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A Nambu–Goldstone Boson from a Possible Renormalization Procedure

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Abstract

Studying the system of a spin–1/2 particle and a spin–1/2 anti-particle with combinations of the Fermi-type interactions (including the spin-spin interaction) and methods to deal with divergent integrals, we obtain two $J^P=0^-$ bound-state solutions and examine electromagnetic form factors of them. Main results are the following: (I) One of the obtained solutions is a positronium-like bound state. It is obtained by taking the cut-off momentum K to be finite, and the em form factor of it has striking features due to K. The cutting off the momentum is found to be unfacourable for understanding available experimental data on the pion em form factor. (II) The other of the obtained solutions is found to be a Nambu-Goldstone boson. It is able to be obtained by taking K of $K\to\infty$ and performing a renormalization operation by virtue of a form factor in the basic equation. The em form factor of it is consistent with experimental data at large Q^2's. This bound-state solution is very interesting.

§1. Introduction

Recently Cardarelli, Grach, Narodetskii, Pace, Salmè and Simula[1)] (CGNPSS) have presented a theoretical pion electromagnetic (em) form factor under the presence of the Godfrey–Isgur (GI) effective $q\bar{q}$ interaction[2)] motivated by QCD and found to be reproducible for masses of observed mesons. They have described the pion as a pseudoscalar S-wave meson and solved their equation for the wave function by expanding the wave function onto a truncated set of harmonic-oscillator-basis states and determining the coefficients of the expansion in a variational principle.

The CGNPSS pion wave function lacks the manifest covariance. The CGNPSS method for finding wave functions is not rigorous to deal with divergent integrals due to the spin-spin interaction in the GI $q\bar{q}$ interaction. Therefore a doubt is thrown on the contribution from the spin-spin interaction to the CGNPSS theoretical pion em form factor.

In this paper, we make a study which is concerned with the doubt stated above and offers a proposal about the contribution from the spin-spin interaction to the pion wave function. Taking combinations of the five Fermi-type interactions including the spin-spin interaction in the system of a spin–1/2 particle and a spin–1/2 anti-particle, and adopting methods to deal with divergent integrals, we obtain two types of manifestly covariant $J^P=0^-$ bound-state solutions and calculate em form factors of them. The basic equation in this work is provided by starting from the work by Katsumori[3] where a non-local form factor[4] with 4 end points is introduced to the ladder Bethe–Salpeter equation in order to cut off the momentum, and by making a modification of it so as to include an additional form factor concerned with a renormalization operation.

We show the following:

(A) By choosing two specific combinations of the Fermi-type interactions, two types of $J^P=0^-$ bound-state solutions are found. One is a positronium-like bound state, and the other is found to be a Nambu-Goldstone boson, judging from the thing that when its rest mass is zero, it is, in a method in the statistical mechanics, found to be a localized zero mode in the most stable state regarded as vacuum. To obtain the former, the cut-off momentum K is taken to be finite and the additional form factor is cleared away in the basic equation. The latter is obtained by the basic equation itself, taking K to be $K\to\infty$ and performing a renormalization operation. The additional form factor in the basic equation removes discords between this renormalizability and the general conclusions about the Fermi-type interactions in quantum field theory. (The additional form factor is set up so as to permit the formation of Nambu-Goldstone bosons but forbid the scattering processes in the inhomogeneous Bethe-Salpeter equation (for the 4-point Green function) covering our basic equation, taking the opinion that the q-$\bar{q}$ (or q-q) scattering does not contribute to hadron-hadron reactions, as suggested by successes in dual resonance models.) (We have also the opinion that the diffraction scattering is entirely understandable by taking account of the absorption to inelastic processes described in the dual resonance model.) The additional form factor yields also new results which are favourable to understaning observed hadron spectrum.

(B) The positronium-like bound-state solution has an intimate connection with the contribution from the spin-spin interaction to the CGNPSS S-wave pion wave function. The solution is dominated by its ${}^1S_0$–component in the probabilistic interpretation made in the rest frame of the bound state. Both of its ${}^1S_0$–component and the spin-spin contribution to the CGNPSS pion wave function depend on the relative momentum $\boldsymbol{p}$ as $(\boldsymbol{p}^2)^{-1/2}$ at large $|\boldsymbol{p}|$'s.

The em form factor of the positronium-like bound-state solution has striking features due to a finite cut-off momentum K. As a function of the squared momentum transfer Q^2, it is continuous but not smooth. It is forced to be zero at Q^2's of $Q^2 \geqq Q^2_{\text{critical}}(K, m, M) \simeq (4K)^2$, where $m \equiv m_a = m_b$ are the masses of the constituent particles and M is the rest mass of the bound state.

Taking the viewpoint that the pion em form factor at large Q^2's is dominated by the contribution from the spin-spin interaction (i.e., the shortest-range part in the $q\bar{q}$ interaction), the em form factor of the positronium-like solution is compared with available experimental data[5] up to $Q^2=9.77$ $(\text{GeV}/c)^2$. To avoid its incompatibility with experiments, K must be taken to be considerably larger than m.

(B′) The specific combination (of the Fermi-type interactions) forming the positronium-like bound-state solution has a partner which forms a $J^P=0^+$ bound-state solution.

(C) The bound-state solution as a Nambu-Goldstone boson is formed by the first term of the interaction with two terms $g_{f2}/4\{(\bar{\psi}\psi)(\bar{\psi}\psi)-(\bar{\psi}\gamma_5\psi)(\bar{\psi}\gamma_5\psi)+1/2(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma^\mu\psi)-1/2(\bar{\psi}\gamma_5\gamma_\mu\psi)(\bar{\psi}\gamma_5\gamma^\mu\psi)\}+g_{f2}'/4\{(\bar{\psi}\psi)(\bar{\psi}\psi)-(\bar{\psi}\gamma_5\psi)(\bar{\psi}\gamma_5\psi)-1/2(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma^\mu\psi)+1/2(\bar{\psi}\gamma_5\gamma_\mu\psi)(\bar{\psi}\gamma_5\gamma^\mu\psi)\}$, each term of which is invariant under the γ_5-gauge transformation $\psi\to e^{ia\gamma_5}\psi$, $\bar{\psi}\to\bar{\psi}e^{ia\gamma_5}$. When g_{f2} and g_{f2}' are chosen to be $g_{f2}=g_{f2}'(<0)$, this interaction is equivalent to that in the Nambu-Jona-Lasinio model[6] (i.e., $-g_0\{(\bar{\psi}\psi)(\bar{\psi}\psi)-(\bar{\psi}\gamma_5\psi)\times(\bar{\psi}\gamma_5\psi)\}(g_0>0)$). The solution is quite analogous to the $J^P=0^-$ bound state in the Nambu-Jona-Lasinio model. Differences between them are only due to differences between methods for obtaining them. Especially it is noted that the solution is obtained by performing a renormalization operation, while a cutoff of the momentum is introduced in the Nambu-Jona-Lasinio model.

The em form factor of the bound-state solution as a Nambu-Goldstone boson behaves as cont$/(Q^2)^{1/2}$ at large Q^2's. This asymptotic behaviour is found to be consistent with available experiments at large Q^2's. We have the best fit of $\overline{\chi^2}=0.73$ to experimental data[5] at Q^2's of $Q^2\gtrsim 1.9$ $(\mathrm{GeV}/c)^2$ by taking the constant to be free, and we have the best fit of $\overline{\chi^2}=0.16$ to experimental data at Q^2's of $Q^2\geqq 3.30$ $(\mathrm{GeV}/c)^2$ by rechoosing the constant.

(C′) In the interaction in (C), the second term is a partner of the first term.*) It forms a $J^P=0^+$ bound-state solution with an overall factor (m_a-m_b), which does not behave like Nambu-Goldstone bosons and is finally forbidden by the additional form factor.**)

(D) From the present study, it is conjectured that a dynamically broken γ_5 invariance suppresses divergences. In phenomena, PCAC (partial conservation of axial-vector current) satisfied by the Nambu-Goldstone boson is supposed to suppress divergences. Later on (in §4), we provide an example where the PCAC consistency condition suppresses divergences at high energies in a phenomenological $\pi\pi\to\pi\pi$ dual Born amplitude with masses and coupling constants as dynamical quantities. Through masses and coupling constants as dynamical quantities, PCAC is able to control phenomena not only at low energies but also at high energies.

In §2, we state the ladder Bethe-Salpeter equation with combinations of the Fermi-type interactions and methods to deal with divergent integrals and obtain bound-state solutions. In §3, we calculate and examine em form factors of the obtained bound-state solutions. In §4, the results are discussed. In the Appendices, some calculations are done.

§2. Bound-state solutions of the ladder Bethe-Salpeter equation with Fermi-type interactions

2.1. *The ladder Bethe-Salpete equation with Fermi-type interactions and methods to deal with divergent integrals*

We start off with the ladder Bethe-Salpeter equation for the system of a spin–1/2 particle a and a spin–1/2 anti-particle b with one of combinations of the Fermi-type interactions and methods to deal with divergent integrals

*) It is noted that there is a combination (of the Fermi-type interactions) which does not form any bound state at least in the ladder approximation.

**) This solution (forbidden finally) is different from the $J^P=0^+$ mesonic state in the Nambu-Jona-Lasinio model which is an excited state due to fluctuations of the vacuum with a structure.[6,7]

$$\psi(x_1, x_2) = ig_f \int S_F{}^a(x_1, x_3) S_F{}^b(x_2, x_4) \Lambda_f F(x_3, x_4, x_5, x_6) f(x_5, x_6) \times \psi(x_5, x_6) d^4x_3 d^4x_4 d^4x_5 d^4x_6, \quad (1)$$

where ψ is an amplitude, $S_F{}^a(=S_F{}^a(x_1, x_3)$ being defined to be

$$S_F{}^a(x_1, x_3) = \mathrm{S_F{}^a}(x_1 - x_3) = \{-i/(2\pi)^4\} \int d^4k\{(i\not{k} + m_a) e^{-ik(x_1-x_3)}/(-k^2 + m_a{}^2 - i\delta)\}$$

with $kx = k^0x^0 - k^1x^1 - k^2x^2 - k^3x^3 = k^0x^0 - \boldsymbol{kx}$, $k^2 = (k^0)^2 - \boldsymbol{k}^2$ and the mass m_a of a) the Feynman propagator of a, $S_F{}^b$ (with the mass m_b of b) the Feynman propagator of b, F the non-local form factor with 4 end points,[3,4] f a form factor concerned with a renormalization operation, and $g_f \Lambda_f$ with a coupling constant g_f and a set of the Dirac matrices Λ_f comes from one of combinations of the Fermi-type interactions.

The factor $f(x_5, x_6)$ is defined as

$$f(x_5, x_6) = f(x_5 - x_6) = \frac{1}{(2\pi)^4} \int e^{-ik(x_5 - x_6)} f(k^2) d^4k \quad (2)$$

with $f(k^2)$ satisfying

$$f(k^2) \geqq 0 \text{ for any } k^2 = (k^0)^2 - \boldsymbol{k}^2. \quad (3)$$

The explicit form of $f(k^2)$ depends on properties of a solution as a Nambu-Goldstone boson and is given after a solution as a Nambu-Goldstone boson is obtained.

The non-local form factor with 4 end points is defined as

$$F(x_3, x_4, x_5, x_6) = \frac{1}{(2\pi)^{16}} \int F(p_3, p_4, p_5, p_6)\, e^{-i(p_3x_3 + p_4x_4 + p_5x_5 + p_6x_6)} d^4p_3 d^4p_4 d^4p_5 d^4p_6 \quad (4)$$

with the restriction

$$p_3 + p_4 + p_5 + p_6 = 0. \quad (5)$$

Because of this restriction, it takes a form

$$F(x_3, x_4, x_5, x_6) = \frac{1}{(2\pi)^{12}} \int G(P', p', p'')\, e^{-i\{P'(X' - X'') + p'x' + p''x''\}} d^4P' d^4p' d^4p'' \quad (6)$$

with $X' = (m_a x_3 + m_b x_4)/(m_a + m_b)$, $X'' = (m_a x_5 + m_b x_6)/(m_a + m_b)$, $x' = x_3 - x_4$, $x'' = x_5 - x_6$ and $P' = p_3 + p_4 = -(p_5 + p_6)$, $p' = (m_b p_3 - m_a p_4)/(m_a + m_b)$, $p'' = (m_b p_5 - m_a p_6)/(m_a + m_b)$, and $G(P', p', p'')$ in this expression is defined as

$$\begin{aligned} G(P', p', p'') &= G(\{\Pi(P', p')\}^2, \{\Pi(P', p'')\}^2)_K \\ &= G(\{\Pi(P', p')\}^2)_K G(\{\Pi(P', p'')\}^2)_K \end{aligned} \quad (7)$$

with

$$G(\{\Pi(P,p)\}^2)_K=\begin{cases}1 & \text{for} \quad \{\Pi(P,p)\}^2\leqq K^2 \\ 0 & \text{for} \quad \{\Pi(P,p)\}^2>K^2,\end{cases} \tag{8}$$

where

$$\{\Pi(P,p)\}^2=-p^2+(Pp)^2/P^2 \tag{9}$$

and K is the cut-off momentum.[3)]

For the Dirac matrices we use

$$\gamma^0=\gamma_0=\beta,\quad \gamma^k=\beta\alpha^k=-\gamma_k \quad (k=1,2,3)$$

with

$$\beta=\begin{pmatrix} I & 0 \\ 0 & -I\end{pmatrix},\quad \alpha^k=\begin{pmatrix} 0 & \sigma^k \\ \sigma^k & 0\end{pmatrix} \quad \text{and} \quad \gamma_5(\equiv\gamma^5)\equiv i\gamma^0\gamma^1\gamma^2\gamma^3,\quad \sigma_{\mu\nu}=i\Big[\gamma_\mu,\gamma_\nu\Big]/2,$$

where I and σ^k are the 2×2 unit and Pauli matrices.

Introducing

$$X=\frac{m_a x_1+m_b x_2}{m_a+m_b},\quad x=x_1-x_2,\quad P=p_1+p_2,\quad p=\frac{m_b p_1-m_a p_2}{m_a+m_b}, \tag{10}$$

and writing $\psi(x_1,x_2)$ (and $\psi(x_5,x_6)$) to be

$$\psi(x_1,x_2)=\psi(x)\,e^{-iPX}\equiv e^{-iPX}\int e^{-ipx}\phi_P(p)\,d^4p/(2\pi)^{3/2}, \tag{11}$$

$$(\psi(x_5,x_6)=\psi(x'')\,e^{-iPX''}\equiv e^{-iPX''}\int e^{-ip''x''}\phi_P(p'')\,d^4p''/(2\pi)^{3/2}),$$

we have the equation in the momentum space

$$\phi_P(p)=-\frac{ig_f}{(2\pi)^8}G(\{\Pi(P,p)\}^2)_K\frac{m_a+\not{p}_1}{m_a^2-p_1^2-i\delta}\frac{m_b-\not{p}_2}{m_b^2-p_2^2-i\delta}$$
$$\times\Big[\iint G(\{\Pi(P,k')\}^2)_K f((k-k')^2)\Lambda_f\phi_P(k)\,d^4k\,d^4k'\Big], \tag{12}$$

where $p_1=m_aP/(m_a+m_b)+p$, $p_2=m_bP/(m_a+m_b)-p$.*) Here the relations $G(\{\Pi(P,-k)\}^2)_K=G(\{\Pi(P,k)\}^2)_K$ and $f((-k)^2)=f(k^2)$ are used.

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*) The equation with a combination of the Fermi-type interactions, that is,

$$\psi(x_1,x_2)=ig_f\int S_F{}^a(x_1-x_3)S_F{}^b(x_2-x_4)\Lambda_f\delta^4(x_3-x_4)\psi(x_3,x_4)$$
$$\times d^4x_3d^4x_4$$

takes, in the momentum space, the form

$$\phi_P(p)=-\frac{ig_f}{(2\pi)^4}\frac{m_a+\not{p}_1}{m_a^2-p_1^2-i\delta}\frac{m_b-\not{p}_2}{m_b^2-p_2^2-i\delta}\int\Lambda_f\phi_P(k)\,d^4k.$$

From Eq. (12), we obtain the relation

$$\int G(\{\Pi(P,p')\}^2)_K f((p-p')^2)\phi_P(p)d^4pd^4p'$$
$$=-\frac{ig_f}{(2\pi)^8}\int G(\{\Pi(P,p)\}^2)_K G(\{\Pi(P,p')\}^2)_K f((p-p')^2)$$
$$\times\frac{m_a+p\!\!\!/_1}{m_a^2-p_1^2-i\delta}\frac{m_b-p\!\!\!/_2}{m_b^2-p_2^2-i\delta}d^4pd^4p'\left[\iint G(\{\Pi(P,k')\}^2)_K f((k-k')^2)\Lambda_f\phi_P(k)d^4kd^4k'\right], \tag{13}$$

which relates the rest mass of the bound state (given by Eq. (12)), that is, $M=\{(P^0)^2-\boldsymbol{P}^2\}^{1/2}$ to g_f, m_a and m_b.

The ortho-normalization condition for the bound-state solution given by Eq. (12) is

$$\int_{-\infty}^{\infty}d\boldsymbol{p}\int_{-\infty}^{\infty}d\boldsymbol{X}\left\{\int_{-\infty}^{\infty}\phi_{P'}^{\dagger}(p)\,dp^0\right\}\left\{\int_{-\infty}^{\infty}\phi_P(p)\,dp^0\right\}e^{-i(\boldsymbol{P}'-\boldsymbol{P})\cdot\boldsymbol{X}}=(2\pi)^3(P^0/M)\delta^3(\boldsymbol{P}'-\boldsymbol{P}). \tag{14}$$

We expand the amplitude $\phi_P(p)$ in terms of the eigenspinors of $\gamma_0{}^a\gamma_0{}^b$,

$$\phi_P(p)=\sum_{t=1}^{4}\sum_{u=1}^{4}\phi_P{}^{tu}(p)\xi_t{}^a\xi_u{}^b,\quad \xi_1=\begin{pmatrix}1\\0\\0\\0\end{pmatrix},\quad \xi_2=\begin{pmatrix}0\\1\\0\\0\end{pmatrix},\quad \xi_3=\begin{pmatrix}0\\0\\1\\0\end{pmatrix},\quad \xi_4=\begin{pmatrix}0\\0\\0\\1\end{pmatrix}, \tag{15}$$

and introduce the combinations*)

$$\phi_P{}^J=\frac{1}{2}(\phi_P{}^{14}-\phi_P{}^{23}+\phi_P{}^{32}-\phi_P{}^{41}), \tag{16a}$$

$$\phi_P{}^{U0}=\frac{1}{2}(-\phi_P{}^{14}+\phi_P{}^{23}+\phi_P{}^{32}-\phi_P{}^{41}), \tag{16b}$$

$$\phi_P{}^{U1}=\frac{1}{2}(-\phi_P{}^{11}+\phi_P{}^{22}+\phi_P{}^{33}-\phi_P{}^{44}), \tag{16c}$$

$$\phi_P{}^{U2}=i\frac{1}{2}(-\phi_P{}^{11}-\phi_P{}^{22}+\phi_P{}^{33}+\phi_P{}^{44}), \tag{16d}$$

$$\phi_P{}^{U3}=\frac{1}{2}(\phi_P{}^{12}+\phi_P{}^{21}-\phi_P{}^{34}-\phi_P{}^{43}), \tag{16e}$$

$$\phi_P{}^{F1}=\frac{1}{2}(-\phi_P{}^{13}+\phi_P{}^{24}-\phi_P{}^{31}+\phi_P{}^{42}), \tag{16f}$$

$$\phi_P{}^{F2}=i\frac{1}{2}(-\phi_P{}^{13}-\phi_P{}^{24}-\phi_P{}^{31}-\phi_P{}^{42}), \tag{16g}$$

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*) These combinations are introduced analogously to the combinations in the work by Moseley and Rosen.[8] Our notation is also analogous to their notation.

$$\phi_P{}^{F3}=\frac{1}{2}(\phi_P{}^{14}+\phi_P{}^{23}+\phi_P{}^{32}+\phi_P{}^{41}), \tag{16h}$$

$$\phi_P{}^{G1}=\frac{1}{2}(-\phi_P{}^{11}+\phi_P{}^{22}-\phi_P{}^{33}+\phi_P{}^{44}), \tag{16i}$$

$$\phi_P{}^{G2}=i\frac{1}{2}(-\phi_P{}^{11}-\phi_P{}^{22}-\phi_P{}^{33}-\phi_P{}^{44}), \tag{16j}$$

$$\phi_P{}^{G3}=\frac{1}{2}(\phi_P{}^{12}+\phi_P{}^{21}+\phi_P{}^{34}+\phi_P{}^{43}), \tag{16k}$$

$$\phi_P{}^{I}=\frac{1}{2}(\phi_P{}^{12}-\phi_P{}^{21}+\phi_P{}^{34}-\phi_P{}^{43}), \tag{16l}$$

$$\phi_P{}^{A0}=\frac{1}{2}(-\phi_P{}^{12}+\phi_P{}^{21}+\phi_P{}^{34}-\phi_P{}^{43}), \tag{16m}$$

$$\phi_P{}^{A1}=\frac{1}{2}(-\phi_P{}^{13}+\phi_P{}^{24}+\phi_P{}^{31}-\phi_P{}^{42}), \tag{16n}$$

$$\phi_P{}^{A2}=i\frac{1}{2}(-\phi_P{}^{13}-\phi_P{}^{24}+\phi_P{}^{31}+\phi_P{}^{42}), \tag{16o}$$

$$\phi_P{}^{A3}=\frac{1}{2}(\phi_P{}^{14}+\phi_P{}^{23}-\phi_P{}^{32}-\phi_P{}^{41}). \tag{16p}$$

In the expressions (16a) ~ (16p), the argument is omitted for simplicity.

2.2. *Two types of $J^P=0^-$ bound-state solutions*

The positronium-like bound-state solution

A specific combination of the Fermi-type interactions forms a $J^P=0^-$ bound-state solution only in the case where K is taken to be finite. To obtain the solution from this combination, we employ the equation which is given by clearing away the factor f in Eq. (1). For this combination, that is,

$$g_{f1}\Lambda_{f1}=\frac{g_{f1}}{16}(1+\gamma_5{}^a\gamma_5{}^b-\gamma_\mu{}^a\gamma^{\mu b}-\gamma_5{}^a\gamma_\mu{}^a\gamma_5{}^b\gamma^{\mu b}-\sigma_{\mu\nu}{}^a\sigma^{\mu\nu b}/2), \tag{17}$$

the basic equation in the momentum space is

$$\phi_P(p)=-\frac{ig_{f1}}{(2\pi)^4}G(\{\Pi(P,p)\}^2)_K\frac{m_a+\not{p}_1}{m_a^2-p_1^2-i\delta}\frac{m_b-\not{p}_2}{m_b^2-p_2^2-i\delta}\times\left[\int G(\{\Pi(P,k)\}^2)_K\Lambda_{f1}\phi_P(k)\,d^4k\right]. \tag{12$'$}$$

As $\Lambda_{f1}\phi_P(k)$ in the integrand of the R.H.S. of Eq. (12′), we have

$$\Lambda_{f1}\phi_P(k)=\frac{1}{2}\phi_P{}^J(k)\{\xi_1{}^a\xi_4{}^b-\xi_2{}^a\xi_3{}^b+\xi_3{}^a\xi_2{}^b-\xi_4{}^a\xi_1{}^b\}, \tag{18}$$

which characterizes Λ_{f1}.

Equation (12′) with $g_{f1}\Lambda_{f1}\,(g_{f1}>0)$ gives, in the case of $m_a=m_b\equiv m$, the solution

$$\phi_P{}^J(p)=\frac{ig_{f1}N}{(2\pi)^4}G(\{\Pi(P,p)\}^2)_K\frac{(C^J)_P}{D_P(p)}\left[p_\nu p^\nu-m^2-\frac{M^2}{4}\right], \tag{19a}$$

$$\phi_P{}^{U\mu}(p)=\frac{ig_{f1}N}{(2\pi)^4}G(\{\Pi(P,p)\}^2)_K\frac{(C^J)_P}{D_P(p)}[mP^\mu]\,(\mu=0,1,2,3), \tag{19b}$$

$$\phi_P{}^{Gj}(p)=\frac{ig_{f1}N}{(2\pi)^4}G(\{\Pi(P,p)\}^2)_K\frac{(C^J)_P}{D_P(p)}[P^jp^0-P^0p^j]\,(j=1,2,3), \tag{19c}$$

$$\{\phi_P{}^{F1}(p),\phi_P{}^{F2}(p),\phi_P{}^{F3}(p)\}=\frac{ig_{f1}N}{(2\pi)^4}G(\{\Pi(P,p)\}^2)_K\frac{(C^J)_P}{D_P(p)}$$
$$\times[-i\{(P^2p^3-P^3p^2),(P^3p^1-P^1p^3),(P^1p^2-P^2p^1)\}], \tag{19d}$$

$$\phi_P{}^I(p)=\phi_P{}^{A0}(p)=\phi_P{}^{A1}(p)=\phi_P{}^{A2}(p)=\phi_P{}^{A3}(p)=0, \tag{19e}$$

where

$$D_P(p)=\left[p_\nu p^\nu+P_\nu p^\nu+\frac{M^2}{4}-m^2+i\delta\right]\left[p_\nu p^\nu-P_\nu p^\nu+\frac{M^2}{4}-m^2+i\delta\right], \tag{20}$$

$$(C^J)_P=\int G(\{\Pi(P,k)\}^2)_K\phi_P{}^J(k)\,d^4k, \tag{21}$$

and N is the normalization constant.

The relation among M, m and g_{f1} is shown in Appendix A. It is noted that M depends also on K.

The normalization condition on the obtained solution is, in the rest frame of the bound state,

$$\int_{-\infty}^{\infty}d\boldsymbol{p}\left\{\int_{-\infty}^{\infty}\phi_{\text{rest}}^{\dagger}(p)\,dp^0\right\}\left\{\int_{-\infty}^{\infty}\phi_{\text{rest}}(p)\,dp^0\right\}=\int_{-\infty}^{\infty}d\boldsymbol{p}\left[\left|\int_{-\infty}^{\infty}\phi_{\text{rest}}^{J}(p)\,dp^0\right|^2\right.$$
$$\left.+\left|\int_{-\infty}^{\infty}\phi_{\text{rest}}^{U0}(p)\,dp^0\right|^2+\sum_j\left|\int_{-\infty}^{\infty}\phi_{\text{rest}}^{Gj}(p)\,dp^0\right|^2\right]=1, \tag{22}$$

where

$$\int_{-\infty}^{\infty}\{\phi_{\text{rest}}^{J}(p),\phi_{\text{rest}}^{U0}(p),\phi_{\text{rest}}^{Gj}(p)\}\,dp^0=\frac{g_{f1}Nm^2}{2(2\pi)^3}[G(\{\Pi(P,p)\}^2)_K]_{P=O}[(C^J)_P]_{P=O}$$
$$\times\left(\frac{\boldsymbol{p}^2+m^2}{m^2},-\frac{M}{2m},\frac{Mp^j}{2m^2}\right)\frac{1}{(\boldsymbol{p}^2+m^2)^{1/2}(\boldsymbol{p}^2+\kappa_r{}^2)}, \tag{23}$$

$$\kappa_r{}^2=m^2-\frac{M^2}{4}. \tag{24}$$

It is noted that $\phi_P{}^{U1}(p)$, $\phi_P{}^{U2}(p)$, $\phi_P{}^{U3}(p)$ (of $\phi_P{}^{U\mu}(p)$) and $\phi_P{}^{Fj}(p)$ vanish in the rest frame of the bound state. The expression (23) implies that M is restricted to be

$$(0<)M<2m. \tag{25}$$

The normalization in the rest frame of the bound state

$$\left|\frac{g_{f1}N}{2(2\pi)^3}[(C^J)_P]_{P=o}\right|^2 \int_{p^2\leq K^2} \frac{p^2+m^2+\dfrac{M^2}{4}}{(p^2+\kappa_r^2)^2}\,dp=1 \tag{26}$$

is accomplished by choosing N as

$$|N|=\left|\frac{g_{f1}[(C^J)_P]_{P=O}}{2(2\pi)^3}\right|^{-1} \times\left[\frac{\kappa_r(K^2+\kappa_r^2)}{4\pi\left\{K\kappa_r\left(K^2+m^2-\dfrac{M^2}{2}\right)-\left(m^2-\dfrac{M^2}{2}\right)(K^2+\kappa_r^2)\,\mathrm{Arctan}\left(\dfrac{K}{\kappa_r}\right)\right\}}\right]^{1/2}, \tag{27}$$

when K is taken to be finite. For the normalization, the relation $G(\{\Pi(P,p)\}^2)_K G(\{\Pi(P,p)\}^2)_K=G(\{\Pi(P,p)\}^2)_K$ is used. It is noted that $(\Pi(P,p)\}^2$ takes the form $\{\Pi(P,p)\}^2|_{p=o}=p^2$ in the rest frame of the bound state.

In the probabilistic interpretation, we make the expansion of

$$\int_{-\infty}^{\infty} dp\left\{\int_{-\infty}^{\infty}\phi^{\dagger}_{\mathrm{rest}}(p)\,dp^0\right\}\left\{\int_{-\infty}^{\infty}\phi_{\mathrm{rest}}(p)\,dp^0\right\},$$

that is,

$$\int_{-\infty}^{\infty} dp\left\{\int_{-\infty}^{\infty}\phi^{\dagger}_{\mathrm{rest}}(p)\,dp^0\right\}\left\{\int_{-\infty}^{\infty}\phi_{\mathrm{rest}}(p)\,dp^0\right\}=w_1+w_2+w_3+w_4(=1) \tag{28}$$

with

$$\begin{aligned}
(w_1,\,w_2,\,w_3=w_4)&=\frac{1}{2}\int_{-\infty}^{\infty}dp\left[\left[\int_{-\infty}^{\infty}\{\phi^{J}_{\mathrm{rest}}(p)-\phi^{U0}_{\mathrm{rest}}(p)\}\,dp^0\right]^2,\right.\\
&\quad\left[\int_{-\infty}^{\infty}\{\phi^{J}_{\mathrm{rest}}(p)+\phi^{U0}_{\mathrm{rest}}(p)\}\,dp^0\right]^2,\ \sum_j\left[\int_{-\infty}^{\infty}\{\phi^{Uj}_{\mathrm{rest}}(p)-\phi^{Gj}_{\mathrm{rest}}(p)\}\,dp^0\right]^2\\
&\left.=\sum_j\left[\int_{-\infty}^{\infty}\{\phi^{Uj}_{\mathrm{rest}}(p)+\phi^{Gj}_{\mathrm{rest}}(p)\}\,dp^0\right]^2\right]\\
&=\frac{1}{2}\,\frac{1}{\dfrac{M^2}{4mK}\mathrm{Arctan}\left(\dfrac{K}{\left(m^2-\dfrac{M^2}{4}\right)^{1/2}}\right)+\dfrac{\left(m^2-\dfrac{M^2}{4}\right)^{1/2}}{m}\{C_1(M^2,m,K)-2C_2(M^2,m,K)\}}\\
&\quad\times\left[\left[\frac{M\left(m+\dfrac{M}{2}\right)}{2mK}\mathrm{Arctan}\left(\frac{K}{\left(m^2-\dfrac{M^2}{4}\right)^{1/2}}\right)\right.\right.
\end{aligned}$$

$$+\frac{\left(m^2-\frac{M^2}{4}\right)^{1/2}}{m}\left\{C_1(M^2,m,K)-\frac{M\left(m+\frac{M}{4}\right)}{2\left(K^2+m^2-\frac{M^2}{4}\right)}-C_3(M^2,m,K)\right\}\Bigg]\Bigg],$$

$$\left[-\frac{M\left(m-\frac{M}{2}\right)}{2mK}\text{Arctan}\left(\frac{K}{\left(m^2-\frac{M^2}{4}\right)^{1/2}}\right)\right.$$

$$\left.+\frac{\left(m^2-\frac{M^2}{4}\right)^{1/2}}{m}\left\{C_1(M^2,m,K)+\frac{M\left(m-\frac{M}{4}\right)}{2\left(K^2+m^2-\frac{M^2}{4}\right)}-C_3(M^2,m,K)\right\}\right]\Bigg],$$

$$\left[\frac{\left(m^2-\frac{M^2}{4}\right)^{1/2}}{m}\{C_3(M^2,m,K)-C_2(M^2,m,K)\}\right]\Bigg], \tag{29}$$

where

$$C_1(M^2,m,K)=1-\frac{\left(m^2-\frac{M^2}{4}\right)^{1/2}}{K}\text{Arctan}\left(\frac{K}{\left(m^2-\frac{M^2}{4}\right)^{1/2}}\right), \tag{30a}$$

$$C_2(M^2,m,K)=\frac{M^2}{8\left(K^2+m^2-\frac{M^2}{4}\right)}, \tag{30b}$$

$$C_3(M^2,m,K)=\frac{m^2}{2\left(K^2+m^2-\frac{M^2}{4}\right)}$$
$$-\frac{4m^3}{M^2K}\text{Arctan}\left(\frac{M^2K}{4\left\{K^2+m\left(m^2-\frac{M^2}{4}\right)^{1/2}\right\}\left\{m+\left(m^2-\frac{M^2}{4}\right)^{1/2}\right\}}\right)$$
$$+\frac{M^2(12m^2+M^2)}{32K\left\{m+\left(m^2-\frac{M^2}{4}\right)^{1/2}\right\}\left\{m^2+\frac{M^2}{4}+m\left(m^2-\frac{M^2}{4}\right)^{1/2}\right\}}\text{Arctan}\left(\frac{K}{\left(m^2-\frac{M^2}{4}\right)^{1/2}}\right). \tag{30c}$$

The equality $w_3=w_4$ is due to $\phi^{Uj}(p)=0$, and this equality is in accordance with the vanishing $w_3\times(-2)+w_4\times(2)=0$ about the baryon number.

From the expression (29), it is found that the obtained solution is dominated by its particle-anti-particle ${}^1S_0$-component

$$\frac{1}{\sqrt{2}}\int_{-\infty}^{\infty}\{\phi^{J}_{\rm rest}(p)-\phi^{U0}_{\rm rest}(p)\}\,dp^0=\frac{1}{\sqrt{2}}\int_{-\infty}^{\infty}\{\phi^{14}_{\rm rest}(p)-\phi^{23}_{\rm rest}(p)\}\,dp^0 \tag{31}$$

at any M of $0<M<2m$, when K is taken to be an arbitrary finite value. The obtained solution is in an intimate connection with the contribution from the spin-spin interaction to the CGNPSS S-wave pion wave function.*) It is noted that both of the dominant component of the obtained solution and the spin-spin contribution to the CGNPSS pion wave function depend on p as $(p^2)^{-1/2}$ at large $|p|$'s.

As a partner of $g_{f1}\Lambda_{f1}$ of the expression (17), there is

$$-g_{f1}'\Lambda_{f1}'=-\frac{g_{f1}'}{16}(1+\gamma_5{}^a\gamma_5{}^b+\gamma_\mu{}^a\gamma^{\mu b}+\gamma_5{}^a\gamma_\mu{}^a\gamma_5{}^b\gamma^{\mu b}-\sigma_{\mu\nu}{}^a\sigma^{\mu\nu b}/2). \tag{32}$$

By Eq. (12′) with $-g_{f1}'\Lambda_{f1}'(g_{f1}'>0)$ instead of $g_{f1}\Lambda_{f1}$, one can obtain a $J^P=0^+$ bound-state solution, when K is taken to be finite.

If g_{f1} and g_{f1}' are taken to be $g_{f1}=g_{f1}'$, the sum of $g_{f1}\times\Lambda_{f1}$ and $-g_{f1}'\Lambda_{f1}'$ possesses the γ_5 invariance property. However each of $g_{f1}\Lambda_{f1}$ and $-g_{f1}'\Lambda_{f1}'$ does not possess the γ_5 invariance property.

The bound-state solution as a Nambu-Goldstone boson

As $g_f\Lambda_f$ in Eq. (12), we take

$$g_{f2}\Lambda_{f2}=\frac{g_{f2}}{4}(1-\gamma_5{}^a\gamma_5{}^b+\gamma_\mu{}^a\gamma^{\mu b}/2-\gamma_5{}^a\gamma_\mu{}^a\gamma_5{}^b\gamma^{\mu b}/2). \tag{33}$$

Then we have a $J^P=0^-$ bound-state solution by taking K to be $K\to\infty$. As seen below, taking K as $K\to\infty$ is necessary for this solution.

Because one has the equality

$$[G(\{\Pi(P,p)\}^2)_K]_{K\to\infty}=1 \tag{34}$$

in the whole domain of $(P,\ p)$, $[G(\{\Pi(P,\ k')\}^2)_K]_{K\to\infty}$ is omitted in Eq. (12) hereafter. However it is noted that the nonlocal form factor is retained to exist in Eq. (12) in spite of any choice for K. It is also noted that the existence of the non-local form factor is necessary to introduce the form factor concerned with a renormalization operation.

As $\Lambda_{f2}\phi_P(k)$ in the integrand of the R.H.S. of Eq. (12) with $g_{f2}\Lambda_{f2}$ as $g_f\Lambda_f$, we have

*) The CGNPSS (equal-time radial) wave function (in the pion rest frame) is given by $H_{q\bar q}w(k^2)|00\rangle\equiv[(m_q^2+k^2)^{1/2}+(m_q^2+k^2)^{1/2}+V_{q\bar q}]w(k^2)|00\rangle=M_{q\bar q}w(k^2)|00\rangle$, where $V_{q\bar q}$ is the GI effective $q\bar q$ interaction, $M_{q\bar q}$ the mass of the pion, and $|00\rangle$ the spin wave function. The contribution from the spin-spin interaction to the CGNPSS wave function is obtained by taking only the spin-spin interaction in $V_{q\bar q}$ in the equation.

$$\Lambda_{f2}\phi_P(k)=-\frac{1}{2}[\phi_P{}^{U0}(k)\{\xi_1{}^a\xi_4{}^b-\xi_2{}^a\xi_3{}^b-\xi_3{}^a\xi_2{}^b+\xi_4{}^a\xi_1{}^b\}$$
$$+\phi_P{}^{U1}(k)\{\xi_1{}^a\xi_1{}^b-\xi_2{}^a\xi_2{}^b-\xi_3{}^a\xi_3{}^b+\xi_4{}^a\xi_4{}^b\}$$
$$-i\phi_P{}^{U2}(k)\{\xi_1{}^a\xi_1{}^b+\xi_2{}^a\xi_2{}^b-\xi_3{}^a\xi_3{}^b-\xi_4{}^a\xi_4{}^b\}$$
$$+\phi_P{}^{U3}(k)\{-\xi_1{}^a\xi_2{}^b-\xi_2{}^a\xi_1{}^b+\xi_3{}^a\xi_4{}^b+\xi_4{}^a\xi_3{}^b\}]. \tag{35}$$

Assuming the relation

$$\int f((k-k')^2)\phi_P{}^{U\mu}(k)d^4kd^4k'=\frac{P^\mu}{P^0}\int f((k-k')^2)\phi_P{}^{U0}(k)d^4kd^4k' \tag{36}$$

in the integral of the R.H.S. of Eq. (12), we obtain a solution and show the consistency of this resultant solution with the assumed relation.

By virtue of the assumed relation (36), Eq. (12) with $g_{f2}\Lambda_{f2}$ gives, in the case of $m_a=m_b\equiv m$, the solution

$$\phi_P{}^J(p)=-\frac{ig_{f2}N'}{(2\pi)^8}\frac{(C^U)_P}{D_P(p)}[-mM^2], \tag{37a}$$

$$\phi_P{}^{U\mu}(p)=-\frac{ig_{f2}N'}{(2\pi)^8}\frac{(C^U)_P}{D_P(p)}\left[P^\mu\left(p_\nu p^\nu+m^2+\frac{M^2}{4}\right)-p^\mu(2P_\nu p^\nu)\right]\quad(\mu=0,1,2,3), \tag{37b}$$

$$\phi_P{}^{Gj}(p)=-\frac{ig_{f2}N'}{(2\pi)^8}\frac{(C^U)_P}{D_P(p)}[2m(P^jp^0-P^0p^j)]\quad(j=1,2,3), \tag{37c}$$

$$\{\phi_P{}^{F1}(p),\phi_P{}^{F2}(p),\phi_P{}^{F3}(p)\}$$
$$=-\frac{ig_{f2}N'}{(2\pi)^8}\frac{(C^U)_P}{D_P(p)}[-2im\{(P^2p^3-P^3p^2),(P^3p^1-P^1p^3),(P^1p^2-P^2p^1)\}], \tag{37d}$$

$$\phi_P{}^I(p)=\phi_P{}^{A0}(p)=\phi_P{}^{A1}(p)=\phi_P{}^{A2}(p)=\phi_P{}^{A3}(p)=0 \tag{37e}$$

with

$$(C^U)_P=\frac{\int f((k-k')^2)\phi_P{}^{U0}(k)d^4kd^4k'}{P^0}. \tag{38}$$

In fact, $\phi_P{}^{U\mu}(p)$ in this solution is consistent with the assumed relation (36). After we note that the assumed relation (36) is equivalent to

$$\int\phi_P{}^{U\mu}(k)d^4k=\frac{P^\mu}{P^0}\int\phi_P{}^{U0}(k)d^4k, \tag{36'}$$

we have, from Eq. (37b),

$$\int[P^j\phi_P{}^{U0}(p)-P^0\phi_P{}^{Uj}(p)]d^4p$$

$$=\frac{g_{f2}N'}{2(2\pi)^7}(C^U)_P P^0\int_{-\infty}^{\infty}dp\left[\frac{p^j+\dfrac{P^j}{2}}{\left\{\left(p+\dfrac{P}{2}\right)^2+m^2\right\}^{1/2}}-\frac{p^j-\dfrac{P^j}{2}}{\left\{\left(p-\dfrac{P}{2}\right)^2+m^2\right\}^{1/2}}\right]\quad (j=1,2,3). \tag{39}$$

By rechoosing the variable p, the integral in the expression (39) is found to vanish.*) Therefore this solution is consistent with the assumed relation (36). We note that for this solution, the choice of K as $K\to\infty$ is necessary. It is also noted that in the relation (36) and $(C^U)_P$ of the expression (39), we assume

$$\int_{-\infty}^{\infty} f(k^2)\,d^4k=\text{a finite constant}\,(\neq 0), \tag{40}$$

$$\int_{-\infty}^{\infty} \phi_P{}^{U0}(k)\,d^4k=\text{a finite constant}\,(\neq 0). \tag{41}$$

These are assumed only for a moment. Later on, we define $f(k^2)$ explicitly and show that these equalities are really satisfied.

The relation among M, m and g_{f2} is given by Eq. (13) with $g_{f2}\Lambda_{f2}$ (as $g_f\Lambda_f$). Only the relation about $\phi_P{}^{U0}(p)$ in the 16 relations in Eq. (13) should be examined. (When the relation about $\phi_P{}^{U0}(p)$ is satisfied, the other 15 relations are satisfied.) The relation about $\phi_P{}^{U0}(p)$ is

$$\begin{aligned}&\int f((p-p')^2)\phi_P{}^{U0}(p)\,d^4p\,d^4p'\\ &=\frac{g_{f2}m^2}{(2\pi)^7}\left[\int_{-\infty}^{\infty} f(p_\nu' p^{\nu\prime})\,d^4p'\right]\int_{-\infty}^{\infty}dp\,\frac{(E^a+E^b)}{E^aE^b\{(E^a+E^b)^2-(P^0)^2\}}\\ &\quad\times\left[\int f((k-k')^2)\phi_P{}^{U0}(k)\,d^4k\,d^4k'\right],\end{aligned} \tag{42}$$

where

$$E^a=\left\{\left(p+\frac{P}{2}\right)^2+m^2\right\}^{1/2},\quad E^b=\left\{\left(p-\frac{P}{2}\right)^2+m^2\right\}^{1/2}. \tag{43}$$

By the help of the equalities (40) and (41), Eq. (42) is reduced to

$$1=\frac{g_{f2}m^2}{(2\pi)^7}\left[\int_{-\infty}^{\infty} f(p_\nu' p^{\nu\prime})\,d^4p'\right]\int_{-\infty}^{\infty}dp\,\frac{(E^a+E^b)}{E^aE^b\{(E^a+E^b)^2-(P^0)^2\}}. \tag{44}$$

Although the integral over p diverges in Eq. (44), this divergence is able to be dealt with by rein-

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*) We take the standpoint that about the expression (39), the calculation

$$\int_{-\infty}^{\infty}\frac{t}{(t^2+a^2)^{1/2}}dt=\lim_{L\to\infty}\int_{-L}^{L}\frac{t}{(t^2+a^2)^{1/2}}dt=0\quad (a\neq 0)$$

is permitted.

terpreting the coupling constant as

$$\frac{1}{G_{f2}} \equiv \frac{2(2\pi)^7}{g_{f2} m^2 \left[\int_{-\infty}^{\infty} f(p_\nu' p^{\nu'}) d^4 p' \right]} - 4\pi \int_0^{\infty} \frac{dp}{(p^2+m^2)^{1/2}}$$
$$= \int_{-\infty}^{\infty} dp \frac{2(E^a+E^b)}{E^a E^b \{(E^a+E^b)^2-(P^0)^2\}} - 4\pi \int_0^{\infty} \frac{dp}{(p^2+m^2)^{1/2}} \quad (p=|p|). \tag{45}$$

It is noted that the R.H.S. of this equation converges. In this equation, G_{f2} is an effective coupling constant. In the rest frame of the bound state, we have

$$\frac{1}{G_{f2}} = -4\pi \frac{(4m^2-M^2)^{1/2}}{M} \text{Arctan} \left(\frac{M}{(4m^2-M^2)^{1/2}} \right) \tag{46}$$

and find that

$$2m > M > 0 \text{ for } \infty > (-G_{f2}) > \frac{1}{4\pi}. \tag{47}$$

The normalization condition on this solution is, in the rest frame of the bound state,

$$\int_{-\infty}^{\infty} dp \left[\left| \int_{-\infty}^{\infty} \phi_{\text{rest}}^J(p)\, dp^0 \right|^2 + \left| \int_{-\infty}^{\infty} \phi_{\text{rest}}^{U0}(p)\, dp^0 \right|^2 + \sum_j \left| \int_{-\infty}^{\infty} \phi_{\text{rest}}^{Gj}(p)\, dp^0 \right|^2 \right] = 1, \tag{48}$$

where

$$\int_{-\infty}^{\infty} \{\phi_{\text{rest}}^J(p), \phi_{\text{rest}}^{U0}(p), \phi_{\text{rest}}^{Gj}(p)\}\, dp^0$$
$$= -\frac{g_{f2} N' \cdot m^2 M}{2(2\pi)^7} [(C^U)_P]_{P=O} \left(\frac{M}{2m}, -1, \frac{p^j}{m} \right) \frac{1}{(p^2+m^2)^{1/2}(p^2+\kappa_r^2)}, \tag{49}$$

as

$$\int_{-\infty}^{\infty} \phi_{\text{rest}}^{Uj}(p)\, dp^0 \text{ and } \phi_{\text{rest}}^{Fj}(p) \quad (j=1, 2, 3)$$

vanish in the rest frame of the bound state. We have

$$|N'| = \left| \frac{g_{f2} M [(C^U)_P]_{P=O}}{2(2\pi)^7} \right|^{-1} \left[\frac{\kappa_r (m+\kappa_r)}{2\pi^2 m^3} \right]^{1/2}. \tag{50}$$

For this normalization, the equalities (40) and (41) are necessary.

We examine properties of this solution. For this purpose, we make the expansion of the expression (48)

$$1 = w_1 + w_2 + (w_{31} + w_{32} + w_{33}) + (w_{41} + w_{42} + w_{43}) \tag{51}$$

with

$$w_1 \equiv \frac{1}{2}\int_{-\infty}^{\infty} dp \left[\int_{-\infty}^{\infty} \{\phi^J_{\text{rest}}(p) - \phi^{U0}_{\text{rest}}(p)\}\, dp^0\right]^2 = \frac{\left(1+\dfrac{M}{2m}\right)^2}{4\left(1+\dfrac{\kappa_r}{m}\right)}, \tag{52a}$$

$$w_2 \equiv \frac{1}{2}\int_{-\infty}^{\infty} dp \left[\int_{-\infty}^{\infty} \{\phi^J_{\text{rest}}(p) + \phi^{U0}_{\text{rest}}(p)\}\, dp^0\right]^2 = \frac{\left(1-\dfrac{M}{2m}\right)^2}{4\left(1+\dfrac{\kappa_r}{m}\right)}, \tag{52b}$$

$$w_{3j} \equiv \frac{1}{2}\int_{-\infty}^{\infty} dp \left[\int_{-\infty}^{\infty} \{\phi^{Uj}_{\text{rest}}(p) - \phi^{Gj}_{\text{rest}}(p)\}\, dp^0\right]^2$$

$$= w_{4j} \equiv \frac{1}{2}\int_{-\infty}^{\infty} dp \left[\int_{-\infty}^{\infty} \{\phi^{Uj}_{\text{rest}}(p) + \phi^{Gj}_{\text{rest}}(p)\}\, dp^0\right]^2 = \frac{\dfrac{\kappa_r}{m}\left(2+\dfrac{\kappa_r}{m}\right)}{12\left(1+\dfrac{\kappa_r}{m}\right)} \quad (j=1, 2, 3). \tag{52c}$$

The decomposition (52a) ~ (52c) is made by taking the eigenstates of the baryon number. (In detail, the particle-arti-particle state is distinguished from the anti-particle-particle state.) (The equality $w_{3j}=w_{4j}$ is due to

$$\int_{-\infty}^{\infty} \phi^{Uj}_{\text{rest}}(p)\, dp^0 = 0,$$

and this equality is in accordance with the vanishing $w_{3j}\times(-2)+w_{4j}\times(2)=0$ about the baryon number.) In the case of $M=0$, it is found that

$$w_1 = w_2 = w_{31} = w_{32} = w_{33} = w_{41} = w_{42} = w_{43} = \frac{1}{8}, \text{*)} \tag{53}$$

which implies, in the statistical mechanics, the probability distribution in the most stable state regarded as vacuum. Therefore this solution is a Nambu-Goldstone boson, because any Nambu-Goldstone boson is related to the vacuum structure through the Goldstone theorem. It is noted that m (i.e., each of m_a and m_b) in this solution is regarded as the bare current mass plus a dynamical mass acquired through a dynamically broken γ_5 invariance in vacuum,

$$m = m_{\text{bare}} + m_{\text{dy}}. \tag{54}$$

We define $f(k^2)$ explicitly and state the validity of the equalities (40) and (41).

The explicit form of $f(k^2)$ is set up on the following assimptions:

(1) The a-b scattering is forbidden. This comes from the opinion that the quark-anti-

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*) On the other hand, about the solution of Eq. (12′) with $g_{f1}\Lambda_{f1}$ as $g_f\Lambda_f$, we have, in the case of $M=0$,

$$(w_1,\ w_2,\ w_{31}=w_{32}=w_{33}=w_{41}=w_{42}=w_{43}) = \left(\frac{1}{2}, \frac{1}{2}, 0\right)$$

from Eq. (29).

quark scattering (or quark-quark scattering) does not contribute to the hadron-hadron reactions, as suggested by successes in dual resonance models. This is equivalent to the assumption that $f(k^2)$ is chosen so as to provide a kind of projection operator which permits the formation of a bound state of a and b acquiring dynamical masses $m_{a,\mathrm{dy}}$ and $m_{b,\mathrm{dy}}$ but forbids the scattering of a and b in the inhomogeneous Bethe-Salpeter equation (covering our basic equation).

(2) The quantity

$$\int_{-\infty}^{\infty} f(k^2)\, d^4k$$

behaves as $(\mathrm{mass})^4$.

(3) About a bound state as a Nambu-Goldstone boson,

$$\int_{-\infty}^{\infty} f(k^2)\, d^4k$$

requires a renormalization operation in estimating it, as a renormalization of m_{dy} is needed judging from the self-consistency equation for the dynamically generated mass in the Nambu-Jona-Lasinio model. (In the model, a cut-off momentum Λ is introduced. However, when Λ is taken as $\Lambda \to \infty$. the dynamically generated mass behaves as $\sim \Lambda \to \infty$.)

(4) The factor $f(k^2)$ is characterized by $(m_a - m_{a,\mathrm{bare}})$ and $(m_b - m_{b,\mathrm{bare}})$ (i.e., each of $(m_{\mathrm{observed}} - m_{\mathrm{bare\ current}})$ about a and b).

(5) In calculating

$$\int_{-\infty}^{\infty} f(k^2)\, d^4k,$$

the smallness of $(m_i - m_{i,\mathrm{bare}})$ $(i=a, b)$ for the scattering is idealized as

$$\lim_{\varepsilon\to 0}\int_{-\infty}^{\infty} [f(k^2)]|_{(m_i - m_{i,\mathrm{bare}})=\varepsilon(>0)}\, d^4k. \tag{55}$$

(6) The interaction (33) (i.e., the interaction under discussion) combining with

$$\int_{-\infty}^{\infty} f(k^2)\, d^4k$$

in the system of two spin-1/2 particles is accommodated to understanding baryon spectrum (more about this later).

We choose $f(k^2)$ as

$$f(k^2) = \frac{1}{\left[\dfrac{\{(k^0)^2 - \boldsymbol{k}^2\}^2}{(m_a - m_{a,\mathrm{bare}})^4} + 1\right]\left[\dfrac{\{(k^0)^2 - \boldsymbol{k}^2\}^2}{(m_b - m_{b,\mathrm{bare}})^4} + 1\right]} = \frac{1}{\left[\dfrac{\{(k^0)^2 - \boldsymbol{k}^2\}^2}{(m - m_{\mathrm{bare}})^4} + 1\right]^2}. \tag{56}$$

Then we have

$$\int_{-\infty}^{\infty} f(k^2)d^4k = 2\pi^2(m-m_{\rm bare})^2\left\{\lim_{\Lambda\to\infty}\int_0^{\Lambda} k\,dk + c(m-m_{\rm bare})^2\right\} \quad (k=|\boldsymbol{k}|) \tag{57}$$

with c of

$$c=\frac{1}{2}\int_0^{\infty} dt \frac{t}{(t^4+1)^{1/2}[\sqrt{2}(t^4+1)^{1/2}+t\{t^2+(t^4+1)^{1/2}\}^{1/2}]} \times\left[\frac{t}{\{t^2+(t^4+1)^{1/2}\}^{1/2}}\left\{1+\frac{2t^2}{(t^4+1)^{1/2}}\right\}-\frac{1}{\sqrt{2}\{t^2+(t^4+1)^{1/2}\}^2}\left\{\frac{2(3t^4+2)}{(t^4+1)}+\frac{5t^2}{(t^4+1)^{1/2}}\right\}\right], \tag{58}$$

which implies

$$\int_{-\infty}^{\infty} f(k^2)d^4k \longrightarrow \begin{cases} \sim\Lambda^4 & \text{when a bound state as a Nambu-Goldstone boson appears,} \\ & \text{i.e., } (m-m_{\rm bare}) \text{ is } m_{\rm dy} \text{ behaving as } \sim\Lambda\to\infty,^{*)} \\ 0 & \text{when the scattering occurs, i.e., } (m-m_{\rm bare}) \text{ continues} \\ & \text{to be very small, }^{*)} \text{ as idealized as } (m-m_{\rm bare})\to 0. \end{cases} \tag{59}$$

For the bound-state solution as a Nambu-Goldstone boson,

$$\int_{-\infty}^{\infty} f(k^2)\,d^4k$$

of (57) is renormalized as

$$\begin{aligned}\int_{-\infty}^{\infty} f(k^2)d^4k &= 2\pi^2 m_{\rm dy}^2\left\{\lim_{\Lambda\to\infty}\int_0^{\Lambda} k\,dk + c m_{\rm dy}^2\right\} \\ &= 2\pi^2 m_{\rm dy}^2\left\{\left(\int_0^{\infty} k\,dk - \int_0^{\Lambda^c} k\,dk\right) + \left(\int_0^{\Lambda^c/m_{\rm dy}} t\,dt + c\right) m_{\rm dy}^2\right\} \\ &\longrightarrow 2\pi^2 m_{\rm dy}^4\left(\int_0^{\Lambda^c/m_{\rm dy}} t\,dt + c\right).\end{aligned} \tag{60}$$

In the last expression, $m_{\rm dy}$ and $\Lambda^c/m_{\rm dy}$ are quantities which are identified with correspondents from experimental informations. Also in Eq. (54), $m_{\rm dy}$ is identified with the correspondent from experiments. It is noted that $m_{\rm dy}$ and $\Lambda^c/m_{\rm dy}$ are related to the rest mass of the bound-state solution through Eq. (45).

We point out two things which justify the assumption (6) stated above.

(i) As shown in Appendix B, the interaction (33) in the system of two spin-1/2 particles A and B forms a $J^P=0^-$ bound-state solution (with a dominant $^3P_0$-component), which has an overall factor (m_A-m_B). It is noted that only in the case of $m_A=m_B$, this bound-state solution does not exist.

*) The self-consistency equation for the dyanmically generated mass in the Nambu-Jona-Lasinio model has two solutions. One is $\sim\Lambda$ for a large cut-off momentum Λ, and the other is a trivial solution which corresponds to the ordinary perturbative result and is constantly zero. This motivates us to conceive the factor $f(k^2)$.

The nonexistence of this bound state only in the case of $m_A=m_B$ conflicts with the approximate flavour symmetry. Therefore, it is favourable to understanding baryon spectrum to forbid the formation of this bound state by introducing

$$\int_{-\infty}^{\infty} f(k^2)\,d^4k$$

so as to combine with this interaction (33).

(ii) It is favourable to understanding the static $SU(6)_{\text{flavour-spin}}$ symmetry to forbid this $J^P=0^-$ bound state (in the system of A and B) by introducing

$$\int_{-\infty}^{\infty} f(k^2)\,d^4k$$

in the combination with this interaction (33).

The validity of the equality (41) is found in connection with the renormalizability of the coupling constant. From Eqs. (37b) and (45), we have

$$\begin{aligned}\int_{-\infty}^{\infty}\phi_P{}^{U0}(p)\,d^4p&=\frac{g_{f2}N'm^2P^0}{(2\pi)^7}(C^U)_P\int_{-\infty}^{\infty}dp\,\frac{(E^a+E^b)}{E^aE^b\{(E^a+E^b)^2-(P^0)^2\}}\\&=\frac{g_{f2}N'm^2P^0}{2(2\pi)^7}(C^U)_P\left\{\frac{1}{G_{f2}}+4\pi\int_0^{\infty}\frac{dp}{(p^2+m^2)^{1/2}}\right\}\quad(p=|\boldsymbol{p}|).\end{aligned}\tag{61}$$

Although the integral in the curly-bracket term diverges, this divergence is dealt with by a renormalization about $\phi_P{}^{U0}(p)$, because the curly-bracket term is independent of p and dimensionless.

We note that the renormalization operations stated above are harmonized with the general conclusions about the Fermi-type interactions in quantum field theory, because

$$\int_{-\infty}^{\infty} f(k^2)\,d^4k$$

is introduced to the inhomogeneous Bethe-Salpeter equations for the *a*-*b* and *A*-*B* systems (covering our basic equations for *a*-*b* and *A*-*B*) so as to combine with the interaction (33) and forbid the scattering processes through the interaction.

The interaction (33) has a partner

$$\frac{g_{f2}'}{4}(1-\gamma_5{}^a\gamma_5{}^b-\gamma_\mu{}^a\gamma^{\mu b}/2+\gamma_5{}^a\gamma_\mu{}^a\gamma_5{}^b\gamma^{\mu b}/2).\tag{62}$$

In Appendix C, we examine Eq. (12), taking the partner (of the interaction (33)) as $g_f\Lambda_f$. Although a $J^P=0^+$ bound-state solution with an overall factor (m_a-m_b) is formed through the partner, this solution is forbidden by

$$\int_{-\infty}^{\infty} f(k^2)\,d^4k.$$

This is favourable, because one need not search partners of Nambu-Goldstone bosons in observed scalar mesons. We note that as shown in Appendix D, the interaction (62) (i.e., the partner of the interaction (33)) combined with

$$\int_{-\infty}^{\infty} f(k^2)\, d^4k$$

in the system of two spin–1/2 particles A and B forms a $J^P=0^+$ bound-state solution with an overall factor (m_A+m_B), which is, in the case of M(i.e., its rest mass) $=0$, a localized zero mode in the most stable state.

§3. Electromagnetic form factors of the obtained bound-state solutions

We calculate em form factors of the obtained bound-state solutions.

The virtual photon is absorbed by a or b in the scattering of the bound state and the electron. It is noted that for each of the obtained bound states, the matrix elements for the absorption of the photon by a and that by b have an identical body form factor, as the masses of a and b are taken to be equal. We calculate a body form factor being common to the matrix element for the absorption of the photon by a or b.

The body form factor $F(Q^2)$ is definitely calculated, when the equal-time wave function in the rest frame of the bound state is definitely given. This is explicitly clear in the light-cone formalism.[9] We adopt this formalism. In the case of $m_a=m_b\equiv m$, the body form factor in the light-cone formalism is given by

$$F(Q^2)=\int_{-\infty}^{\infty} dk_\perp \int_0^1 d\xi \frac{(M_0M_0')^{1/2}}{4\xi(1-\xi)} \frac{\xi(1-\xi)M_0^2+k_\perp\cdot(k_\perp'-k_\perp)}{\xi(1-\xi)M_0M_0'} \frac{w^\dagger(k)w(k')}{4\pi} \tag{63}$$

with

$$M_0^2=\frac{m^2+k_\perp^2}{\xi(1-\xi)},\quad M_0'^2=\frac{m^2+k_\perp'^2}{\xi(1-\xi)},\quad k_\perp'\equiv k_\perp+(1-\xi)Q_\perp, \tag{64}$$

$$k^2\equiv k_\perp^2+k_n^2,\quad k_n\equiv(\xi-1/2)M_0,\quad k'^2\equiv k_\perp'^2+k_n'^2,\quad k_n'\equiv(\xi-1/2)M_0'. \tag{65}$$

(It is noted that the spin-quantization axis is chosen so as to make the plus component of the four-momentum transfer vanish.[9]) The wave function $w(k)$ in the expression (63) is defined so as to satisfy the condition

$$\int_0^{\infty} k^2 w^+(k)w(k)\,d|k|=1$$

and is able to be identified with the equal-time wave function in the rest frame of the bound state which satisfies this condition.

3.1 *The body form factor of the positronium-like bound-state solution*

We have

$$w^{\dagger}(k)w(k')=[G(\{\Pi(P,k)\}^2)_K]_{P=O}[G(\{\Pi(P',k')\}^2)_K]_{P'=O}$$
$$\times\frac{(K^2+\kappa_r^2)}{K^3\left[1+\dfrac{\left(m^2-\dfrac{M^2}{2}\right)}{K^2}\left\{1-\dfrac{(K^2+\kappa_r^2)}{K\kappa_r}\text{Arctan}\left(\dfrac{K}{\kappa_r}\right)\right\}\right]}$$
$$\times\frac{(k^2+m^2)(k'^2+m^2)+\dfrac{M^2}{4}(k\cdot k'+m^2)}{(k^2+m^2)^{1/2}(k^2+\kappa_r^2)(k'^2+m^2)^{1/2}(k'^2+\kappa_r^2)} \tag{66}$$

with

$$[G(\{\Pi(P,k)\}^2)_K]_{P=O}=\begin{cases}1 \text{ for } k^2\leqq K^2\\ 0 \text{ for } k^2>K^2,\end{cases}$$
$$[G(\{\Pi(P',k')\}^2)_K]_{P'=O}=\begin{cases}1 \text{ for } k'^2\leqq K^2\\ 0 \text{ for } k'^2>K^2\end{cases} \tag{67}$$

for the positronium-like bound-state solution.

It is found that the body form factor $F^{\text{posi}}(Q^2)$ of the positronium-like bound-state solution has striking features due to a finite cut-off momentum K. These features are easily seen, when the static body form factor

$$F_{\text{st}}^{\text{posi}}(q^2)=\int dr\rho_{\text{st}}^{\text{posi}}(r)\,e^{iq\cdot r}\left(\int dr\rho_{\text{st}}^{\text{posi}}(r)=1,\ q^2=Q^2,\ r=x/2\right) \tag{68}$$

is calculated by assuming $(m/K)^2\ll 1$ and $(M/K)^2\ll 1$. We have

$$F_{\text{st}}^{\text{posi}}(q^2)=\begin{cases}1-\dfrac{3|q|}{8K} & \text{for}\quad 0<|q|<2K\\ \dfrac{|q|}{8K}-1+\dfrac{2K}{|q|} & \text{for}\quad 2K<|q|<4K\\ 0\quad \text{for}\quad 4K<|q| & \end{cases} \tag{69}$$

by using

$$\int_0^{\infty}\frac{\sin(ax)\sin(bx)\sin(xy)}{x^3}dx=\begin{cases}\dfrac{\pi}{2}by & \text{for}\quad 0<y<a-b\\ \dfrac{\pi}{2}by-\dfrac{\pi}{8}(a-b-y)^2 & \text{for}\quad a-b<y<a+b\\ \dfrac{\pi}{2}ab & \text{for}\quad a+b<y<\infty\ (a\geqq b>0,\ y>0).\end{cases} \tag{70}$$

[10]

From the comparison of $F^{\text{posi}}(Q^2)$ at large Q^2's with available experimental data[5] up to $Q^2=9.77\ (\text{GeV}/c)^2$, it is found that the cut-off momentum K must be taken to be considerably larger than $m(\simeq 0.3\text{ GeV})$,

$$K/m \gtrsim 5. \tag{71}$$

This seems to be unfavoureble for the introduction of the cut-off momentum.

3.2. *The body form factor of the bound-state solution as a Nambu-Goldstone boson*

As $w^{+}(k)w(k')$ of the bound-state solution as a Nambu-Goldstone boson, we have

$$w^{\dagger}(k)w(k') = \frac{\kappa_r(m+\kappa_r)}{2\pi m}\frac{4(k\cdot k'+m^2)+M^2}{(k^2+m^2)^{1/2}(k^2+\kappa_r^2)(k'^2+m^2)^{1/2}(k'^2+\kappa_r^2)}\,. \tag{72}$$

It is found that the body form factor $F^{\text{N-Gb}}(Q^2)$ of the bound-state solution as a Nambu-Goldstone boson behaves asymptotically as

$$F^{N-Gb}_{\text{asymp}}(Q^2) \simeq \frac{\text{const}}{\left[\dfrac{Q^2}{\left\{\dfrac{\kappa_r(m+\kappa_r)}{m}\right\}^2}\right]^{1/2}}\,. \tag{73}$$

Taking the viewpoint that the pion em form factor at large Q^2's is dominated by the contribution from the spin-spin interaction (i.e., the shortest-range part in the $q\bar{q}$ interaction), $F^{\text{N-Gb}}_{\text{asymp}}(Q^2)$ is compared with available experimental data[5] at large Q^2's (i.e., data at 1.94, 1.99, 2.01, 3.30, 3.33, 3.99, 6.30 and 9.77 $(\text{GeV}/c)^2$). It is found that the best fit of $\overline{\chi^2}=0.73$ is obtained by taking $(\text{const})\{\kappa_r(m+\kappa_r)/m\}$ to be free, and, if data at Q^2's of $Q^2 \geqq 3.30$ $(\text{GeV}/c)^2$ are adopted, the best fit of $\overline{\chi^2}=0.16$ is obtained by rechoosing $(\text{const})\{\kappa_r(m+\kappa_r)/m\}$.

§4. Discussion

Studying the system of a spin–1/2 particle and a spin–1/2 anti-particle with combinations of the Fermi-type interactions (including the spin-spin interaction) and methods to deal with divergent integrals, we have obtained two $J^P=0^-$ bound-state solutions and examined em form factors of them. As the main results, we stress the following:

(I) The em form factor of the solution with a cut-off momentum K has striking features due to K. Therefore it is incompatible with available experimental data on the pion em form factor.

(II) The solution as a Nambu-Goldstone boson is obtained by performing a renormalization operation. The em form factor of it is consistent with experimental data at large Q^2's. We suppose that in constructing the wave function for the pion, its high-momentum component (mainly due to the spin-spin interaction in the $q\bar{q}$ interaction) can be searched in a manner with a renormalization operation, judging from the present study.

From the present study, we conjecture that a dynamically broken γ_5 invariance or the PCAC satisfied by the Nambu-Goldstone boson suppresses divergences. We present an example where the PCAC consistency condition suppresses divergences at high energies.

Munakata, Sakamoto and the author[11] obtained a uniquely determined $\pi^-\pi^+\to\pi^-\pi^+$ Born amplitude

$$-\lambda_{1,1}(1-\beta^2)\sum_{n=1}^{\infty}\frac{1}{(n-1)!(2n-1+\beta)(2n-3+\beta)}\left[\frac{\Gamma(n-\alpha_s)\Gamma(n-\alpha_t)}{\Gamma(n-\alpha_s-\alpha_t)}+(1-\beta)\right.$$
$$\left.\times\frac{\Gamma(n-\alpha_s)\Gamma(n-\alpha_t)}{2\Gamma(n+1-\alpha_s-\alpha_t)}\right]\quad(\alpha_s\equiv\alpha_{\rho-f}(s)=\alpha_0+\alpha's,\ \beta\equiv2-3\alpha_0-4\alpha'm_\pi^2)$$

by starting from the most general Veneziano-type amplitude

$$\sum_{n=1}^{\infty}\sum_{k=n}^{2n}\Gamma(n-\alpha_s)\Gamma(n-\alpha_t)/\Gamma(k-\alpha_s-\alpha_t)$$

and imposing the restrictions (a) absence of odd daughters, (b) the local duality relations[12] between the s- and t-channel resonance families N and N' ($N, N'=0, 1, 2, \cdots$),[12] and (c) the convergence condition at $s\to\infty$ and fixed t. The showed that the obtained amplitude has good properties, (α) it contains neither negative-norm states nor tachyons in a domain of $(\alpha' m_\pi^2, \alpha_0)$ around the physical m_π, α' and α_0, (β) when m_π, α' and α_0 are taken to be the physical values and $\lambda_{1,1}$ is adjusted by observed $\rho\to2\pi$ width, it predicts partial decay widths of low-lying resonances consistently with experiments, and (γ) in the limit of $m_\pi=0$ and $\alpha_0=1$, it satisfies Adler's PCAC consistency condition. When (γ) is adopted as a restriction instead of (c), one has, in the limit of $m_\pi=0$ and $\alpha_0=1$, a uniquely determined amplitude which converges at $s\to\infty$ and fixed t.

Appendix A

—The relation among M, m, g_{f1} and K in the positronium-like bound-state solution—

The relation among M, m, g_{f1} and K in the positronium-like bound-state solution is given by

$$\int G(\{\Pi(P,p)\}^2)_K\phi_P(p)d^4p=-\frac{ig_{f1}}{(2\pi)^4}\int[G(\{\Pi(P,p)\}^2)_K]^2\frac{m_a+p_1}{m_a^2-p_1^2-i\delta}\frac{m_b-p_2}{m_b^2-p_2^2-i\delta}d^4p$$
$$\times\left[\int G(\{\Pi(P,k)\}^2)_K\Lambda_{f1}\phi_P(k)d^4k\right],\qquad(\mathrm{A}\cdot1)$$

which is obtained from Eq. (12′). In this relation (A·1), $[G(\{\Pi(P,p)\}^2)_K]^2$ is equivalent to $G(\{\Pi(P,p)\}^2)_K$. Only the relation about $\phi_P{}^J(p)$ in the 16 relations of (A·1) should be examined. (When the relation about $\phi_P{}^J(p)$ is satisfied, the other 15 relations are satisfied.) In the case of $m_a=m_b\equiv m$, we have, in the rest frame of the bound state,

$$1=\frac{g_{f1}}{2(2\pi)^3}\int_{p^2\leqq K^2}\frac{(p^2+m^2)^{1/2}}{(p^2+\kappa_r^2)}dp,\qquad(\mathrm{A}\cdot2)$$

which implies

$$2m>M>0 \text{ for } \left[\frac{K}{m}\left\{1+\left(\frac{K}{m}\right)^2\right\}^{1/2}+\ln\left[\frac{K}{m}+\left\{1+\left(\frac{K}{m}\right)^2\right\}^{1/2}\right]\right]^{-1}<\frac{g_{f1}m^2}{2(2\pi)^2}$$
$$<\left[\frac{K}{m}\left\{1+\left(\frac{K}{m}\right)^2\right\}^{1/2}-\ln\left[\frac{K}{m}+\left\{1+\left(\frac{K}{m}\right)^2\right\}^{1/2}\right]\right]^{-1}. \tag{A·3}$$

Appendix B

—The $J^P=0^-$ bound-state solution in the system of two spin–1/2 particles with interaction (33)—

The basic equation for the system of two spin–1/2 particles A and B with interaction (33) is

$$\phi_P(p)=-\frac{ig_{f2}}{(2\pi)^8}[G(\{\Pi(P,p)\}^2)_K]_{K\to\infty}\frac{m_A+\not{p}_1}{m_A{}^2-p_1{}^2-i\delta}\frac{m_B+\not{p}_2}{m_B{}^2-p_2{}^2-i\delta}$$
$$\times\left[\iint[G(\{\Pi(P,k')\}^2)_K]_{K\to\infty}f((k-k')^2)\Lambda_{f2}\phi_P(k)d^4kd^4k'\right] \tag{A·4}$$

with

$$[G(\{\Pi(P,p)\}^2)_K]_{K\to\infty}=[G(\{\Pi(P,k')\}^2)_K]_{K\to\infty}=1. \tag{A·5}$$

We have the bound-state solution in the rest frame of the bound state

$$\int_{-\infty}^{\infty}\{\phi^J_{\text{rest}}(p),\phi^{U0}_{\text{rest}}(p)\}\,dp^0=\frac{g_{f2}N_{AB}(m_A-m_B)}{16(2\pi)^7}[(C^U_{AB})_P]_{P=O}\left\{1,-\frac{(m_A-m_B)}{M}\right\}$$
$$\times\frac{M^2\{(E_A+E_B)^2-(m_A+m_B)^2\}+(m_A+m_B)^2\{(E_A-E_B)^2-(m_A-m_B)^2\}}{E_AE_B(E_A+E_B)(p^2+\kappa_R{}^2)}, \tag{A·6a}$$

$$\int_{-\infty}^{\infty}\{\phi^{Uj}_{\text{rest}}(p),\phi^{Gj}_{\text{rest}}(p)\}\,dp^0=-\frac{g_{f2}N_{AB}(m_A-m_B)}{8(2\pi)^7}[(C^U_{AB})_P]_{P=O}\{p^j(m_A+m_B)\}$$
$$\times\frac{\left\{\{(E_A-E_B)^2-M^2\},\left\{-\frac{M^2(E_A+E_B)^2-(m_A{}^2-m_B{}^2)^2}{M(m_A+m_B)}\right\}\right\}}{E_AE_B(E_A+E_B)(p^2+\kappa_R{}^2)}, \tag{A·6b}$$

$$\int_{-\infty}^{\infty}\phi^I_{\text{rest}}(p)\,dp^0=\int_{-\infty}^{\infty}\phi^{A\mu}_{\text{rest}}(p)\,dp^0=\int_{-\infty}^{\infty}\phi^{Fj}_{\text{rest}}(p)\,dp^0=0 \tag{A·6c}$$

with

$$(C^U)_P=\frac{\int f((k-k')^2)\phi_P{}^{U0}(k)d^4kd^4k'}{P^0},\quad E_A=(p^2+m_A{}^2)^{1/2},\quad E_B=(p^2+m_B{}^2)^{1/2},$$
$$\kappa_R{}^2=\frac{\{(m_A+m_B)^2-M^2\}\{M^2-(m_A-m_B)^2\}}{4M^2}. \tag{A·7}$$

This solution has an overall factor (m_A-m_B). As known from Eqs. (A·6a) and (A·6b), its rest mass M is restricted to be

$$|m_A-m_B|<M<m_A+m_B. \tag{A·8}$$

In the case of $M=|m_A-m_B|$, its probability distribution is

$$w_{\frac{1}{2}}=\frac{1}{2}\int_{-\infty}^{\infty}dp\left[\int_{-\infty}^{\infty}\{\phi_{\rm rest}^{J}(p)\mp\phi_{\rm rest}^{U0}(p)\}\,dp^0\right]^2\simeq 0, \tag{A·9a}$$

$$\begin{aligned} w_{31}+w_{32}+w_{33}&=\frac{1}{2}\sum_j\int_{-\infty}^{\infty}dp\left[\int_{-\infty}^{\infty}\{\phi_{\rm rest}^{Uj}(p)-\phi_{\rm rest}^{Gj}(p)\}\,dp^0\right]^2 \\ =w_{41}+w_{42}+w_{43}&=\frac{1}{2}\sum_j\int_{-\infty}^{\infty}dp\left[\int_{-\infty}^{\infty}\{\phi_{\rm rest}^{Uj}(p)+\phi_{\rm rest}^{Gj}(p)\}\,dp^0\right]^2\simeq\frac{1}{2}, \end{aligned} \tag{A·9b}$$

when m_A and m_B are taken to be $|{\rm m_A}-{\rm m_B}|/(m_A+m_B)\ll 1\,(m_A\neq m_B)$. This distribution is quite different from the distribution in Eq. (53). By virtue of

$$\int_{-\infty}^{\infty}f(k^2)\,d^4k,$$

this solution is forbidden.

Appendix C

—*The* $J^P=0^+$ bound-state solution in the system of a spin–1/2 particle a and a spin–1/2 antiparticle b with interaction (62) being the partner of the bound-state solution as a Nambu-Goldstone boson—

Equation (12) with interaction (62) gives the partner of the bound-state solution as a Nambu-Goldstone boson. The explicit expression for this partner in the rest frame of the bound state is obtained from Eqs. (A·6a)~(A·6c) by making the substitution

$$\begin{aligned} &\left[m_A,\,m_B,\,g_{f2},\,\left\{\int_{-\infty}^{\infty}\left(\phi^J(p),\,\phi^{U0}(p),\,\phi Uj(p),\,\phi^{Gj}(p),\,\phi^I(p),\,\phi^{A0}(p),\,\phi^{Aj}(p),\,\phi^{Fj}(p)\right)_{\rm rest}dp^0\right\}\right] \\ &\longrightarrow\left[m_a,\,m_b,\,g_{f2}',\,\left\{\int_{-\infty}^{\infty}(\phi^I(p),\,\phi^{A0}(p),\,\phi^{Aj}(p),\,\phi^{Fj}(p),\,\phi^J(p),\,\phi^{U0}(p),\,\phi^{Uj}(p),\right.\right. \\ &\left.\left.\qquad\phi^{Gj}(p))_{\rm rest}\,dp^0\right\}\right]. \end{aligned}$$

This partner has an overall factor (m_a-m_b). The probability distribution of this partner in the case of $M=|m_a-m_b|$ and $|{\rm m_a}-{\rm m_b}|/(m_a+m_b)\ll 1\,(m_a\neq m_b)$ is known from Eqs. (A·9a) and (A·9b) by making the above substitution, and this partner is forbidden by virtue of

$$\int_{-\infty}^{\infty}f(k^2)\,d^4k.$$

Appendix D

—The $J^P=0^+$ bound-state solution in the system of two spin-1/2 particles A and B with interaction (62)—

In the system of two spin-1/2 particles A and B, the interaction (62) gives the $J^P=0^+$ bound-state solution, whose explicit expression in the rest frame of the bound state in the case of $m_A=m_B\equiv m$ is obtained from Eq. (49) (and

$$\int_{-\infty}^{\infty}\{\phi^{Uj}(p),\ \phi^{Fj}(p,\ \phi^{I}(p),\ \phi^{A0}(p),\ \phi^{Aj}(p)\}_{\rm rest}dp^0=(0,0,0,0,0))$$

by making the substitution

$$\Big[m(\equiv m_a=m_b),\ g_{f2},\ \Big\{\int_{-\infty}^{\infty}(\phi^{J}(p),\ \phi^{U0}(p),\ \phi^{Gj}(p);\ \phi^{Uj}(p),\ \phi^{Fj}(p),\ \phi^{I}(p),\ \phi^{A0}(p),\ \phi^{Aj}(p))_{\rm rest}\,dp^0\Big\}\Big]$$

$$\longrightarrow\Big[m(\equiv m_A=m_B),\ g_{f2}',\ \Big\{\int_{-\infty}^{\infty}(\phi^{I}(p),\ \phi^{A0}(p),\ \phi^{Fj}(p);\ \phi^{Aj}(p),\ \phi^{Gj}(p),\ \phi^{J}(p),\ \phi^{U0}(p),\ \phi^{Uj}(p))_{\rm rest}\,dp^0\Big\}\Big].$$

The probability distribution of this solution in the case of $M=0$ is known from Eqs. (52a)～(52c) and (53) by the same substitution. In the case of $M=0$, this solution is a localized zero mode in the most stable state regarded as vacuum.

It is a remaining problem to study connections of the above solution with properties of the baryons.

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