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Scalar Mesons in a Duality Scheme Based on the Quark-Orbital Regge Trajectory

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Decay properties of the scalar mesons are studied in a duality scheme based on the quark-orbital Regge trajectory. It is suggested that the $\delta'(\sim 1300)$, $\kappa(\sim 1425)$, $\bar{\kappa}$, $\varepsilon(\sim 1300)$ and $\varepsilon'(\sim 1550)$ constitute a nonet with a nearly ideal mixing angle.

It will be worthwhile to accommodate the hadron spectrum in the quark model to some kind of duality scheme. The author[1] has recently proposed an attempt on this problem. He has presented semi-local duality relations involving all the states of the harmonic-oscillator spectrum of $SU(6)\otimes O(3)_L$ multiplets. The basic ideas to introduce them were as follows: (i) From studies of the hadron dynamics and the hadron spectrum in the quark model, it is supposed that the dynamics of hadrons is governed essentially by the quark-orbital Regge trajectory (QORT).[2,3] We intend to realize a duality between the QORT's. (ii) We are interested in a new-type of semi-local duality relations recently presented by Hoyer and Uschersohn,[4] modifying the finite-energy sum rules (FESR's) by the requirements of a symmetric treatment of direct and crossed channels and a self-consistency. Such relations may be easily generalized to describe the duality between QORT's, judging from the fact that the FESR's or the superconvergence relations often lead to the symmetry of resonance couplings, when they are saturated by resonances in an $SU(6)$ multiplet.[5]

The proposed relations have been compared with experiments for some processes.[1] And they are found to be promising. The relations for N(excitation quantum number)$=0$ states are in good agreement with experiments. Those including $N=1$ states are also consistent with experiments available now.

In this note, we make another test of this semi-local duality relation. As experimental informations on $N\neq 0$ states are still poor now, a rather formal test is done here, differently from that in Ref. 1). Decay properties of the scalar mesons ($N=1$, ${}^3P_0$ mesons) in this scheme are discussed considering the $0^- - 0^-$ scattering, and compared with the $SU(3)$ predictions.

We consider the $0^-0^- \to 0^-0^-$ processes where the u-channe is exotic. The relation for a process, under discussion, is

$$\sum_{P_L\in\{P_L\}} R^{(s)}_{P_L}(t)|_{t=m^2_{L'}} = \sum_{P'_{L'}\in\{P'_{L'}\}} R^{(t)}_{P'_{L'}}(s)|_{s=m^2_L}, \quad L,\ L'=0,\ 1,\dots . \tag{1}$$

Here $\{P_L\}$ ($\{P'_{L'}\}$) is the s-(t-)channel particle group consisting of parent particles with a fixed quark-orbital angular momentum $L(L')$ and their (L-plane) daughters in the process. $R^{(s)}_{P_L}(t)$ ($R^{(t)}_{P'_{L'}}(s)$) is the residue of the contribution of s-(t-) channel state $P_L(P'_{L'})$ to the scattering amp itude, in the narrow-width approximation. And m^2_L ($m^2_{L'}$) is defined to be the average squared-mass of $\{P_L\}$ ($\{P'_{L'}\}$).

Here, we study the scalar mesons ($N=1$, $^3P_0$ mesons) by the relations (1) for $(L, L')=(1, 0)$ (and (0, 1)). As the decay properties of vector ($N=0$, $^3S_1$) and tensor ($N=1$, $^3P_2$) mesons are known, these relations determine those of $N=1$, $^3P_0$ mesons. In the study here, (differently from that in Ref. 1)), we assume and use the universality that

$$\begin{aligned}&l.h.s. \text{ and } r.h.s. \text{ of } (L,\ L') \text{ relations (1)}\\ &\quad =(\text{a universal constant depending only on } (L,\ L'))\times n.\end{aligned} \tag{2}$$

Here, n is 1 for the $\pi^-K^+\to\pi^-K^+$ and $K^-K^\circ\to K^-K^\circ$ processes, and 2 for the $\pi^-\pi^+\to\pi^-\pi^+$ and $K^-K^+\to K^-K^+$ processes. The extra factor 2 for the $\pi^-\pi^+\to\pi^-\pi^+$ and $K^-K^+\to K^-K^+$ processes come from the identical-particle effect. This universality is expected, if the dynamics of hadrons is really governed essentially by the QORT. And, in fact, the corresponding universal feature is found in the comparison with experiments.[1b)]

Now, we discuss the $N=1$, $^3P_0$ mesons by the relation (2) in the case of $(L, L')=(1, 0)$ and $(0, 1)$. The assignment of observed vector and tensor mesons to the theoretical $N=0$, $^3S_1$ and $N=1$, $^3P_2$ states is done as usual. As for the scalar mesons, there remain some problems in their assignments. Here, to the $N=1$, $^3P_0$ states, the $\varepsilon(1300)$, $\kappa(1400)$[6)] and the K^-K° resonance with mass $\sim$1300 MeV[7)] are assigned. They are supposed to constitute a nonet, together with another $I=0$ scalar meson of mass $\sim$1550 MeV.[8)] The lower-mass and narrow scalar mesons $\delta(980)$ and $S^*(980)$[6)] are assumed to be $2q2\bar{q}$ mesons, according to Jaffe,[9)] and are outside the present scheme.

The universal constant for the $(L, L')=(1, 0)$ case is taken to be $1.47\times(16\pi)$, which is the average of the $r.h.s.$ values of (1, 0) relations (1) for the $\pi^-\pi^+\to\pi^-\pi^+$, $\pi^-K^+\to\pi^-K^+$ and $K^-K^\circ\to K^-K^\circ$ processes. The used data to determine the constant are taken from Particle Data Group Tables[6)] and Ref. 7). Here, Sakurai's universality is assumed to estimate the $\rho K\bar{K}$ coupling.

Using the universal constant and subtracting the contributions from the $N=1$, $^3P_2$ mesons, we predict the partial decay widths of the $N=1$, $^3P_0$ mesons. The used data are taken also from Refs. 6) and 7). The results are listed in Table I.

We take the $SU(3)$ $0^+\to0^-0^-$ couplings similarly to those done by Morgan.[10)] Some of them, which are relevant to the present work, are

Table I. Some predictions from the present model.

process	relation[a]	prediction
$\pi^-K^+\to\pi^-K^+$	$[K^{**}, \kappa]=[\rho]$	$\Gamma_{\kappa K\pi}=300\pm140$ MeV
$K^-K^\circ\to K^-K^\circ$	$[A_2, \delta']=[\Phi]$[b,c]	$\Gamma_{\delta' K\bar{K}}=220\pm 80$ MeV
$\pi^-\pi^+\to\pi^-\pi^+$	$[f, \varepsilon]=[\rho]$[d]	$\Gamma_{\varepsilon\pi\pi}=300\pm300$ MeV

a) The notation for the relations is the same as in Ref. 1b). The values of these relations are taken universally as $1.47\times(16\pi)$.

b) δ' denotes the $I=1$ scalar meson of mass $\sim$1300 MeV.[7]

c) It is noted that in the quark diagram (the duality diagram) the t-channel of the $K^-K^\circ\to K^-K^\circ$ process is the $s\bar{s}$ state.

d) We have assumed that the contribution of ε' (an $I=0$ scalar meson with mass $\sim$1550 MeV[8]) to this process is negligible.

$$
\begin{aligned}
g_{\kappa K\pi}&=\frac{3}{\sqrt{10}}\,g_8,\\
g_{\delta' KK}&=\sqrt{\frac{3}{5}}\,g_8,\\
g_{\varepsilon\pi\pi}&=-\sqrt{\frac{3}{5}}\sin\theta_S\,g_8+\sqrt{\frac{3}{8}}\cos\theta_S\,g_1,\\
g_{\varepsilon KK}&=\frac{1}{\sqrt{5}}\sin\theta_S\,g_8+\frac{1}{\sqrt{2}}\cos\theta_S\,g_1,\\
g_{\varepsilon'\pi\pi}&=-\sqrt{\frac{3}{5}}\cos\theta_S\,g_8-\sqrt{\frac{3}{8}}\sin\theta_S\,g_1,\\
g_{\varepsilon' KK}&=\frac{1}{\sqrt{5}}\cos\theta_S\,g_8-\frac{1}{\sqrt{2}}\sin\theta_S\,g_1.
\end{aligned}
\tag{3}
$$

Here, δ' denotes the $I=1$ scalar meson of mass $\sim$1300 MeV[7], and ε' does the $I=0$ one with mass $\sim$1550 MeV.[8] The mixing angle θ_S is defined by

$$
\begin{aligned}
\varepsilon&=S_8\sin\theta_S+S_1\cos\theta_S,\\
\varepsilon'&=S_8\cos\theta_S-S_1\sin\theta_S.
\end{aligned}
\tag{4}
$$

The coupling constants (3) are related, by our definition, to the partial decay widths Γ_{Sab} for the $S(0^+)\to a(0^-)b(0^-)$ as

$$\Gamma_{Sab}=g^2_{Sab}q_{s=m^2_S} \tag{5}$$

The coupling constants determined by the predicted widths in Table I are shown in Table II. They are consistent with the $SU(3)$ coupling relations (3), if θ_S, g_8 and g_1 are as, for example,

$$\cos\theta_S=\sqrt{2/3},\quad g_8^2=0.55\quad\text{and}\quad g_1=-\frac{3}{\sqrt{5}}\,g_8. \tag{6}$$

The mixing angle in Eq. (6) is compatible with the masses of the $\delta'(\sim 1300)$, $\kappa(\sim 1425)$, $\varepsilon(\sim 1300)$ and $\varepsilon'(\sim 1550)$ and the Gell-Mann-Okubo mass formula $4m_\kappa^2 = m_{\delta'}^2 + 3(m_{\varepsilon'}^2 \cos^2\theta_S + m_\varepsilon^2 \sin^2\theta_S)$. The $SU(3)$ coupling constants for the θ_S, g_8 and g_1 in Eq. (6) are also shown in Table II.

The $SU(3)$ couplings in Table II are also consistent with the remaining (1, 0) and (0, 1) semi-local duality relations $[A_2, f, f', \delta', \varepsilon, \varepsilon'] = [\rho, \omega, \Phi]$, $[K^*] = [f, \varepsilon]$ and $[\rho] = [f', \varepsilon']$, which are for the $K^-K^+ \to K^-K^+$, $\pi^-K^+ \to \pi^-K^+$ and $K^-K^0 \to K^-K^0$ processes, respectively. In the estimation with the scalar couplings and experimental data,[6,7,8] each pair of the *l.h.s.* and *r.h.s.* values of these relations is found to be 16π $(1.53\times 2, 1.49\times 2)$, 16π $(1.49, 1.19)$ and 16π $(1.73, 1.66)$ (in $(\mathrm{GeV})^2$) respectively.

Thus, from the present study, it is suggested that the δ' (~ 1300), κ (~ 1425), $\bar{\kappa}$, ε (~ 1300) and ε' (~ 1550) constitute a nonet with a nearly ideal mixing angle, as the tensor meson nonet $(A_2, K^{**}, \bar{K}^{**}, f, f')$.

Table II. The predictions for the $0^+ \to 0^-0^-$ coupling constants from the present model and the $SU(3)$ symmetry.

	predictions		experiment
	present model	$SU(3)$[a]	
$g^2_{\kappa K\pi}$	0.49 ± 0.23	0.50	0.40 ± 0.09[b]
$g^2_{\delta' K\bar{K}}$	0.52 ± 0.19	0.33	
$g^2_{\varepsilon\pi\pi}$	0.47 ± 0.47	0.69[c]	
$g^2_{\varepsilon'\pi\pi}$	~ 0 (assumed)	0.014	
$g^2_{\varepsilon K\bar{K}}$		0.15	
$g^2_{\varepsilon' K\bar{K}}$		0.46	

a) θ_S, g_8 and g_1 are taken to be $\cos\theta_S = \sqrt{2/3}$, $g_8^2 = 0.55$ and $g_1 = -(3/\sqrt{5})g_8$.
b) This comes from the data in Ref. 6).
c) The relative signs of the $g_{\varepsilon\pi\pi}$ etc. are $g_{\varepsilon\pi\pi} = -1.12g_8$, $g_{\varepsilon'\pi\pi} = -0.16g_8$, $g_{\varepsilon K\bar{K}} = -0.52g_8$ and $g_{\varepsilon' K\bar{K}} = 0.91g_8$.

References

1) a) T. Ino, Prog. Theor. Phys. **61** (1979), 1863.
   b) T. Ino, to be published in Memoirs of the Faculty of Science, Shimane University Vol. 13 (1979).
2) M. Bando, S. Machida, H. Nakkagawa and K. Yamawaki, Prog. Theor. Phys. **47** (1972), 626.
3) H. Nakkagawa, K. Yamawaki and S. Machida, Prog. Theor. Phys. **48** (1972), 939.
4) P. Hoyer and J. Uschersohn, Nucl. Phys. **B135** (1978), 445.
   P. Hoyer, Nucl. Phys. **B127** (1977), 476.
5) For a review see, e.g., M. Fukugita and K. Igi, Phys. Reports **31C** (1977), 237.
6) Particle Data Group Tables 1978, Phys. Letters **75B** (1978), 1.
7) A. D. Martin et al., Phys. Letters **74B** (1978), 417.
8) P. Estabrooks: "Where and What are the Scalar Mesons," Contributed paper (No. 1051) to *the 19th International Conference on High Energy Physics, Tokyo,* 1978.
9) R. L. Jaffe, Phys. Rev. **D15** (1977), 267.
10) D. Morgan, Phys. Letters **51B** (1974), 71.