} \Delta\theta \qquad (7).$$

Substituting the numerical values, $n = 1$, $\theta = 3°48'$ (for water), and $d_o = 2\times10^{-3}$cm in it, we get

$$\frac{\Delta V}{\Delta\theta} = -4\times10^2 \text{ (m/sec/deg)} \qquad (8).$$

This means that $\Delta V = 6.7$ (m/sec) for $\Delta\theta = 1'$. When $n = 3$, we get $\Delta V = 2$ (m/s) for $\Delta\theta = 1'$. Therefore, if the angle measurement would be performed with the same accuracy, the rotary grating method could give better value than Debye-Sears method.

In the preceding discussion of Debye-Sears method, it is assumed that a spectrometer system is used. When a photographing system is used, its accuracy is another problem. But, as photographing needs considerable time for developing, the rotary grating method has a advantage over it in respect to the necessary time.

5. Treatment of Data and Experimental Error

Angle measurements are performed ten times, taking upper and lower coincidences (or right and left coincidences) alternately. Grouping these ten values, we take their arithmetic mean and calculate the probability error. We cite two instances.

Example 1. Velocity of ethyl alcohol at 30°C, density $\rho = 0.7842$ (g/cm^3)

Angle θ	Velocity
15° 26′	1128
15° 20′	1135
15° 31′	1121
15° 28′	1125
15° 27′	1126
15° 17′	1138
15° 21′	1133
15° 22′	1132
15° 25′	1129
15° 26′	1128

Mean value of velocity	1129.5(m/s)
Probabiliy error	±1.1(m/s)
Max. angle deviation from the mean value	7′
Max. velocity deviation from the mean value	8.5(m/s)
Reference value from Weissler's measurement[3]	V=1127.4(m/s)
	ρ=0.7809(g/cm³)

Example 2. Velocity of methyl alcohol at 30°C, $\rho=0.7869$(g/cm³)

Angle θ	Velocity
15° 58′	1091
15° 58′	1091
15° 52′	1097
15° 50′	1099
16° 00′	1088
15° 53′	1096
15° 56′	1093
15° 59′	1089
16° 00′	1088
15° 52′	1097

Mean value of velocity	1092.9 (m/s)
Probability error	±0.9 (m/s)
Max. angle deviation from the mean value	6′
Max. velocity deviation from the mean value	6 (m/s)
Reference value from Weissler's measurement	V=1088.9 (m/s)
	ρ=0.7816 (g/cm³)

In these examples, measured values are scattered in some degree, but their arithmetic means coincide fairly with acknowledged values.

6. Effect of the Deviation from the Exact Parallelism of the Light Beam

We test how the divergency and convergency of the light beam will affect the velocity measurement. The position of the slit of collimator is moved both ahead of and to the rear of the position which gives exact parallelism of light beam and the velocity is measured by adjusting a telescope to it. The position which gives exact parallelism, is chosen zero position and the sense in which the slit is moved from the light source, posi-tive. Namely, the slit position which gives divergent ray becomes positive. Fig. 3 gives the graph which represents the relation between the velocity change and the slit position. The measurement is made using distilled water at 30°C. The velocity which corresponds to zero position is 1510. 5(m/s).

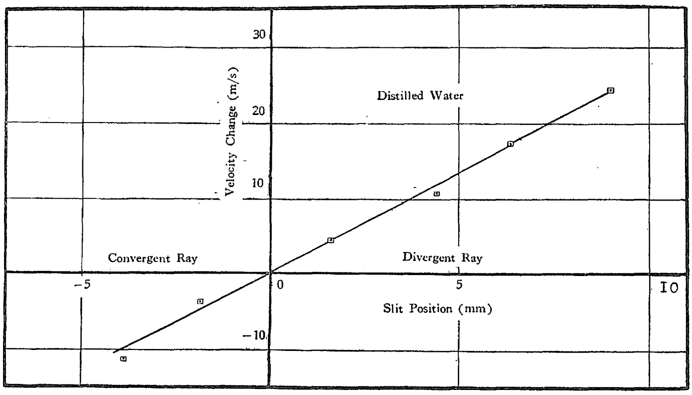

Fig. 3 The Relation between Velocity Change and Slit Position

This graph shows the linear relation between the velocity change and the slit position. It can be concluded that the values of velocity becomes higher when it is measured by divergent ray and lower when measured by convergent ray.

7. Influence of the Beam Forming Cylinder and Reflector

We measure the velocities of distilled water at 30°C with and without beam forming cylinder and compare the results of the both cases. We do the same thing with and without reflector. The following are respectively the weighted mean values taken from the three groups.

I. The case with beam forming cylinder and reflector

$V = 1511.0 \pm 0.7$ (m/s)

II. The case without beam forming cylinder, but with reflector

$V = 1510.5 \pm 1.3$ (m/s)

III. The case without beam forming cylinder and reflector

$V = 1508.9 \pm 0.5$ (m/s)

Comparing I and II, we can conclude that the beam forming cylinder has no effect on the measured value of sound velocity, namely, that the friction between the liquid and wall of the sample container does not affect the value of sound velocity. Comparing II and III, it seems that the wavelength becomes slightly shorter when there is no reflector than when there is. In order to confrim the point, the following experiment is done. The horizontal aperture is attached on the head of the collimator and the velocity is measured without reflector by passing a restricted light beam through the part of liquid which is directly under the free surface. In that case, the diffraction lines becomes vague and unstable. Therefore, the measured values are less reliable than the foregoing values. But they almost show lower values and the result of this experiment supports the foregoing conclusion. It seems that the shortening of the wavelength is related with the curving up of the free surface and occurs only when the ultrasonic waves are considerably intense.

8. Effect of the Inclination of the Slit to the Wave Front

We consider how the velocity value will change when the slit inclines some angle φ to the wave front. As it is the same when the ultrasonic grating is rotated by angle φ refering to the fixed slit, we can apply to this case the rotary grating formula (1), and get

$$\Lambda = \frac{\Lambda_0}{\sin\,(90^\circ - \varphi)} = \frac{\Lambda_0}{\cos\varphi} \qquad (9),$$

where Λ is the measured wavelength and Λ_0 is the true wavelength. While in formula (1), the angle is measured from the position at which the slit is perpendicular to the grating lines, it is measured in this case from the position at which the slit is pararell to the wave front. Therefore, $\theta = 90^\circ - \varphi$.

From the formula (9), we can see that measured velocity shows always higher value when the slit inclines to the wave front. So, we should correct the measured value, by the multplication of cos φ. But it is difficult to know the angle of inclination φ or to adjust the slit exactly parallel to the wave front. In order to keep the error which occurs from this cause, within 1/1000, we must restrict the angle angle φ within $2^\circ 30'$. As a practical problem, we have no good idea for the present except that the slit is so adjusted as to give the acknowledged value referring to the standard liquid.

9. Conclusion

a. Accuracy of the apparatus

The arithmetic mean of the probability errors which were obtained from many groups of measurement referring to distilled water is ± 1.6m/s. This means that the accuracy of the apparatus is 1/1000. It is said that the accuracy of the ultrasonic interferometer ordinarily adjusted is 1/1000. Therefore, it can be said that the rotary grating ultrasonic velocity meter has reached the stage of the practical use.

b. Defect of the method

It has been pointed out that the fundamental defect of the method is the use of considerably intense ultrasonic wave. In the foregoing experiments, we used in fact the considerably intense ultrasonic wave in order to clarify the procedures of the method, but it is possible to practise this method by using weak ultrasonic waves which produce only the first order diffraction line .

10. Acknowledgement

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