Some Experiments on Submerged V-notch Weir

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もぐり三角ゼキに関する一実験

福　間　　順

Introduction

Measurement of discharge is the most fundamental work to be carried out in any hydraulic experiment and hydrological observation. Standard sharp-crested weirs, such as V-notch weirs, rectangular weirs, and Cippoletti weirs are largely used for the purpose in the open channels. Such weirs have been well calibrated and their discharge equations have been already formulated in many styles by the forerunners.

In some cases such as hydrological field investigations, the weirs should be set in low level because of the effects caused by backwater and other factors. In consequence, flow on the weirs shows often characteristics of submergence under the unexpected circumustances.

Few formulae on submerged weirs can be found out except only VILLEMONTE'S and KOZENY'S formulae limited for the application. Especially, there has been no paper dealing with submerged V-notch weir even up to present.

This report describes some experimental results and presents the discharge equation to estimate the discharge volume from the submerged V-notch weir.

Experiments and Results

The experiments were made in the hydraulic laboratory, Shimane University. A right angle weir as V-notch was set in the open channel. Necessary water volume was taken out from the head tank and tailwater depth was regulated by adjusting the sluice

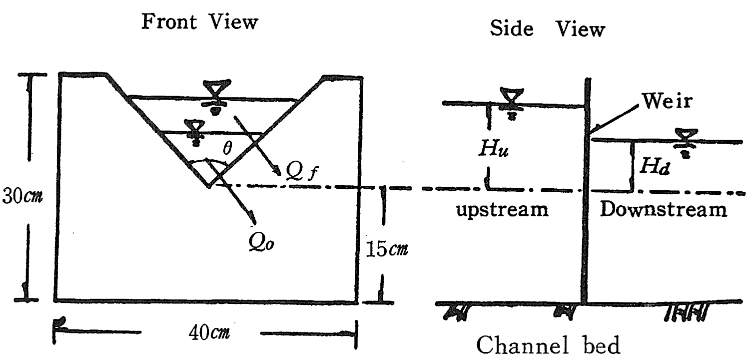

Fig. 1 Sketch of V-notch weir used in the experiment.

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gate at the end of the channel. Such a case of submerged flow is shown schematically in **Fig. 1**.

1 Critical Value of H_d/H_u Related to Degree of Submergence

Submerged state, gennerally defined as follows that flow can not transform itself into jet flow from ordinary flow, will appear at $H_d = H_u$ under the rough view for the definition, because the flow over the weir must show the critical depth interpreted by BÉLANGER's theorem right up the weir, in which, H_u is upstream depth and H_d is downstream. Let the value of Q/Q_{af} be correlated with the value of H_d/H_u, in which Q is equal to the real discharge volume and Q_{af} is the same as in the corresponding free flow, and submerged state, defined expediently concerning discharge volume only, rises when H_d/H_u is 0.3 ~0.4 as seen in **Fig. 2**. This value is much less than that of BÉLANGER's theorem.

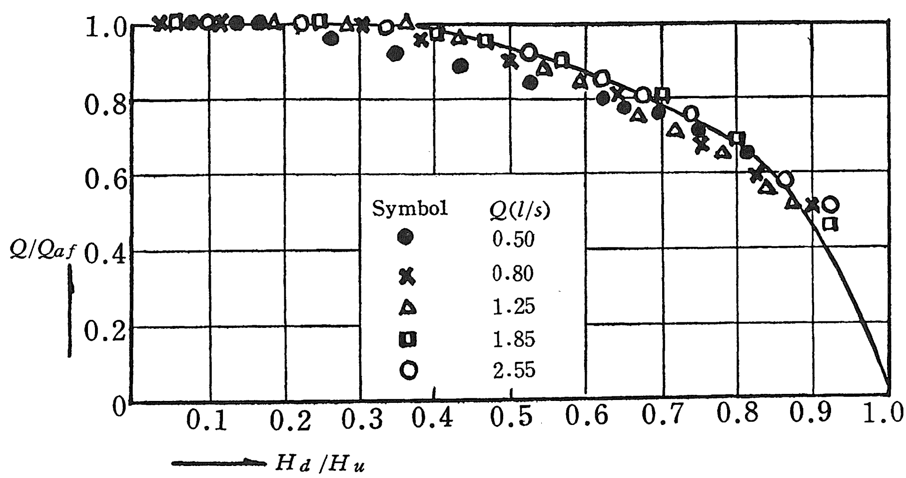

Fig. 2 Relation between Q/Q_{af} and H_d/H_u

It is presumed that the difference between theoretical value and experimental value is due to non-linear distribution of pressure and non-uniformity of velocity in the rotational flow of nappe.

2 Vertical Distribution of Velocity

Horizontal velocities in each depth were measured at center point of the weir by means of Plandtl-type pitot-static tube $\phi = 4$ mm in diameter. It is known that the Plandtl tube is quite insensitive to the angle of attack and that the tube gives the resultant velocity, which is assumed to be horizontal in this experiment. Typical distributions of velocity for the run at $H_d/H_u = 0$, 0.366, 0.412, and 0.897 are shown in **Fig. 3**, where H is distance from the deepest point of notch, V is velocity at each point and V_{max} is the maximum of velocity measured on the same line.

With reference to **Fig. 3**, it is explained that the vertical distribution becomes gradually uniform as the degree of submergence is promoted. Furthermore, it is especially emphasized that the comparatively uniform part of velocity is found out in each case. This uniformity very resembles the distribution of velocity observed often in the investigations of submerged orifice flow.

3 Inducement of Discharge Equation

If the real discharge volume Q is considered to be the total volume of Q_f and Q_0, in which, Q_f is the discharge volume from the upper part of notch as free flow and Q_0 is from the lower part as submerged orifice flow, then the following equations are given by simple integration of BERNOULLI's equation.

$$Q = Q_f + Q_0 \qquad \cdots\cdots\cdots \text{Eq. 1}$$

$$Q_f = 2C_1\sqrt{2g}\,\tan\frac{\theta}{2}\left(\frac{4}{15}H_u^{5/2} - \frac{2}{3}H_u H_d^{3/2} + \frac{2}{5}H_d^{5/2}\right) \qquad \cdots\cdots\cdots \text{Eq. 2}$$

$$Q_0 = C_2\sqrt{2g}\,\tan\frac{\theta}{2}H_d^2\,(H_u - H_d)^{1/2} \qquad \cdots\cdots\cdots \text{Eq. 3}$$

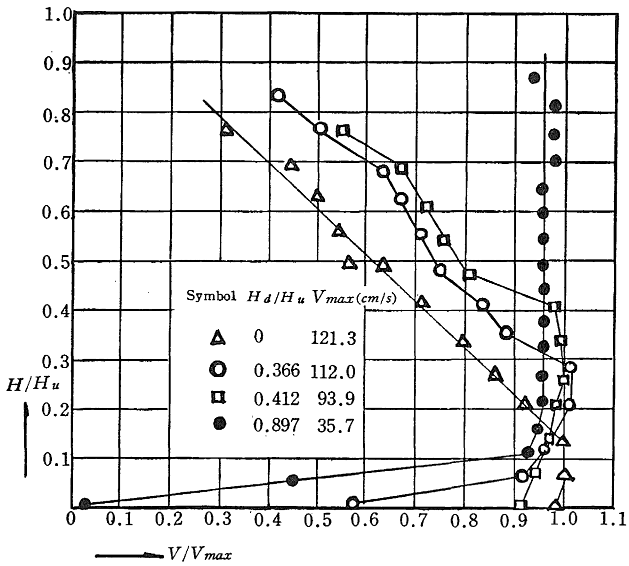

Fig. 3 Typical velocity profiles correalted with the degrees of submergence.

Fig. 4-1 Deformation of stream line with variation of the degrees of submergence $Hd/Hu = 0.312$ $Q = 1.82(s/l)$

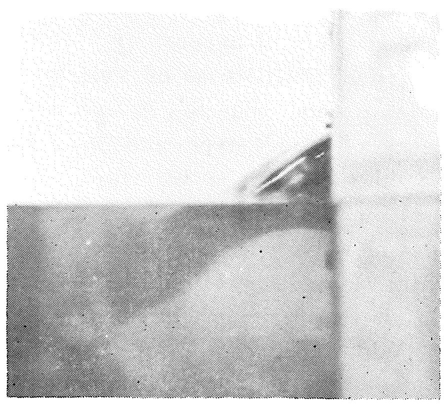

Fig. 4-2 Deformation of stream line with variation of the degrees of submergence. $Hd/Hu = 0.341$ $Q = 1.85(l/s)$

in which C_1 and C_2 are the coefficients of velocity for each part, g is the acceleration from gravity and θ is the opening angle of notch ($\theta=90°$ in the experiment).

If dimensionless number K is defined as the modified coefficent of discharge volume from the submerged weir under the assumption that C_1 and C_2 are equal together, then Q can be written as follows in the simple function of H_u and H_d

$$Q=K\cdot F(H_u,\ H_d) \qquad \cdots\cdots\cdots \text{Eq. 4}$$

in which

$$F(H_u,\ H_d)=2\sqrt{2g}\ \left\{\tan\frac{\theta}{2}\left(\frac{4}{15}H_u^{5/2}-\frac{2}{3}H_uH_d^{3/2}+\frac{2}{5}H_d^{5/2}\right)\right.$$

$$\left.+\frac{1}{2}\tan\frac{\theta}{2}H_d^2\,(H_u-H_d)^{1/2}\right\} \qquad \cdots\cdots\cdots \text{Eq. 5}$$

Namely, the discharge volume from the submerged weir may be calculated with the applications of the induced equations in substituting H_u and H_d into Eq. 5.

The value of K calculated from Eq. 5 with use of the experimental data obtained are shown plotted in **Fig. 5**. It is seen in **Fig. 5** that the plots are considerably systematic even in each degree of submergence. A careful study of **Fig. 5** shows that K is the value of velocity coefficent itself for free flow at low degree of submergence, reaches gradually a maximum value of near 0.80 and decreases finitely to the value of near 0.60 which may be corresponded to the value of velocity coefficent for submerged

Table 1. An example of experimental data gained in $Q=0.80(l/s)$

Depth on downstream $H_d(cm)$	Depth on upstream $H_u(cm)$	Values of H_d/H_u	Discharge volume as free flow $Q_{af}(l/s)$	Values of Q/Q_{af}	Values of K
0.11	4.90	0.022	0.80	1.000	0.636
0.91	4.90	0.186	0.80	1.000	0.723
1.16	4.90	0.237	0.80	1.000	0.735
1.31	4.90	0.267	0.80	1.000	0.772
1.48	4.90	0.302	0.80	1.000	0.780
1.91	4.98	0.384	0.84	0.952	0.805
2.35	5.09	0.462	0.88	0.909	0.806
2.59	5.14	0.504	0.90	0.889	0.805
3.48	5.42	0.642	1.00	0.800	0.742
3.80	5.53	0.687	1.05	0.762	0.731
4.34	5.77	0.752	1.18	0.678	0.668
4.91	6.05	0.812	1.30	0.615	0.645
5.14	6.17	0.833	1.35	0.593	0.606
5.37	6.32	0.850	1.45	0.552	0.621
5.79	6.46	0.896	1.50	0.533	0.633

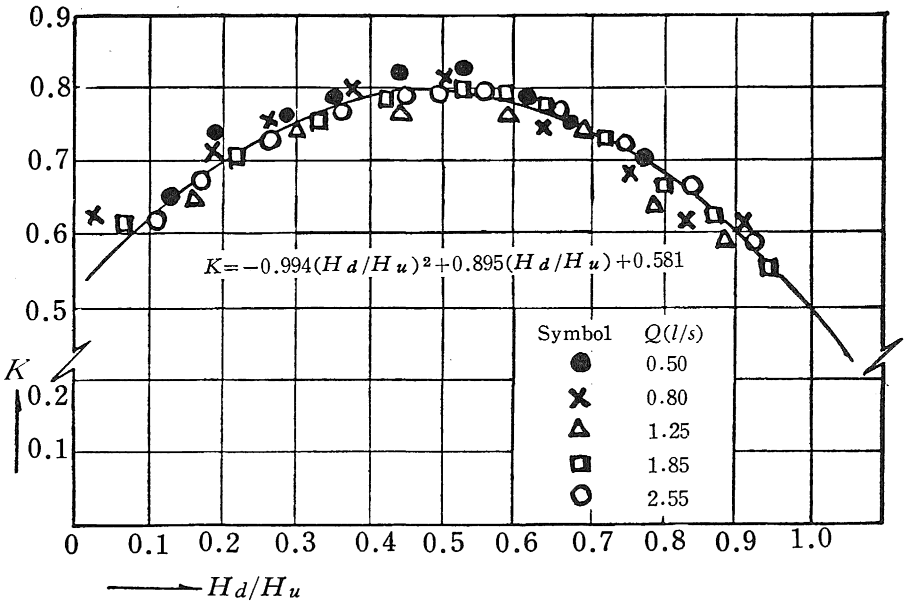

Fig. 5　Relation between value of K and H_d/H_u

orifice flow.

Supposing that K is expressed in the quadratic function of H_d/H_u, then the following equation has been induced as a result of the experimental data dealt with the method of least squares.

$$K=-0.994(H_d/H_u)^2+0.895(H_d/H_u)+0.581 \qquad \cdots\cdots\cdots \text{Eq. 6}$$

Eqs. 5 and 6 can give finitely the value of discharge volume from the submerged V-notch weir concerned. An example of calculation applied is shown in **Table 2** where the application errors are within 4.0 % through the wide range of submergence and oonsiderably allowable in most cases.

Table 2.　An example of calculation errors due to application of the induced discharge equations in real discharge volume $Q=1.85(l/s)$

Values of H_d/H_u	0.921	0.865	0.800	0.726	0.643	0.571	0.517	0.430	0.340	0.064
Calculated values of K	0.562	0.622	0.661	0.706	0.746	0.768	0.778	0.782	0.770	0.634
Calculated discharge volume $Q(l/s)$	1.85	1.83	1.82	1.79	1.78	1.83	1.78	1.83	1.90	1.88
Values of relative error(%)	0.0	1.0	1.7	3.2	3.0	1.0	3.8	1.0	2.7	1.7

Summary

The author intended to induce a simple and applicable discharge equation for the submerged V-notch weir. As the results of the experimental investigations, the following conclusions are summarized ;

1 It is possible to estimate the discharge volume with use of Eqs. 5 and 6. Applicability of the induced equations can cover the considerably wide range of H_d/H_u.

2 An approximate discharge volume can be regarded as equal to the discharge volume for free flow within $H_d/H_u=0.35$.

Many interesting problems, such as the more theoretical anlysis of the critical value of H_d/H_u and the value of K are still suspended out of the examination in this paper.

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摘　　要

流量測定は各種水理実験および水文調査遂行のために欠くことのできない基礎的作業である．流量測定法のうち，刃型ゼキの使用があげられるが，水文調査などの現地観測を行なう場合，各種の制約によってセキ上の流れはもぐり状態となることが多い．しかしもぐり刃型ゼキの流量計算式をとり扱った報告はきわめて少なく，とくにもぐり刃型三角ゼキにいたってはほとんど見当らない．筆者はもぐり三角ゼキの流量計算式を誘導することを目的として若干の実験を行なった結果，以下の点を明らかにするとともに所期の目的を達することができた．

(1)　流量のみに着目した狭義のもぐり状態は，H_d/H_u を parameter とすると，$H_d/H_u>0.35$ の場合に出現している．かかるもぐり状態の流量算出は式(4)，(5)および(6)によって可能である．$H_d/H_u<0.35$ の場合には流量は完全越流として計算しても実用上その誤差は無視できる．

(2)　セキ直上の流速分布はもぐり度合いが進行するにしたがって等分布化する傾向がある．**Fig. 3.** このことからセキ越流量をセキ上部からの自由流下量，セキ下部からのもぐりオリフィス流量の２者から成っているものとして，新らしく流量係数 K を定義し流量計算式(4)，(5)，(6)，を導いた．本式は，その適用誤差および範囲計算の簡便性にかんがみて実用的にじゅうぶんその適用に耐えられる．

本実験から２，３の興味ある問題，たとえば H_d/H_u の厳密解，流量係数 K の挙動など今後考究すべき問題点を提示することができた．