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A Duality botween the Quark-Orbital Trajectories and Its Constraints for Mesons and Baryons

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Using the new method of imposing semi-local duality constraints recently proposed by Hoyer and Uschersohn, we present a duality between the quark-orbital trajectories. Its constraints for meson-meson and meson-baryon scattering are tested and found to be consistent with experiments. It is especially remarked that the scalar mesons are successfully accommodated to the duality scheme, and both of the meson-meson and meson-baryon scattering are understandable in the common way. This gives a support for the supposition that the dynamics of hadrons is governed essentially by the quark-orbital trajectory, made by Bando et al. and Nakkagawa et al.

§1. Introduction

Recently, Hoyer and Uschersohn[1] (HU) have proposed a new method of imposing semi-local duality constraints for the meson-meson scattering, starting from the finite-energy sum rules (FESR's) and modifying them by the requirements of a symmetric treatment of direct and crossed channels and a self-consistency. The duality introduced by HU is significantly different from the one realized in the dual resonance model,[2] and is meaningful for a better understanding of the duality phenomena. HU's method is more interesting than other models. The FESR's, which are taken as the starting point by HU, are successful in a wide range of hadronic processes.[3] While, dual models[2] and the dual unitarization framework[4] meet well-known considerable difficulties, when the baryons are introduced.

In this paper, we use HU's method to obtain a duality scheme where the involved states are just the harmonic-oscillator spectrum of $SU(6)\otimes O(3)_L$ multiplets.*) This spectrum reflects the real hadron spectrum very well.

By studying the hadron dynamics and the hadron spectrum in the quark model, Bando et al.[5] and Nakkagawa et al.[6] have presented a supposition that the dynamics of hadrons is governed essentially by the spatial motion at the quark level, that is, by the quark-orbital trajectory in the Regge terminology, not by the usual J-plot (Chew-Frautschi plot) trajectory. On the basis of the supposition, our work will be done.

According to Nakkagawa et al., in the symmetry limit of $SU(6)\otimes O(3)_L$ the quark-

*) The preliminary results of the present work are reported in the reference 22).

orbital trajectories for the mesons or baryons reduce to a single universal trajectory (and its daughters) for each $SU(6)$ multiplet. This is also assumed.

In § 2, a duality scheme is presented. And, the connection among this scheme and other models is stated. In § 3, the duality constraints for meson-meson scattering are compared with experiments. In § 4, the meson-baryon scattering is discussed in the scheme. We summarize the results and make some discussions in § 5.

§ 2. Semi-local duality relations

HU have stressed the necessity of a symmetric treatment of the s- and t-channel states. This treatment is indeed required by the crossing symmetry in some processes. The semi-local duality relations, obtained by HU modifying the FESR's, are as follows, e.g. for the $\pi^-\pi^+\to\pi^-\pi^+$ scattering:

$$R_k^{(s)}(t)|_{t=m_l^2}=R_l^{(t)}(s)|_{s=m_k^2}, \quad k,\ l=\rho,\ f,\ g,\ h,\dots . \tag{2.1}$$

Here, $R_k^{(s)}(t)$ ($R_l^{(t)}(s)$) is the residue of the contribution of s-channel state k (t-channel one l) to the scattering amplitude, in the narrow resonance approximation.

Their relations are in a good agreement with experiments. There remains, however, a question: what set of s- and t-channel states satisfies such a semi-local duality relation generally, including the meson-baryon scattering?

It is noted that the relations (2.1) are satisfied by the B_4 amplitude[7]

$$A(\pi^-\pi^+)=\lambda\frac{\Gamma(1-\alpha_{\rho-f}(s))\Gamma(1-\alpha_{\rho-f}(t))}{\Gamma(1-\alpha_{\rho-f}(s)-\alpha_{\rho-f}(t))}, \tag{2.2}$$

when all the contributions from the daughters appearing in the amplitude (2.2) are omitted.*)**) Here, $\alpha_{\rho-f}$ denotes the exchange-degenerate $\rho-f$ trajectory.

We try to generalize and modify HU's relations to involve all the states of the harmonic-oscillator pattern of $SU(6)\otimes O(3)_L$ multiplets on the assumptions stated in § 1.***) In other words, we intend to gain (semi-local) duality relations which realize a duality between the quark-orbital trajectories. Judging from the fact that the FESR's or the superconvergence relations often lead to the symmetry of resonance couplings when they are saturated by resonances in an $SU(6)$ multiplet,[8] this trial is supposed to be promising.

We consider reactions $ab\to cd$ where a channel (the u-channel) is exotic. a, b, c

*) The relations (2.1) are satisfied also by a more general Veneziano-type amplitude having satellite terms symmetric under $s\leftrightarrow t$.

**) Through this work, we neglect the possible non-degeneracy of Regge slopes,[20] and assume the universal slope, for simplicity.

***) HU's model systematizes the scalar mesons δ, S^* etc. as the daughters of the vector mesons. And HU found their contributions to be small, and neglected them. Thus, our attempt is different from HU's model.

and d are taken as the $J^P=0^-$ mesons in this section and § 3, for simplicity. (Then, the $SU(6)$ singlet meson is forbidden to exchange.) Let $\{P_L\}$ ($\{P'_{L'}\}$) be the particle group consisting of s-channel parent particles with a fixed quark-orbital angular momentum L and their satellites (L-plane daughter particles) in a process $ab\to cd$ (that of t-channel parents with L' and their satellites). The duality scheme, which we want to present here, is a set of constraints on the scattering amplitude for $ab\to cd$ as

$$\sum_{P_L\in\{P_L\}} R^{(s)}_{P_L}(t)|_{t=m^2_{L'}} = \sum_{P'_{L'}\in\{P'_{L'}\}} R^{(t)}_{P'_{L'}}(s)|_{s=m^2_L}, \quad L,\ L'=0,\ 1, \dots , \tag{2,3}$$

where m^2_L ($m^2_{L'}$) is defined to be the average squared-mass of $\{P_L\}$ ($\{P'_{L'}\}$).

Eq. (2.3) takes a rigid form in the symmetry limit of $SU(6)\otimes O(3)_L$ by the help of the universal quark-orbital trajectory for the 35-plet mesons α_L:

$$\sum_{P_L\in\{P_L\}} R^{(s)}_{P_L}(t)|_{t=\alpha_L^{-1}(L')} = \sum_{P'_{L'}\in\{P'_{L'}\}} R^{(t)}_{P'_{L'}}(s)|_{s=\alpha_L^{-1}(L)}. \tag{2.4}$$

The relations (2.4) are satisfied by the amplitude suggested by Bando et al.[5]

$$f(s,\ t)\sim[a+b(s+t)]\frac{\Gamma(-\alpha_L(s))\Gamma(-\alpha_L(t))}{\Gamma(-\alpha_L(s)-\alpha_L(t))}\ . \tag{2.5}$$

In the amplitude (2.5), the momentum dependence of the factor $[a+b(s+t)]$ comes from the spin effects of quarks.

The connection between the semi-local dualities (2.1) and (2.4) and that between the B_4 formula (2.2) and the amplitude (2.5) are, of course, similar. In fact, the amplitude (2.2) can be interpreted as an approximation of the amplitude (2.5). If we retain only the $J=L+1$ states having the highest Regge intercept and impose the PCAC consistency condition,[9] Eq. (2.5) becomes an amplitude formally equivalent to Eq. (2.2). Here, we have assumed that $\alpha_{\rho-f}(m_\pi^2)=1/2$. The comparison of the scheme proposed here and other models is summarized in the last of this section in Fig. 1.

The reasons why we take the scheme (2.3) (or (2.4)) instead of the model as (2.5) are as the following:

(A) The model (2.5) needs, presumably, an infinite number of satellite terms in order to assure a correct residue for each of involved states. And, a new principle to intro-

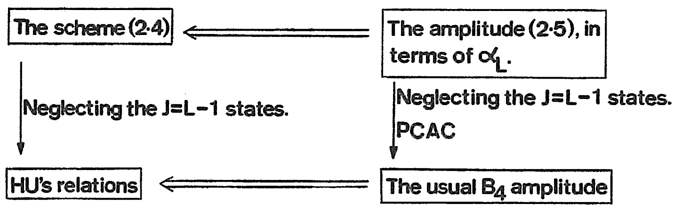

Fig. 1. Comparison of the model presented here with other ones, for the $\pi\pi\to\pi\pi$ scattering.

duce satellite terms is necessary.

(B) The models using the beta or generalized beta function are based on the exchange-degeneracy of the corresponding even-parity and odd-parity trajectories. Such exchange-degeneracies hold, however, only approximately for the baryons even in the symmetry limit of $SU(6)\otimes O(3)_L$.[6] (The universal trajectory for the 56-plet baryons is $\alpha_L(56)=-1.3+0.90s$; while, that for the 70-plet baryons is $\alpha_L(70)=-1.7+0.90s$.[6]) Thus, the model as (2.5) may not be so successful for the meson-baryon scattering, as for the meson-meson scattering.

The scheme (2.3) is free from the above difficulties (A) and (B). If we can find a well-defined quantity for m_L^2 in the relation (2.3), then our aim will be accomplished.

In the rest of this work, we define m_L^2, on trial, as the average squared-mass of $\{P_L\}$, $m_L^2=\Sigma_{P_L\in\{P_L\}}m_{P_L}^2/(\text{no. of } P_L\text{'s in } \{P_L\})$.

It will be found in the comparison with experiments that the relation (2.3) is indeed an improvement of HU's relation. HU's relations are in a good agreement with experiments. The relations (2.3) are valid at a higher accuracy level.

For convenience' sake, the relations (2.3) are denoted simply by $[P_L\text{'s}]=[P'_{L'}\text{'s}]$ hereafter. For example, for the $\pi^-\pi^+\to\pi^-\pi^+$ scattering, they are $[\rho]=[\rho]$, $[\rho]=[f, \varepsilon]$, $[f, \varepsilon]=[\rho]$,.... In the relations, the left-hand sides are always the s-channel states, and the right-hand sides the t-channel ones. We call each of the relations the relation (2.3) in the case of $(N, N')=(0, 0)$ (for the $\pi^-\pi^+\to\pi^-\pi^+$ scattering), that in the case (0, 1), that in the case (1, 0),... respectively, as the particles in each group have the same excitation quantum number.

§3. Relations for the moson-meson scattering

We compare the semi-local duality relations (2.3) with experiments, taking the 0^-–0^- scattering.

The assignment of observed resonances to the theoretical states is done as usual, as in the recent work on the meson spectroscopy in the quark shell model by Martin and Reinders.[10] As for the 0^+ mesons, there remain some problems in their assignments. According to Jaffe,[11] the lower-mass 0^+ states are assumed to be $2q2\bar{q}$ mesons. Thus, to the $N=1$, ${}^3P_0$ states, the $\varepsilon(1300)$, $\kappa(1400)$[12] and the K^-K° resonance with mass $\sim$1300 MeV[13] are assigned. (They are considered to constitute a nonet, together with another $I=0$ scalar meson.*)) The narrow 0^+ resonances $\delta(980)$ and $S^*(980)$,[12] which are assumed to be $2q2\bar{q}$ mesons, are outside the present scheme.

As decay properties of some ${}^3P_0$ mesons are known in addition to those of ${}^3P_2$ and ${}^3S_1$ mesons, we can test the relations (2.3) for N, $N'=0, 1$.

First, we discuss the relations (2.3) in the case of $(N, N')=(0, 0)$. Next, the ones in the cases (1, 0) and (0, 1) are compared with experiments. Some of the relations

*) There is an indication that there exists an $I=0$ scalar meson with mass $\sim$1550 MeV.[14]

in the cases (1, 1), (2, 0) and (0, 2) are also studied.

3.1 *Relations in the case of* $(N, N')=(0, 0)$

In this case, the relation (2.3) is essentially identical with HU's relation*) and agrees with experiments very well. We treat this case briefly, putting our emphasis on a formal discussion about the symmetry breaking predicted by the relation (2.3), which is behind HU's discussion.

The $[\rho]=[\rho]$ for the $\pi^-\pi^+\to\pi^-\pi^+$ process is an identity. The $[K^*]=[\rho]$ for $\pi^-K^+\to\pi^-K^+$ means that

$$2f_{K^*K\pi}^2 q_s^2 P_1(z_s)|_{s=m_{K^*}^2, t=m_\rho^2} = 2f_{\rho\pi\pi} f_{\rho K\bar{K}} q_t q_t' P_1(z_t)|_{t=m_\rho^2, s=m_{K^*}^2}. \quad ^{**)} \tag{3.1}$$

Here, $P_1(z)$ is the Legendre polynomial of $z=\cos\theta$ of the first degree, and θ_s and q_s (θ_t, q_t and q_t') are the c.m. scattering angle and momentum in the s-channel (t-channel). From Eq. (3.1), we have

$$\frac{f_{K^*K\pi}^2}{f_{\rho\pi\pi} f_{\rho KK}} = \frac{2m_{K^*}^2 + m_\rho^2 - 2m_\pi^2 - 2m_K^2}{2m_\rho^2 + m_{K^*}^2 - 2m_\pi^2 - 2m_K^2 + \Delta}, \tag{3.2}$$

where $\Delta = (m_K^2 - m_\pi^2)^2 / m_{K^*}^2$. We assume the semi-local duality between the (ρ, ω) and the Φ for the $\bar{K}K\to\bar{K}K$ process, suggested by the B_4 model.[7] Then, the $[\rho]=[\Phi]$ for $K^-K^\circ\to K^-K^\circ$ gives

$$\frac{f_{\Phi KK}^2}{f_{\rho KK}^2} = 2\frac{2m_\Phi^2 + m_\rho^2 - 4m_K^2}{2m_\rho^2 + m_\Phi^2 - 4m_K^2}. \tag{3.3}$$

Eqs. (3.2) and (3.3) predict the symmetry breaking for the vector meson decays corresponding to that in the vector meson masses. Assuming the universality $f_{\rho\pi\pi} = f_{\rho KK}$,[15] supported by experiments,[16] and taking observed masses[12] as input, we get

$$f_{K^*K\pi}^2 = 1.08 f_{\rho\pi\pi}^2, \quad f_{\Phi KK}^2 = 1.35 \times 2 f_{\rho\pi\pi}^2. \tag{3.4}$$

(Compare these with $f_{K^*K\pi}^2 = f_{\rho\pi\pi}^2$ and $f_{\Phi KK}^2 = 2f_{\rho\pi\pi}^2$ from the $SU(3)$ symmetry with Okubo's nonet ansatz.) Alternatively, the decay widths for $K^*\to K\pi$ and $\Phi\to K\bar{K}$ are predicted, taking also that for $\rho\to\pi\pi$ as input. The predictions are listed in Table I, together with input values (of masses and a partial width being underlined) taken from Ref. 12). The predictions from the $SU(3)$ symmetry are also shown for comparison.

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*) Strictly speaking, HU's relation includes a factor which is not used in the relation (2.3). When the resonance spacing is different in the s- and t-channels, HU multiplied each residue by the trajectory slope in that channel. While, the relation (2.3) does not include such a factor. The effect of such a factor is small for the mesons in the non-charmed SU (3) sector, even if this factor is necessary.

**) Our definitions of couplings relevant to normalization are as follows: $H_{\text{int}} = f_{\rho\pi\pi}\boldsymbol{\rho}_\mu\cdot\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi} + if_{\rho K\bar{K}}\boldsymbol{\rho}_\mu\cdot K^\dagger\tfrac{1}{2}\boldsymbol{\tau}\overleftrightarrow{\partial}_\mu K + if_{K^*K\pi}[K_\mu^{*\dagger}\tfrac{1}{2}\boldsymbol{\tau}K\overleftrightarrow{\partial}_\mu\cdot\boldsymbol{\pi} - \text{h.c.}] + i\tfrac{1}{2}f_{\Phi K\bar{K}}\Phi_\mu K^\dagger\overleftrightarrow{\partial}_\mu K + i\tfrac{1}{2}f_{\omega K\bar{K}}\omega_\mu K^\dagger\overleftrightarrow{\partial}_\mu K.$

Table I. Summary of relevant experimental data and some predictions for partial decay widths. Data are taken from Refs. 12), 13) and 18).[a)]

state (J^P)	experiments: mass (MeV)	experiments: partial width (MeV)		theoretical partial width: present model		($SU(3)$)
$\rho(1^-)$	776 ± 3	$\rho\to\pi\pi$	155 ± 3			
$K^*(1^-)$	892.2 ± 0.4	$K^*\to K\pi$	49.5 ± 1.5		47.3 ± 0.9	(43.8)
$\Phi(1^-)$	1019.6 ± 0.2	$\Phi\to K\bar{K}$	3.4 ± 0.3		4.21 ± 0.08	(3.12)
$K^{**}(2^+)$	1434 ± 5	$K^{**}\to K\pi$	49.1 ± 6.7			
$\kappa(0^+)$	1400–1450 (1425)[b)]	$\kappa\to K\pi$	200–300 (250 ± 50)[b)] 541 ± 106[18)]		410 ± 170	
$A_2(2^+)$	1312 ± 5	$A_2\to K\bar{K}$	4.8 ± 0.8			
$\delta'(0^+)$	~1300[13)]	Γ_{tot}	~250[13)]	$\delta'\to K\bar{K}$	170 ± 150	
$f(2^+)$	1271 ± 5	$f\to\pi\pi$	145 ± 16			
$\varepsilon(0^+)$	~1300	Γ_{tot}	200–400 (300)[b)]	$\varepsilon\to\pi\pi$	170 ± 380	
$g(3^-)$	1688 ± 20	$g\to\pi\pi$	43 ± 18			
$\rho'(1^-)$	~1600	$\rho'\to\pi\pi$	~75			
$\rho''(1^-)$				$\rho''\to\pi\pi$	~210	

a) Used data are taken from Ref. 12) (Particle Data Group Tables 1978), except for the data for the δ' which are from Ref. 13). $\Gamma_{\kappa\to K\pi}$ from Ref. 18) is for the comparison.
b) As for the κ and ε, these values are used.

Considering the $I=0$ $\bar{K}K\to\bar{K}K$ amplitude, we have another independent relation. This, together with Eq. (3.3), gives $f^2_{\omega KK}=f^2_{\rho KK}$, when it is assumed that $m^2_\omega=m^2_\rho$.

3.2 *Relations in the cases of* $(N, N')=(1, 0)$ *and* $(0, 1)$

Now, we test the new relations (2.3) in these cases. The used experimental values are also seen in Table I.

We start with the $[K^{**}(1430), \kappa]=[\rho]$ for the process $\pi^-K^+\to\pi^-K^+$, i.e.,

$$16\pi\times\frac{1}{3}\left(\frac{5\Gamma_{K^{**}K\pi}m^2_{K^{**}}}{q_s}P_2(z_s)\Big|_{s=m^2_{K^{**}}}+\frac{\Gamma_{\kappa K\pi}m^2_\kappa}{q_s}P_0(z_s)\Big|_{s=m^2_\kappa}\right)\Big|_{t=m^2_\rho}$$

$$=2f_{\rho\pi\pi}f_{\rho KK}q_tq'_tP_1(z_t)\Big|_{t=m^2_\rho,s=M^2}, \tag{3.5}$$

where M^2 is the average squared-mass of $\{K^{**}, \kappa\}$, $M^2=(m^2_{K^{**}}+m^2_\kappa)/2$. Using Eq. (3.1), the r.h.s. of Eq. (3.5) is found equal to

$$16\pi\times\frac{1}{3}\left(\frac{2M^2+m^2_\rho-2m^2_\pi-2m^2_K}{2m^2_{K^*}+m^2_\rho-2m^2_\pi-2m^2_K}\right)\left[\frac{3\Gamma_{K^*K\pi}m^2_{K^*}}{q_s}P_1(z_s)\right]\Big|_{s=m^2_{K^*},t=m^2_\rho}. \tag{3.6}$$

Experimentally the l.h.s. of Eq. (3.5) and the quantity (3.6) are $4.25\times(16\pi/3)$ and $4.78\times(16\pi/3)$ (in $(\text{GeV})^2$), respectively. Thus, the present relation $[K^{**}(1430), \kappa]=[\rho]$ is found to be in a better agreement with experiments than HU's relation

$[K^{**}(1430)]=[\rho]$ whose l.h.s. and r.h.s. are $3.43\times(16\pi/3)$ and $4.81\times(16\pi/3)$.*) The present relation is valid at the 11% accuracy level and well within the experimental errors $\lesssim 20\%$; while, HU's relation $[K^{**}(1430)]=[\rho]$ is at the 29% level, and only in a fair agreement with experiments. (It is noted that if the κ is systematized as the daughter of the vector meson K^*, then the resultant relation has a hardly poorer validity.)

Conversely, employing Eqs. (3.5) and (3.6) to predict the width for $\kappa\to K\pi$, we obtain $\Gamma_{\kappa K\pi}=0.41$ GeV.

Other relations in the present cases are also found to be valid at a high-accuracy level, similarly as the relation (3.5). The results of numerical estimations are listed in Table II. The predicted partial widths of scalar mesons are also shown in Table I.

For the estimations, we have made some assumptions as follows:

(a) For the $K^-K^\circ\to K^-K^\circ$ process, the semi-local duality relation $[A_2, \delta']=[\Phi]$ is assumed analogously to the $[\rho]=[\Phi]$.**) Here, δ' denotes the K^-K° resonance with $J^P=0^+$ and mass ~ 1300 MeV.[13] The partial width $\Gamma_{\delta' K\bar{K}}$ is not known. We have assumed that $\Gamma_{\delta',\mathrm{tot}}\sim 250$ MeV[13] is mainly due to the $\delta'\to K\bar{K}$ decay.

(b) There are no precise experimental data for the $\Gamma_{\varepsilon\pi\pi}$. We have assumed that the ε mainly decays into $\pi\pi$.

(c) The $f'\pi\pi$ coupling is very small.[12] And, we have neglected the contributions of the f' to the $\pi K\to\pi K$ and $\pi\pi\to\pi\pi$ processes.

Table II. The comparison of the $(N, N')=(1, 0)$ (and (0, 1)) relations with experiments. For each relation, the contributions from the tensor and scalar mesons are also shown dividually.

process	relation	experiments l.h.s./16π	experiments r.h.s./16π ((GeV)^2)
$\pi^+K^+\to\pi^-K^+$	$[K^{**}, \kappa]=[\rho]$	1.42(=1.14+0.28)	1.59
$K^-K^\circ\to K^-K^\circ$	$[A_2, \delta']=[\Phi]$	1.53(=1.03+0.50)	1.40
$\pi^-\pi^+\to\pi^-\pi^+$	$[f, \varepsilon]=[\rho]$ ($[\rho]=[f, \varepsilon]$)	2.94(=2.67+0.27)	2.82

We note that the estimations of the $[A_2, \delta']=[\Phi]$ and $[f, \varepsilon]=[\rho]$ ($[\rho]=[f, \varepsilon]$) relations are done without the help of $(N, N')=(0, 0)$ relations.

As seen in Table II, HU-type semi-local duality relations are indeed improved by the introduction of the scalar mesons based on the view that the dynamics of hadrons is governed by the quark-orbital trajectory. Moreover, the relations in Table II are found to show a tendency to have a rather centred value $\sim 1.47\times(16\pi)$ (GeV)^2. (The

*) It is clear that this conclusion is not affected by the introduction of an extra factor used by HU (see the footnote on page 63).

**) It is noted that in the quark diagram (the duality diagram)[17] the t-channel of the $K^-K^\circ\to K^-K^\circ$ process is the $s\bar{s}$ state.

extra factor 2 for the $\pi\pi\to\pi\pi$ process comes from the identical-particle effect.) This is also thought to be a reflection of the universality suggested by this view.

In the present view, a good agreement of HU's relations (or, the B_4 amplitudes) with experiments is due to the fact that the contributions of the scalar mesons are in general small compared with those of the tensor mesons.

We have four other independent relations in the present cases. One is given by the $\pi^-K^+\to\pi^-K^+$ amplitude, and three by the $\bar{K}K\to\bar{K}K$ amplitude. These relations do not, however, provide us with conclusive results because of lack of precise experimental data for relevant scalar meson widths and for the width $\Gamma_{f'K\bar{K}}$.

3.3 *Relations in the cases of* $(N, N')=(1, 1)$, $(2, 0)$ *and* $(0, 2)$

We briefly discuss the new relations (2.3) in the cases not treated in the previous subsections, as the present experimental situation is not enough to test them.

As for the (1, 1) case, the dominance of the tensor mesons is more remarkable than in the cases of (1, 0) and (0, 1). And, very precise data are necessary to examine which of the present scheme and HU's relations is more favourable.

As for the relations in the (2, 0) and (0, 2) cases, our interest is the roles of the (L-plane) daughters. We assume the $\rho'(1600)$[12] to be a radially excited state ($N=2$, $^3S_1$ state), as usual.[19] Then, we have the $[g, \rho', \rho'']=[\rho]$ (or, $[\rho]=[g, \rho', \rho'']$) for the $\pi^-\pi^+\to\pi^-\pi^+$ process. Here, ρ'' represents the theoretical $I=1$, $N=2$, $^3D_1$ state, whose existence is not confirmed yet. Assuming $m_{\rho''}$ to be equal to m_g, and with the relevant data in Table I as input, we find that the relation is satisfied if the contribution of the ρ'' is of $\Gamma_{\rho''\pi\pi}\simeq 210$ MeV. (We note that the l.h.s. and the r.h.s. of HU's relation $[g]=[\rho]$ are $2.70\times(16\pi)$ and $4.57\times(16\pi)$ (in $(\mathrm{GeV})^2$).) The contribution of the (L-plane) daughter ρ' is of 10% in the l.h.s. of the relation $[g, \rho', \rho'']=[\rho]$.

§4. Semi-local duality relations for the meson-baryon scattering

In this section, we generalize the semi-local duality relation (2.3) for the meson-baryon scattering, and compare the resultant relation with experiments.

The generalization can be easily done by postulating the set of constraints (2.3) for each of the invariant amplitudes. In fact, the obtained relations for the $\bar{K}$–N scattering are consistent with experiments, as shown below.

Now, we consider the $\bar{K}N\to\bar{K}N$ scattering where the u-channel is exotic, and construct the relations for the case of $(N, N')=(0, 0)$. The s-channel $N=0$ states are ($Y_1^*(1385, J^P=3/2^+)$, $\Sigma(J^P=1/2^+)$, $\Lambda(J^P=1/2^+)$), and the t-channel $N=0$ ones (ρ, ω). The Φ, forbidden by the Okubo-Zweig-Iizuka rule, is neglected. Normalizing the residues of the Y_1^*, Σ, Λ, ρ and ω in the $A^{(I_s=1,0)}$ and $B^{(I_s=1,0)}$ amplitudes by using the couplings in usual $SU(3)$ fits,[21] we have

$$-6G^{2*}_{Y_1^*NK}\frac{(m_{Y_1^*}+m_N)r_1}{4m_{Y_1^*}(E_Y+m_N)}+2G^2_{\Sigma NK}(m_\Sigma-m_N)$$

$$=\left\{-\frac{G^T_{\rho N\bar{N}}G_{\rho K\bar{K}}}{4}+G^T_{\omega N\bar{N}}G_{\omega K\bar{K}}\right\}\frac{r_2}{m_N}, \tag{4.1a}$$

$$2G^2_{\Lambda NK}(m_\Lambda-m_N)=\left\{3\frac{G^T_{\rho N\bar{N}}G_{\rho K\bar{K}}}{4}+G^T_{\omega N\bar{N}}G_{\omega K\bar{K}}\right\}\frac{r_3}{m_N}; \tag{4.1b}$$

and

$$6G^{2*}_{Y_1^*NK}\frac{r_1}{4m_{Y_1^*}(E_Y+m_N)}-2G^2_{\Sigma NK}=\frac{(G^T_{\rho N\bar{N}}+G^V_{\rho N\bar{N}})G_{\rho K\bar{K}}}{2}$$

$$-2(G^T_{\omega N\bar{N}}+G^V_{\omega N\bar{N}})G_{\omega K\bar{K}}, \tag{4.1c}$$

$$-2G^2_{\Lambda NK}=-3\frac{(G^T_{\rho N\bar{N}}+G^V_{\rho N\bar{N}})G_{\rho K\bar{K}}}{2}-2(G^T_{\omega N\bar{N}}+G^V_{\omega N\bar{N}})G_{\omega K\bar{K}}, \tag{4.1d}$$

where

$$r_1=M_t^2+2\left\{E_Y^2-m_N^2+\frac{(m_{Y_1^*}-m_N)(E_Y+m_N)^2}{3(m_{Y_1^*}+m_N)}\right\}, \tag{4.2}$$

$$E_Y=\frac{\{(m_{Y_1^*}+m_N)^2-m_K^2\}}{2m_{Y_1^*}}-m_N, \tag{4.3}$$

$$r_2=M_s^2-m_N^2-m_K^2+\frac{m^2_{\rho,\omega}}{2},\quad r_3=m_\Lambda^2-m_N^2-m_K^2+\frac{m^2_{\rho,\omega}}{2}, \tag{4.4}$$

and M_t^2 and M_s^2 are the average squared-masses of the (ρ,ω) and (Y_1^*,Σ), respectively. Here, we have assumed that $m_\rho^2=m_\omega^2\equiv m^2_{\rho,\omega}$.

The l.h.s. and the r.h.s. of each of Eqs. (4.1a)~(4.1d) are found to be comparable to each other as $4\pi(-2.60$ GeV, 6.00 GeV; -1.38, $-34.0)=4\pi(-3.38$ GeV, 5.05 GeV; 0.37, $-34.7)$ in our evaluation using coupling constants[21] from $SU(3)$ fits to low-energy experimental data.*) Here, we have assumed that $M_s^2=(m^{2*}_{Y_1}+m_\Sigma^2)/2$.

As for the vector-meson couplings it is to be noted that the $SU(3)$ is obeyed in the Sachs-type vertex functions $G^E_{vN\bar{N}}(t)=G^V_{vN\bar{N}}(t)+(t/4m_N^2)G^T_{vN\bar{N}}(t)$ and $G^M_{vN\bar{N}}(t)=G^V_{vN\bar{N}}(t)+G^T_{vN\bar{N}}(t)$ at the poles, (and $G^V_{vN\bar{N}}\equiv G^V_{vN\bar{N}}(m_v^2)$, $G^T_{vN\bar{N}}\equiv G^T_{vN\bar{N}}(m_v^2)$), and $G^V_{\Phi N\bar{N}}$ and $G^T_{\Phi N\bar{N}}$ are assumed to be zero.

*) The used values are $(G^2_{\Sigma N\bar{K}}, G^2_{\Lambda N\bar{K}})=4\pi(1.4, 17.0)$ and $(G^T_{\rho N\bar{N}}, G^V_{\rho N\bar{N}}, G_{\rho K\bar{K}}; G^T_{\omega N\bar{N}}, G^V_{\omega N\bar{N}}, G_{\omega K\bar{K}})$ $=\sqrt{4\pi}(8.90, 1.48, 1.69; 0.00, 4.00, 1.05)$ taken directly from Ref. 21), and $G^{2*}_{Y_1^*N\bar{K}}=G^2_{\Delta N\pi}/6=4\pi\times14.9/6$ given by the exact $SU(3)$ invariance and the observed width of $\Delta(1232)$. It is noted that $G_{\rho K\bar{K}}$ and $G_{\omega K\bar{K}}$ are related to $f_{\rho K\bar{K}}$ and $f_{\omega K\bar{K}}$ in §3 as $G_{\rho K\bar{K}}=f_{\rho K\bar{K}}$ and $G_{\omega K\bar{K}}=f_{\omega K\bar{K}}/2$.

§5. Summary of results and some further discussions

We have generalized and modified HU's semi-local duality relations to involve all the states of the harmonic-oscillator pattern of $SU(6)\otimes O(3)_L$ multiplets on the standpoint that the dynamics of hadrons is governed by the quark-orbital trajectory. The obtained relations for meson-meson scattering have been found to be in a better agreement with experiments than HU's relations. It is also found that the generalization is successful for the meson-baryon scattering. Thus, the present results support the supposition made by Bando et al. and Nakkagawa et al.

We would like to make some further discussions.

(I) Zero intercepts of almost all quark-orbital trajectories are negative.[6] Therefore, the difficulty concerning the appearance of tachyons and the convergence domain ($\alpha(s)<0$, $\alpha(t)<0$) of integral representations in the Veneziano-like model disappears, if the dual resonance model is meaningful not for the whole T-matrix but for the spatial part of it, as in the amplitude (2.5).[6]

The model presented here suggest that the difficulty concerning the appearance of ghosts in the dual resonance model may be also resolved by the introduction of the quark-orbital trajectory to the dual models.

(II) If one accepts the amplitude (2.5) seriously, then the pole-pole duality, that is,

$$\text{the sum of } s\text{-channel poles} = \text{the sum of } t\text{-channel poles}, \tag{5.1}$$

is doubtful.

We note that even if the relation (5.1) is not valid, the semi-local duality proposed here may retain its validity.

There remain some problems in the present model. We must find a well-defined quantity for the average squared-mass in the relation (2.3). We must generalize our model for the processes where the s-, t- and u-channels are non-exotic. These will be discussed elsewhere.

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