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Feasibility of Timoshenko's bending theory for measuring the shear modulus of wood

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Abstract

In a previous paper we examined the applicability of Timoshenko's bending theory, which describes the effect of shearing force in bending deflection, when measuring the shear modulus of wood, and found that it is difficult to obtain the shear modulus by Timoshenko's theory because of the distorted shear stress condition around the loading point. When the deflection is measured at the point distant from the loading point, we thought that the shear modulus would be obtained by Timoshenko's theory properly. Here, we conducted the three-point bending tests with measuring the deflection at the midpoint between the loading point and a support, and examined whether Young's and shear moduli can be measured properly.

Akamatsu (Japanese red pine, *Pinus densiflora* D. Don) and balsa (*Ochroma lagopus* Sw.) were used for the testing materials. First the Young's modulus and the shear modulus were measured by free-free flexural vibration tests. Then the three-point static bending tests with varying the depth/span ratios were simulated by the finite element method (FEM). From the FEM analyses, the load-deflection behavior is effectively described by Timoshenko's bending theory when the deflection is measured at the opposite point against the loading point. Finally the static bending tets were conducted with a dial gage set below the specimen to measure the deflection at the opposite point against the loading point, and the Young's and shear moduli were calculated by Timoshenko's bending equation.

From the testing results, we concluded that it is difficult to obtain the shear modulus properly by original Timoshenko's theory even when the measured points of dflections were variously changed, and that the modification of the original equation should be needed.

Key words: shear modulus, static bending test, FEM analysis, Timoshenko's bending theory, measured point.

1. Introduction

When measuring Young's and shear moduli of wood by three-point bending tests, the basic theory is often dependent on Timoshenko's bending theory. Young's modulus can be given properly by this theory, whereas shear modulus is derived as extremely small because the distorted stress condition around the loading point is not considered in Timoshenko's theory. In our previous paper, we modified Timoshenko's theory with the assumption that the distorted shear stress condition produces the extra deflection.[1] We thought that this modification is needed because the deflection is measured at the loading point where the stress condition is most seriously distorted, and that Timoshenko's theory is applicable when the deflection is measured at the region out of the distorted area. In this paper, we conducted three-point bending tests by measuring the deflection at the midpoint between the loading point and a support, and examined the feasibility of Timoshenko's bending equation for the measurement of Young's and shear moduli of wood.

2. Theories

Figure 1 shows the deflected beam subjected to the load imposed at the midpoint between the supports. When the load P is imposed at the center of the beam with the span of l, the deflection at the point of $x=x$ caused by the bending moment, y_b, can be written as follows:[2]

$$y_b=\frac{P}{12EI}x\left(\frac{3}{4}l^2-x^2\right), \tag{1}$$

where E and I are the Young's modulus and the second moment of cross-sectional area of the beam, respectively. When the depth/span ratio of the beam is large, the effect of the shearing force should be taken into account. According to Timoshenko's theory, the deflection at the same point caused by shearing force, y_s, can be written as follows:

$$y_s=\frac{sPx}{2GA}, \tag{2}$$

where G and A are the shear modulus and the cross-sectional area of the beam, respectively, and x is the Timoshenko's shear factor. When s is defined as the ratio of the maximum shear stress to the average shear stress, it is given as 1.5 for the beam with a rectangular cross section. On the other hand, s is derived as 1.2 from the calculation of strain energy.[2] The shearing force

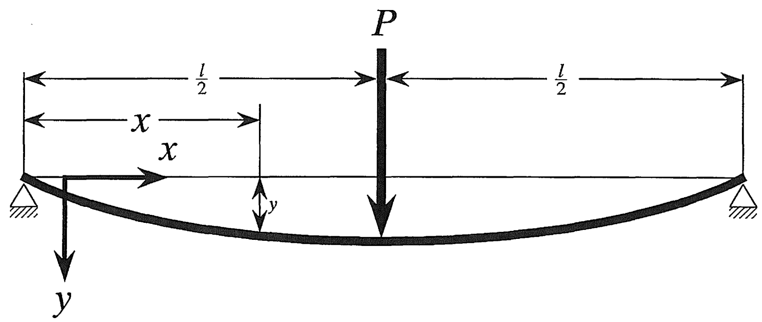

Fig. 1. Three-point bending diagram.

which does not distribute homogeneously in the shearing plane is leveled by this factor. The total flexural displacement at $x=x$, y, is given as follows:

$$y=y_b+y_s=\frac{P}{12EI}x\left(\frac{3}{4}l^2-x^2\right)+\frac{sPx}{2GA}. \tag{3}$$

When the beam has a rectangular cross section whose depth is h, Eq. (3) is transformed as follows:

$$y=\frac{P}{12I}x\left(\frac{3}{4}l^2-x^2\right)\left\{\frac{1}{E}+\frac{sh^2}{G\left(\frac{3}{2}l^2-2x^2\right)}\right\}=\frac{P}{12E_sI}x\left(\frac{3}{4}l^2-x^2\right). \tag{4}$$

where E_s is the "apparent" Young's modulus represented as follows:

$$\frac{1}{E_s}=\frac{1}{E}+\frac{sh^2}{G\left(\frac{3}{2}l^2-2x^2\right)}. \tag{5}$$

When the deflection is measured at the loading point ($x=l/2$), Eqs. (4) and (5) are represented as:

$$y=\frac{Pl^3}{48E_sI}, \tag{6}$$

and

$$\frac{1}{E_s}=\frac{1}{E}+\frac{s}{G}\left(\frac{h}{l}\right)^2. \tag{7}$$

Equation (7) is well-known as the conventional "Timoshenko's bending equation".

3. Experiment

3.1 Specimens

Akamatsu (Japanese red pine, *Pinus densiflora* D. Don) and balsa (*Ochroma lagopus* Sw.) were used for the specimens. Specimens were conditioned at 20°C and 65% relative humidity (RH) before and during the tests.

3.2 FEM simulation of static bending

Static bending tests were simulated by the finite element method (FEM) to determine the proper points for measuring the deflection.

The program used was "MSC/NASTRAN Ver. 67", which is a library program of the Computer Center of The University of Tokyo. Figure 2 shows the finite element mesh and the boundary conditions we used. The finite elements were divided by the dimensions of 1 mm × 1 mm. The Young's moduli in the longitudinal direction E_L and the shear moduli G_{LT} were given

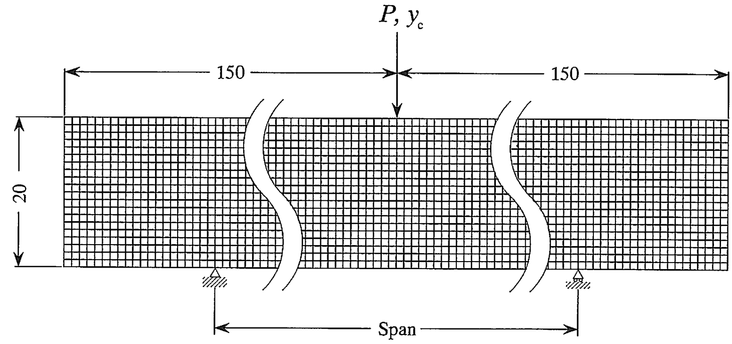

Fig. 2. Finite element mesh used for the calculations (unit: mm).
Note: Meshes are uniformly divided to the dimensions of 1 mm × 1 mm.
Span = 60 to 260 mm at the interval of 50 mm.

by the flexural vibration testing data. The Young's moduli in the tangential direction E_T and the Poisson's ratios ν_{LT} used in the calculations were given from a citation; $E_T=0.65$ GPa for akamatsu and 0.06 GPa for balsa; $\nu_{LT}=0.60$ for akamatsu and 0.23 for balsa.[3] Supported points were variously changed as the same manner of the static bending tests. A fixed displacement (0.5 mm $=y_c$) was given at the top of the center of beam, and the nodal force at the displaced point, P, was obtained. By substituting the nodal displacement and P into Eq. (1), the displacement caused by the bending deflection, y_b was calculated. This y_b is subtracted from the nodal displacement y, and the additional deflection was obtained. On the other hand, the additional deflection was calculated by substituting P into Eq. (2). The additional deflections obtained by the different procedures were compared for each nodal point, and the proper point for applying Timosheneko's bending theory was determined.

3.3 Static bending tests

Beam specimens were cut with the dimensions of 500 mm (longitudinal direction) × 30 mm (radial direction) × 10, 20, and 30 mm (tangential direction). Six specimens were used for one test condition.

Specimen was supported by the spans varied from 130 to 480 mm at the interval of 50 mm, and the vertical load whose velocity was 5 mm/min was applied to the center of the longitudinal-radial (LR) surface with a loading head whose radius was 15 mm. The deflections were measured at the loading point and the opposite side of the loading point which was determined by the FEM calculations. The determined procedure is mentioned after. The deflection at the loading point was obtained from the moving distance of the cross head, whereas that at the opposite side of the loading point was measured by a dial gage set below the specimen. The load/deflections relations were recorded by a X-Y recorder.

From the linear segment of load/deflection diagrams, the apparent Young's moduli corresponding to the measured points of deflections were obtained. The values of E and G/s were separated each other from the $1/E_s - h/l$ relationships by the method of least squares.

3.4 Flexural vibration tests

For examining the Young's moduli and shear moduli obtained by our proposal, these moduli were independently measured by the free-free flexural vibration test.

The test beam was suspended by two threads at the nodal positions of the free-free vibration corresponding to its resonance mode. Specimen was excited in the direction of the thickness at one end by a hammer. The resonance frequencies whose mode was from 1st to 4th were measured by the FFT (fast frequency transform) digital signal analyzer, and the Young's modulus E^v and shear modulus G^v were obtained from the Timoshenko-Goens-Hearmon method whose detail was described in a previous paper.[4)]

4. Results and discussion

4.1 Flexural vibration tests

Table 1 shows the Young's moduli and shear moduli obtained from the vibration tests. As described in several previous works, these values were precise enough. Thus, we examined the validity of the static bending tests by comparing with those obtained by the vibration tests.

4.2 FEM calculations

Figure 3 shows the comparisons of the additional deflections calculated by the different procedures mentioned above. When s equals 1.2, the predicted additional deflection is always larger than that calculated by Timoshenko's bending theory wherever the deflection is measured. On the contrary, these additional deflections rather agree with each other in the small depthh/span ratio range when $s=1.5$. In measuring the shear modulus properly, however, the large depth/span ratio range where the effect of shearing force is remarkable is important. From this viewpoint, it is difficult to use the Timoshenko's equation when the deflection is measured at the top of the specimen the large discrepancy exists between the additional deflections from the different procedures. On the contrary, these additional deflections agree with each

Table 1 Young's and shear moduli obtained by the vibration tests and the static bending tests based on Timoshenko's bending theory (unitl: GPa).

Species	Vibration test		Loading point		Oppsite of loading point	
	$E_L{}^v$	$G_{LT}{}^v$	$E_L{}^L$	$G_{LT}{}^L$	$E_L{}^B$	$G_{LT}{}^B$
Akamatsu	16.5	1.30	18.8	0.29	16.0	0.81
Balsa	3.18	0.20	2.90	0.05	2.71	0.08

Notes: E and G represent the Young's modulus and shear modulus, respectively. Subscripts L and T represent the longitudinal and tangential directions, respectively. Superscript v represents the vibration tests, and L and B express the measured points of deflection at the static bending tests, loading point and the opposite point against the loading point, respectively.

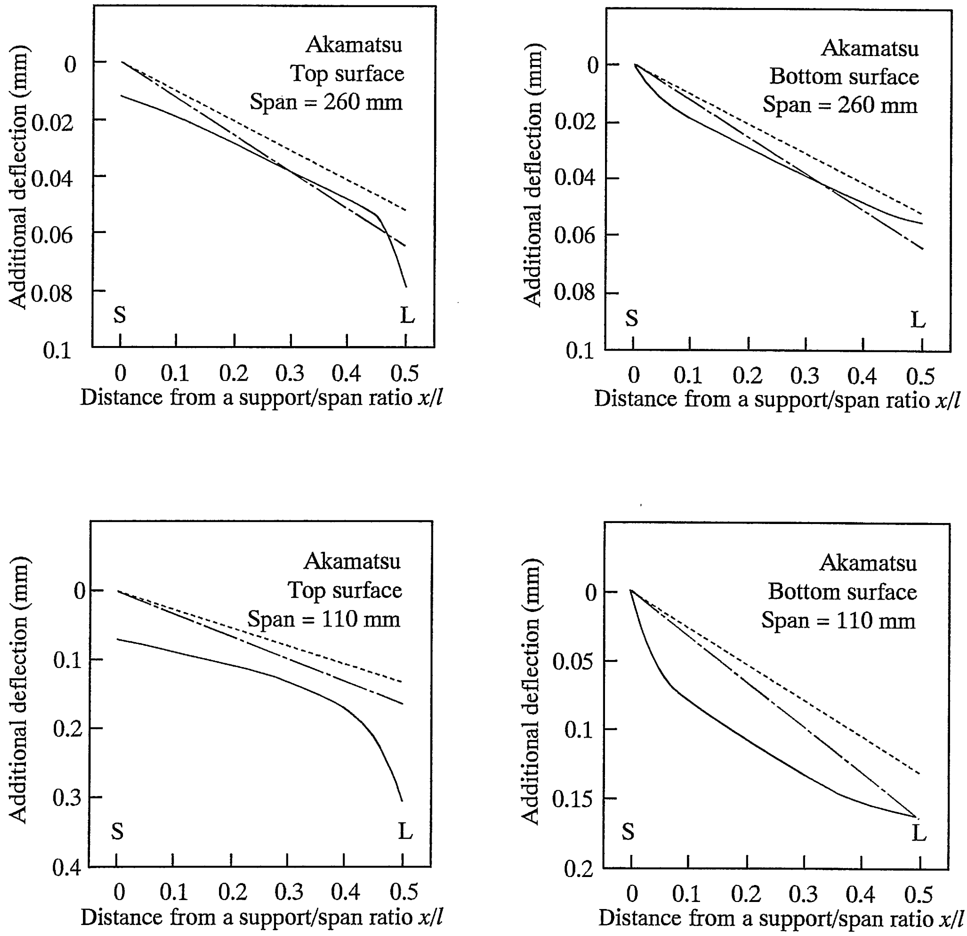

Fig. 3. Additional deflection corresponding to the distance from a support/beam length.
Notes: S and L represent a supporing point and loading point, respectively.
Legend: Solid line: FEM calculation, Dashed line: Timoshenko's bending theory ($s=1.2$), Semi solid line: Timoshenko's bending theory ($s=1.5$).

other when the deflection is calculated at the opposite side against the loading point in the large depth/span ratio range. This tendency is applicable for every calculation results.

Thus, we thought that Timoshenko's bending theory would be applicable when the Timoshenko's shear factor equals 1.5 and the deflection is measured at the point opposite against the loading point.

4.3 Static bending tests

Table 1 also shows the Young's and shear moduli obtained by substituting 1.5 into s of

Timoshenko's bending theory. As pointed out in the previous paper, the additional deflection is remarkably large at the loading point, and the shear modulus derived from the load/deflection at the loading point data is evaluated very small. From the FEM calculation results, we expected the applicability of Timoshenko's bending theory on the data given by the load/deflection at the opposite side against the loading point. However, the calculated shear modulus is about half of that obtained from the vibration tests. Although the additional deflection caused by the penetration of loading nose is reduced effectively by measuring the deflection at the bottom, the stress distortion that cannot be predicted by Timoshenko's theory have an influence on the deflection at the point far from the loading nose. Thus, we think that it is difficult to use Timoshenko's bending theory wherever the deflection is measured, and the modification similar to that done in the previous work should be made.

4.4 Modification of Timoshenko's theory

In our previous paper, we modified Timoshenko's theory as follows:

$$\frac{1}{E_s}=\frac{1}{E}+\frac{s'}{G}\cdot\left(\frac{h}{l}\right)^2, \tag{8}$$

where

$$s'=1.2+\alpha\frac{h}{l}. \tag{9}$$

When the deflection is measured at the loading point, we derived a constant value of 35 to α. From the data obtained here, however, the value of α should be smaller than 35 when the deflection is measured at the opposite side against the loading point. Table 2 shows the Young's and shear moduli calculated by substituting 5 into α of Eqs. (8) and (9) when the deflection was measured at the opposite point. At first, we expected that the value of α was determined by the regression procedure. When α was not fixed, however, the shear modulus and α converged to the anomalous values.

Table 2 Young's and shear moduli calculated by the modified equations (unit: GPa).

Measured point	Loading point		Opposite of loading point	
Species	E_L^L	G_{LT}^L	E_L^B	G_{LT}^B
Akamatsu	15.3	1.25	15.3	1.24
Balsa	2.70	0.17	2.50	0.17

Notel: E and G: See Table 1. Subscripts L and T, and superscripts L and B: Same as in Table 1.

5. Conclusion

We examined the feasibility of Timoshenko's bending theory on the measurement of Young's and shear moduli of wood.

From the results, we considered that it is difficult to evaluate the shear modulus by Timoshenko's theory even when the measured point was varied, and some modification should be made for obtaining the proper shear modulus by bending tests.

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