On the Velocity Distribution in the Turbulent Wake behind a Body of Revolution.

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I Introduction

The velocity distribution at a considerable distance downstream in the turbulent wake behind a body of revolution was calculated by Swain[1] on the momentum transfer theory and by Goldstein[2,3] on the vorticity transfer theories. We shall consider the same problem by assuming the apparent turbulent coefficient of diffusion of the appropriate form.

II The Fundamental Theory

In a wake behind a body of revolution, in which the mean motion is assumed symmetrical about its axis, let (r, θ, x) be cylindrical polar coordinates, x being measured along the axis of symmetry and r being at a distance from it. If U_o is the velocity of the undisturbed stream, $U_o - U$ the velocity in the wake parallel to the axis, and τ the shearing stress, then far downstream the equation for U is approximately

$$U_o \frac{\partial U}{\partial x} = -\frac{1}{\rho}\frac{1}{r}\frac{\partial}{\partial r}(r\tau), \qquad (1)$$

where ρ is the density of a fluid.

If we now introduce the apparent turbulent coefficient of diffusion E, we have the following expression for the shearing stress τ

$$\tau = -\rho E \frac{\partial U}{\partial r}. \qquad (2)$$

With the same assumptions as in the two-dimensional case——geometrical and mechanical similarity in different sections of the wake——, we use the following expression for U

$$\frac{U}{U_o} = \frac{f(\eta)}{x^{2/3}}, \qquad (3)$$

where

$$\eta = \frac{r}{x^{1/3}} \qquad (4)$$

We assume that the apparent turbulent coefficient of diffusion E takes the form

$$E = \varepsilon \frac{\eta^n}{x^m}, \qquad (5)$$

where m and n are undetermined constants.

With Eqs. (2), (3), (4) and (5), the differential equation (1) is transformed in

$$-\frac{U_o}{3x^{5/3}}\left\{2f(\eta)+\eta f'(\eta)\right\}=\frac{\varepsilon}{x^{m+4/3}}\,\frac{(\eta^{n+1}f'(\eta))'}{\eta}, \qquad (6)$$

in which the prime denotes the differentiation with respect to η. From the similar condition in different sections, we have

$$m=\frac{1}{3}. \qquad (7)$$

With this value of m Eq. (6) is integrated to

$$-\frac{U_o}{3\varepsilon}\eta^2 f(\eta)=\eta^{n+1}f'(\eta)+C.$$

As the value of $f(\eta)$ is finite and the value of $f'(\eta)$ zero at $\eta=0$, the constant C must be zero. Then,

$$-\frac{U_o}{3\varepsilon}\eta^{1-n}=\frac{f'(\eta)}{f(\eta)}. \qquad (8)$$

Integrating Eq. (8), we have

$$f(\eta)=Ae^{-\frac{U_o}{3\varepsilon(2-n)}\eta^{2-n}}, \qquad (9)$$

where A is the integration constant.

Then, we obtain the expession for U:

$$\frac{U}{U_o}=A\,x^{-2/3}\,e^{-\alpha\eta^{2-n}} \qquad (10)$$

in which

$$\alpha=\frac{U_o}{3\varepsilon(2-n)}. \qquad (11)$$

If we denote the velocity U at $\eta=0$ by U_m, we have

$$U=U_m\,e^{-\alpha\eta^{2-n}}. \qquad (12)$$

If we denote by η_* the value of η, at which the value of U is half the value of U_m, we have

$$\alpha=\frac{0.69315}{\eta_*^{2-n}}. \qquad (13)$$

With this value of α, the expression of U takes the form

$$\frac{U}{U_m}=e^{-0.69315\left(\frac{\eta}{\eta_*}\right)^{2-n}}. \qquad (14)$$

III Comparison with the experimental results and other solutions.

We have not yet determined the value of n in the expression of U. We must determine it in such a manner that the distribution of U/U_m given by Eq. (14) agrees with the experimental distribution of U/U_m given by Hall and Hislop.[4] It seems to us that the value of $n=1/5$ gives the best agreement with the experimental results in the range $0 \leqq \eta/\eta* \leqq 1.25$. Therefore, if we take $n=1/5$ the expression of U becomes

$$\frac{U}{U_m} = e^{-0.69315\left(\frac{\eta}{\eta*}\right)^{9/5}} \qquad (15)$$

The comparison of the above theoretical curve (15) with the results of measurements by Hall and Hislop is shown in Fig. 1, where U/U_m is plotted against $\frac{r}{R} = \frac{\eta}{\eta*}$ and $r=R$ when $U=1/2 U_m$. The full line curve is the theoretical curve, the small circles are the experimental mean results.

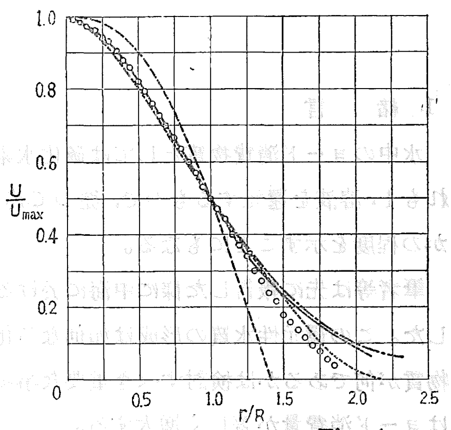

Fig. 1

The comparison of the result (15), with those of the momentum transfer theory and vorticity transfer theories is also shown in Fig 1, where the broken line---- is given by the vorticity transfer theory, the chain-dotted line-·-·- by the modified vorticity transfer theory, the dotted line ······ by the momentum transfer theory. From this figure, we see that Eq. (15) of U shows best agreement with the experimental results in these theoretical curves.

III Conclusion

With the assumptions of geometrical and mechanical similarity in different sections of the wake and the expression (5) of the apparent turbulent coefficient of diffusion, the velocity distribution at a considerable distance downstream in the turbulent wake behind a body of revolution was calculated. The expression of U takes the form (15) and shows the best agreement with the experimental results in the theoretical curves hitherto obtained.

References

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4. Hall, A. A, and Hislop, G. S., Velocity and Temperature Distribution in the Turbulent wake behind a Heated Body of Revolution, Proc.. Camb. Phil Soc., Vol.34, pp.345 350, 1938