

Rank-Correlation among the Four Class-Records
in Colleges (Part 2)

SINGU, Tadao

(Laboratory of Mathematics and Statistics)

大学生の成績の順位相関について (その2)

新 宮 忠 雄

(数学および統計学研究室)

§ 101. Introduction

In (6) I wrote on the rank-correlation among the four class-records in the Simane Agricultural College. It was only for those graduated in 1955 through 1958, so here I will discuss for those graduated in 1959 and also for those graduated from the Simane Women's Junior College in 1955 through 1959.

In this treatise I omit all data, formulae, calculations which were once written in (6) already. For this purpose I use numbers of sections, and tables from 101 instead of from 1 in this treatise.

Table 101. Notation for Each Class

Entered in	Graduated in	Name of the class		No-tation
		Course	Time	
1955	1959	A	5th	A 5
"	"	F	"	F 5
"	"	E	"	E 5
1953	1955	D	1st	D 1
"	"	C	"	C 1
1954	1956	D	2nd	D 2
"	"	C	"	C 2
1955	1957	D	3rd	D 3
"	"	C	"	C 3
1956	1958	D	4th	D 4
"	"	C	"	C 4
1957	1959	D	5th	D 5
"	"	C	"	C 5

D : Dietetics and hygienics
C : Clothings

§ 102. Data

Although the data which I showed as in tables 2-13 should be written here for the rankings, I have few spaces to do so. Accordingly, I have to omit them against my intention and can cite the table 114 only.

§ 103. Calculation of Correlation Coefficients

The test statistic for the criterion by the rank-correlation coefficient is

$$r_s = 1 - \frac{6 \sum_{i=1}^N d^2}{N(N^2 - 1)}$$

Table 114. $\sum_{i=1}^N d_i^2$

Class	<i>N</i>	$\Sigma(X-Y)^2$	$\Sigma(X-Z)^2$	$\Sigma(X-V)^2$	$\Sigma(Y-Z)^2$	$\Sigma(Y-V)^2$	$\Sigma(Z-V)^2$
A 5	28	2106.5	1660	2096	271.5	262.5	538
F 5	20	826	817.5	1054	259.5	78	475.5
E 5	25	1743	1496	2308	810	304	1790
D 1	28	1340	894	1804	712	384	1358
C 1	26	974.5	522	896.5	489	212	918
D 2	25	1700.5	1727	1821.5	194.5	105	505.5
C 2	27	1304	2099.5	1199.5	625.5	134.5	1163
D 3	32	3854	3780	4103	1217.5	110.5	1852
C 3	25	2749.5	2750	2768.5	794.5	38	990.5
D 4	32	3869	3696	4187	614	128	1064
C 4	33	3194	3719	3122	1197	102	1564
D 5	31	2317	2723	2514	1309	88.5	1785.5
C 5	33	1008	1299.5	1210	484.5	210	1094.5

Table 115. Rank-Correlation Coefficients Obtained

Class	<i>r_{er.v.}</i>		<i>r_{XY}</i>	<i>r_{XZ}</i>	<i>r_{XV}</i>	<i>r_{YZ}</i>	<i>r_{YV}</i>	<i>r_{ZV}</i>	<i>N</i>
A 5	0.448	0.317	0.424 (6) *	0.546 (4)**	0.426 (5) *	0.926 (2)**	0.928 (1)**	0.853 (3)**	28
F 5	0.534	0.377	0.379 (5) *	0.385 (4) *	0.208 (6)⊙	0.805 (2)**	0.941 (1)**	0.642 (3)**	20
E 5	0.475	0.336	0.330 (4)⊙	0.425 (3) *	0.112 (6)⊙	0.688 (2)**	0.883 (1)**	0.312 (5)⊙	25
D 1	0.448	0.317	0.633 (4)**	0.755 (3)**	0.506 (6)**	0.805 (2)**	0.895 (1)**	0.628 (5)**	28
C 1	0.465	0.329	0.667 (6)**	0.822 (3)**	0.694 (4)**	0.833 (2)**	0.928 (1)**	0.686 (5)**	26
D 2	0.475	0.336	0.346 (4) *	0.336 (5)⊙	0.299 (6)⊙	0.925 (2)**	0.960 (1)**	0.806 (3)**	25
C 2	0.456	0.323	0.602 (5)**	0.359 (6) *	0.634 (4)**	0.809 (2)**	0.959 (1)**	0.645 (3)**	27
D 3	0.417	0.296	0.294 (5)⊙	0.307 (4) *	0.248 (6)⊙	0.777 (2)**	0.980 (1)**	0.661 (3)**	32
C 3	0.475	0.336	.8943 (2)**	.8942 (3)**	.8935 (4)**	0.694 (5)**	0.999 (1)**	0.619 (6)**	25
D 4	0.417	0.296	0.291 (5)⊙	0.323 (4) *	0.233 (6)⊙	0.887 (2)**	0.977 (1)**	0.805 (3)**	32
C 4	0.410	0.291	0.466 (5)**	0.379 (6) *	0.478 (4)**	0.800 (2)**	0.983 (1)**	0.739 (3)**	33
D 5	0.424	0.301	0.533 (4)**	0.451 (6)**	0.493 (5)**	0.736 (2)**	0.982 (1)**	0.640 (3)**	31
C 5	0.410	0.291	0.832 (3)**	0.783 (6)**	0.798 (5)**	0.919 (2)**	0.965 (1)**	0.817 (4)**	33

§ 104. Discussion on the Rank-Correlation Coefficients

Analysis of the Table 116 :

The grand total of *p*'s for each *r_s* shows that *r_{XY}*, *r_{XZ}* and *r_{XV}* seem very much larger than the other three, or extremely larger than *r_{YZ}* and *r_{YV}*. To discuss such differences

Table 116. Totals and Averages of p 's

	r_{XY}	r_{XZ}	r_{XV}	r_{YZ}	r_{YV}	r_{ZV}	size of sample
grand total	114	111	132	53	25	90	25
grand average	4.56	4.44	5.28	2.12	1.00	3.60	
total of A, F & E's	71	65	82	30	15	52	15
" D & C's	43	46	50	23	10	38	10
" A's	23	21	29	10	5	17	5
" F's	26	24	25	10	5	15	5
" E's	22	20	28	10	5	20	5
" D's	22	22	29	10	5	17	5
" C's	21	24	21	13	5	21	5
" A5, F5 & E5	15	11	17	6	3	11	3
" D1 & C1	10	6	10	4	2	10	2
" D2 & C2	9	11	10	4	2	6	2
" D3 & C3	7	7	10	7	2	9	2
" D4 & C4	10	10	10	4	2	6	2
" D5 & C5	7	12	10	4	2	7	2
average of A, F & E's	4.73	4.33	5.47	2.00	1.00	3.47	
" D & C's	4.30	4.60	5.00	2.30	1.00	3.80	
" A's	4.60	4.20	5.80	2.00	1.00	3.40	
" F's	5.20	4.80	5.00	2.00	1.00	3.00	
" E's	4.40	4.00	5.60	2.00	1.00	4.00	
" D's	4.40	4.40	5.80	2.00	1.00	3.40	
" C's	4.20	4.80	4.20	2.60	1.00	4.20	
" A5, F5 & E5	5.00	3.67	5.67	2.00	1.00	3.67	
" D1 & C1	5.00	3.00	5.00	2.00	1.00	5.00	
" D2 & C2	4.50	5.50	5.00	2.00	1.00	3.00	
" D3 & C3	3.50	3.50	5.00	3.50	1.00	4.50	
" D4 & C4	5.00	5.00	5.00	2.00	1.00	3.00	
" D5 & C5	3.50	6.00	5.00	2.00	1.00	3.50	

statistically I use the criteria for testing for extreme mean and the method of analysis of variance as I did in (6).

$\bar{X}_1=5.28$, $\bar{X}_2=4.56$, $\bar{X}_k=1.00$ for the hypothesis H: $\bar{p}(r_{XV})$ is not significantly larger than other \bar{p} 's. We get $r_{10}=0.168$, and this value is smaller than its critical value 0.560 for 5% level of significance again. Therefore we accept H. Also for the hypothesis H': $\bar{p}(r_{YV})$ is not significantly smaller than other \bar{p} 's, we have $\bar{X}_1'=1.00$, $\bar{X}_2'=2.12$, $\bar{X}_k'=5.28$, $r_{10}'=0.254 < 0.560$. Therefore we accept H', too.

As for other totals of p 's the results are the same to this, so I will omit the discussion for this table anymore. Instead of it I will use the method of analysis of variance applied to the table 115.

We can analyze the data in four ways: one is done in the table 117 just as in the table 17, it is only for the Agricultural College graduates; one is done in the table 118, it is only for the Women's College graduates; and others are done in the tables 119 and 120, they are both college graduates combined. In the cases of the tables 119 and 120, the year-effect is rather meaningless, though. Because the graduates had not been examined by

the same problems at the entrance examination even though they graduated in the same year. For example, D1, C1, A3, F3 and E3 were examined by the same problems at their entrance examination. Therefore I feel less importance in it than in the other two.

N.B. In the case of the table 120, T-effect means the college-effect instead of the course-effect, so instead of H_3 we take

H_3' : no college (T)-effect.

Also, in this case the formulae for the analysis of variance slightly differs from those written in (6). Because $m \neq \text{constant}$, $m_{hi1}=9$ (Agricultural College) and $m_{hi2}=6$ (Women's College).

Table 117. Analysis of Variance

Source	Sum of squares	d. f. (d)	Mean square	F-ratio	(to accept H use left columns; to reject H use right columns)			
					Level of significance (α)		Table value of $F(d, 60)_{1-\alpha}$	
Groups	$S_1=3.0585$	1	3.0585	$F_1=147.04$		0.5%		8.49
Years	$S_2=0.9007$	4	0.2252	$F_2=10.83$		0.5%		4.14
Courses	$S_3=0.5370$	2	0.2685	$F_3=12.91$		0.5%		5.80
RS-interaction	$S_4=0.2356$	4	0.0589	$F_4=2.84$	2.5%	5%	3.01	2.53
RT-interaction	$S_5=0.2614$	2	0.1307	$F_5=6.27$		0.5%		5.80
ST-interaction	$S_6=0.3148$	8	0.0394	$F_6=1.90$	5%	10%	2.10	1.77
RST-interaction	$S_7=0.1360$	8	0.0170	$F_7=0.82$	50%		0.93	
Residual	$S_8=1.2456$	60	0.0208					
Total	$S_9=6.6895$	89						

Conclusions : The result is almost the same to the Table 17. The difference happened at the RS-interaction, namely, its level of significance might be slightly higher.

Table 118. Analysis of Variance

Source	Sum of squares	d. f. (d)	Mean square (M)	F-ratio $\begin{cases} F = M/M_8 \\ F' = M/M_7 \end{cases}$	(to accept H use left columns; to reject H use right columns)			
					Level of significance (α)		Table value of $F \begin{cases} F_{1-\alpha}(d,40) \\ F_{1-\alpha}(d,4) \end{cases}$	
Groups	$S_1=1.2364$	1	$M_1=1.2364$	$F_1=100.85$ $F_1'=33.63$		0.5%		8.83 31.33
Years	$S_2=0.1635$	4	$M_2=0.0409$	$F_2=3.33$ $F_2'=1.11$	2.5%	1%	3.13	3.83 1.00
Courses	$S_3=0.2773$	1	$M_3=0.2773$	$F_3=22.62$ $F_3'=7.54$		0.5%		8.83 4.54
RS-interaction	$S_4=0.3310$	4	$M_4=0.0827$	$F_4=6.74$ $F_4'=2.25$		0.5%		4.37 2.06
RT-interaction	$S_5=0.2969$	1	$M_5=0.2969$	$F_5=24.22$ $F_5'=8.08$	2.5%	5%	12.22	8.83 7.71
ST-interaction	$S_6=0.1399$	4	$M_6=0.0350$	$F_6=2.85$ $F_6'=0.95$	2.5%	5%	3.13	2.61 1.00
RST-interaction	$S_7=0.1471$	4	$M_7=0.0368$	$F_7=3.00$	2.5%	5%	3.13	2.61
Residual	$S_8=0.4903$	40	$M_8=0.0123$					
Total	$S_9=3.0824$	59						

Conclusions: The group-effect is extremely significant, as in the Agricultural College, though the course-effect and the year-effect are not significant, and the group-course-interaction is weakly significant. The most difference is that the triple interaction is of weak significance. If we assume the level of significance for this interaction to be 2.5% we can use F_1 through F_6 , but if we assume it to be 5% we have to use F' instead of F . See(7).

Table 119. Analysis of Variance

Source	Sum of squares	d. f. (d)	Mean square (M)	F-ratio $\begin{cases} F = M/M_8 \\ F' = M/M_7 \end{cases}$	(to accept H use left columns; to reject H use right columns)			
					Level of significance (α)		Table value of $F_{1-\alpha}(d,100)$ $F_{1-\alpha}(d,16)$	
Groups	$S_1=4.2356$	1	$M_1=4.2356$	$F_1=245.17$		0.5%		8.28
				$F_1'=122.17$		0.5%		10.58
Years	$S_2=0.5120$	4	$M_2=0.1280$	$F_2=7.37$		0.5%		3.99
				$F_2'=3.69$	2.5%	5%	3.73	3.01
Courses	$S_3=0.9535$	4	$M_3=0.2384$	$F_3=13.73$		0.5%		3.99
				$F_3'=6.88$		0.5%		5.64
RS-interaction	$S_4=0.2948$	4	$M_4=0.0737$	$F_4=4.25$		0.5%		3.99
				$F_4'=2.13$	10%	25%	2.33	1.50
RT-interaction	$S_5=0.6176$	4	$M_5=0.1544$	$F_5=8.32$		0.5%		3.99
				$F_5'=4.45$	1%	2.5%	4.77	3.73
ST-interaction	$S_6=1.0069$	16	$M_6=0.0629$	$F_6=3.62$		0.5%		2.29
				$F_6'=1.82$	10%	25%	1.93	1.41
RST-interaction	$S_7=0.5548$	16	$M_7=0.0347$	$F_7=2.00$	1%	2.5%	2.21	1.96
Residual	$S_8=1.7359$	100	$M_8=0.0174$					
Total	$S_9=9.9111$	149						

Numbers written in 'italics' are not suitable now.

Conclusions: Again as in the previous case the RST-interaction is significant, and I had to use F' instead of F for the test of other effects. See (7). The most significant ones are the group-effect and the course-effect. Next come the group-course-interaction and the triple interaction. The year-effect is weakly significant and the interaction between group and year is not significant as that between year and course.

Table 120. Analysis of Variance

Source	Sum of squares	d. f. (d)	Mean square	F-ratio	(to accept H use left columns; to reject H use right columns)			
					Level of significance (α)		Table value of $F_{1-\alpha}(d,130)$	
Groups	$S_1=4.2356$	1	4.2356	$F_1=143.21$		0.5%		8.16
Years	$S_2=0.5120$	4	0.1280	$F_2=4.33$		0.5%		3.90
Colleges	$S_3=0.1393$	1	0.1393	$F_3=4.71$	2.5%	5%	5.14	3.92
RS-interaction	$S_4=0.2948$	4	0.0737	$F_4=2.49$	2.5%	5%	2.88	2.44
RT-interaction	$S_5=0.0598$	1	0.0598	$F_5=2.02$	10%	25%	2.74	1.34
ST-interaction	$S_6=0.5522$	4	0.1381	$F_6=4.67$		0.5%		3.90
RST-interaction	$S_7=0.2724$	4	0.0681	$F_7=2.30$	5%	10%	2.44	1.99
Residual	$S_8=3.8450$	130	0.0296					
Total	$S_9=9.9111$	149						

Conclusions: Again the group-effect is most significant, but in this case almost every effect or interaction is more or less significant. The only non-significant ones are the group-college-interaction and the triple interaction.

As the result of the analysis of variance done in the tables 117—120 I show the next table.

Table 121. Level of Significance to Accept H (left columns), and Level of Significance to Reject H (right columns)

Hypotheses		Table 17		Table 117		Table 118		Table 119		Table 120	
No.		Agr. Coll. (1-4)		Agr. Coll. (1-5)		W's Coll. (1-5)		Both Colls. (1-5)		Both Colls. (1-5)	
1	No group (R)-effect		0.5%		0.5%		0.5%		0.5%		0.5%
2	No year (S)-effect		0.5%		0.5%	25%	50%	2.5%	5%		0.5%
3	No course (T)-effect (or college-effect)		0.5%		0.5%	5%	10%		0.5%	2.5%	5%
4	No RS-interaction	1%	2.5%	2.5%	5%	10%	25%	10%	25%	2.5%	5%
5	No RT-interaction		0.5%		0.5%	2.5%	5%	1%	2.5%	10%	25%
6	No ST-interaction	10%	25%	5%	10%	50%		10%	25%		0.5%
7	No RST-interaction	50%		50%		2.5%	5%	1%	2.5%	5%	10%

§ 105. Conclusion

As for the Agricultural College graduates the conclusion is almost the same to § 5, (6), namely, almost the half of the correlations between the remarks in the entrance examination and those at the graduation are independent, though almost all of the correlations between the three kinds of remarks at the graduation are closely correlated.

As for the Women's College graduates still this conclusion holds. Especially Y is closely correlated with V (almost perfectly) just as in the case of the other college, and the correlation between Y and Z is always at the next.

Yet there is some difference between two colleges. In the former the RST-interaction is always not significant, but in the latter or even in testing about both colleges the triple interaction seems rather significant. Instead of this the year-effect is not significant in the Women's College. For this reason I can say that this college had been making the entrance examination by one subject from each of mathematics, natural sciences, and social studies every year.

Beside this treatise I have already tested the correlation between the remarks of mathematics at the entrance examination and these four kinds, but as I have no more space left I will write on it later.

REFERENCES

- (6) Singu, Tadao : "Rank-correlation among the four class-records in a college," Bulletin of the Simane Agricultural College, 7, 139—148, 1959.
- (7) Singu, Tadao : "On a formula in the analysis of variance," Bulletin of the Simane Agricultural College, 4, 129—131, 1956.

Errata for (6)

P. 142, in the formula of r_s : $(N-1)$ should be (N^2-1) .

P. 145, line 1 and line 4: 'is significantly' should be 'is not significantly.'

摘

(6)において島根農科大学の第1期から第4期までの卒業生について調べた成績の相関を、同大学および島根女子家政短期大学の第1期から第5期までの卒業生に拡張した。その結果は第115表、第116表および第121表に示してある。農大の方では(6)とあまり変化はみあたらないが、女子大の方は大分異なっている。すなわち期別効果、科別効果をはじめとして有意でないものがある反面、RST交互作用が有意であったりしている。これは在学期間が農大の4年に対して2年であるために、入学後の浮沈が前者に比べてすくないためと、入試が毎年数学、理科、社会各々1科目選択制をとっているためによ

要

ると思われる。しかしながらいずれの場合にも、またどのように分析してみても、在学中の成績間の相関は著しく強い。

特にYとVは最強で、YとZがこれについている。これに対し、入試成績と在学中の3種の成績との相関ははるかにすくない。これが入学試験の不適当なためか、学生の入試のための勉強法と入学後の勉強法との著しい相違のためかはまだわからないが、教育者として今後大いに検討を要する問題ではあるまいか。なお入試中数学の成績と他の4種の成績との相関についても調査したが、余白がないので稿を改めて発表したい。