Mem. Fac. Sci. Eng. Shimane Univ.
Series A
32, pp. 65-69 (1998)

A Resort for Convergence about Spinor-Spinor Systems with Short-Range Interactions

Taketoshi INO

Department of Material Science, Shimane University, Matsue

(Received September 18, 1998)

Abstract

The one-gluon-exchange interaction includes a contact term, which includes the spin-spin interaction. Such a contact interaction leads to divergent integrals. About combinations of the Fermi-type interactions including the spin-spin interaction, we present a new resort for convergence. In addition to the non-local form factor with 4 end points introduced by Kristensen and Møller, a factor for convergence is introduced. Under a specific combination of the Fermi-type interactions, the wave function is able to be normalized and the eigenvalue is able to be obtained even in the case where the cut-off momentum is taken to be infinity, as the factor for convergence works.

§1. Introduction

The one-gluon-exchange interaction (by gluon with mass approaching to zero at very small separtion between q and q (or $\bar{q}$)) includes a contact term, which includes the spin-spin interaction. Such a contact interaction leads to the difficulty of divergent integrals.

In this paper, we present a resort for convergence about combinations of the Fermi-type interactions including the spin-spin interaction, by modifying the work by Katsumori[1] where a non-local form factor with 4 end points proposed by Kristensen and Møller[2] is introduced to the Bethe-Salpeter (BS) equation in the ladder approximation. We introduce newly a factor for convergence. We point out an advantage brought about by introduing the factor for convergence.

§2. The ladder BS equation with Fermi-type interactions and a resort for convergence

We propose the ladder BS equation for the system of two spin-1/2 particles a and b with combinations of the Fermi-type interactions and a resort for convergence

$$\psi(x_1, x_2) = ig\int S_F{}^a(x_1, x_3) S_F{}^b(x_2, x_4) \Lambda F(x_3, x_4, x_5, x_6) f(x_5, x_6) \times \psi(x_5, x_6) d^4x_3 d^4x_4 d^4x_5 d^4x_6, \quad (1)$$

where ψ is an amplitude, $S_F{}^a$ and $S_F{}^b$ are the Feynman propagators of a and b, F is the non-local form factor with 4 end points, f is a factor for convergence introduced in this work, and $ig\Lambda$

with a coupling constant g and a set of the Dirac matrices Λ comes from one of combinations of the Fermi-type interactions.

The non-local form factor with 4 end points is defined as

$$F(x_3, x_4, x_5, x_6)=\frac{1}{(2\pi)^{16}}\int F(p_3, p_4, p_5, p_6)\, e^{-i(p_3x_3+p_4x_4+p_5x_5+p_6x_6)}d^4p_3d^4p_4d^4p_5d^4p_6 \tag{2}$$

where $px=p^0x^0-p^1x^1-p^2x^2-p^3x^3$, and the condition

$$p_3+p_4+p_5+p_6=0 \tag{3}$$

is imposed on it. Because of this condition, it takes a form

$$F(x_3, x_4, x_5, x_6)=\frac{1}{(2\pi)^{12}}\int G(P', p', p'')\, e^{-i(P'(X'-X'')+p'x'+p''x'')}d^4P'd^4p'd^4p'' \tag{4}$$

with

$$X'=\frac{m_ax_3+m_bx_4}{m_a+m_b},\quad x''=\frac{m_ax_5+m_bx_6}{m_a+m_b},\quad x'=x_3-x_4,\quad x''=x_5-x_6, \tag{5a}$$

$$P'=p_3+p_4=-(p_5+p_6),\quad p'=\frac{m_bp_3-m_ap_4}{m_a+m_b},\quad p''=\frac{m_bp_5-m_ap_6}{m_a+m_b}, \tag{5b}$$

and $G(P', p', p'')$ in the expression (4) is defied as

$$\begin{aligned}G(P',p',p'')&=G(\{\Pi(P',p')\}^2, \{\Pi(P',p'')\}^2)_K\\&=G(\{\Pi(P',p')\}^2)_K G(\{\Pi(P',p'')\}^2)_K\end{aligned} \tag{6}$$

with

$$G(\{\Pi(P,p)\}^2)_K=\begin{cases}1 & \text{for } \{\Pi(P,p)\}^2\leqq K^2\\ 0 & \text{for } \{\Pi(P,p)\}^2>K^2,\end{cases} \tag{7}$$

where

$$\{\Pi(P,p)\}^2=-p^2+(Pp)^2/P^2 \quad (p^2=(p^0)^2-\boldsymbol{p}^2,\ P^2=(P^0)^2-\boldsymbol{P}^2), \tag{8}$$

and K is the cut-off momentum.[1] It is noted that in the rest frame of the bound state, $\{\Pi(P,p))^2$ reduces to $\mathbf{p}^2$, that is,

$$\{\Pi(P,p)\}^2|_{\boldsymbol{P}=0}=\boldsymbol{p}^2. \tag{9}$$

(It is also noted that $M\equiv(P^2)^{1/2}(=\{(P^0)^2-\mathbf{P}^2)^{1/2})$ is the rest mass of the bound state.)

The factor for convergence $f(x_5, x_6)$ introduced in this work is defined as

$$f(x_5, x_6)=f(x_5-x_6)=\frac{1}{(2\pi)^4}\int e^{-ik(x_5-x_6)}f(k^2)d^4k \tag{10}$$

with

$$f(k^2)=e^{-c^2(k^2)^2} \quad (c>0), \tag{11}$$

where c is a constant.

Introducing

$$X=\frac{m_a x_1+m_b x_2}{m_a+m_b}, \quad x=x_1-x_2, \quad P=p_1+p_2, \quad p=\frac{m_b p_1-m_a p_2}{m_a+m_b}, \tag{12}$$

and inserting the expressions (4) and (10), the Feynman propagator

$$S_F(x)=(i\not\partial+m)\Delta_F(x), \quad i\Delta_F(x)=\frac{1}{(2\pi)^4}\int\frac{e^{-ekx}}{-k^2+m^2-i\delta}d^4k, \tag{13}$$

and

$$\psi(x_1,x_2)=\psi(x)\,e^{-iPX}\equiv e^{-iPX}\int e^{-ipx}\phi_P(p)\,d^4p/(2\pi)^{3/2}$$

and

$$\psi(x_5,x_6)\equiv e^{-iPX''}\int e^{-ip''x''}\phi_P(p'')\,d^4p''/(2\pi)^{3/2}$$

into Eq. (1), we have

$$\begin{aligned}\phi_P(p)=&-\frac{ig}{(2\pi)^8}G(\{\Pi(P,p)\}^2)_K\frac{\not p_1+m_a}{p_1^2-m_a^2+i\delta}\frac{\not p_2+m_b}{p_2^2-m_b^2+i\delta}\\&\times\left[\iint G(\{\Pi(P,k')\}^2)_K f((k-k')^2)\Lambda\phi_P(k)\,d^4k\,d^4k'\right],\end{aligned} \tag{14}$$

where

$$p_1=\frac{m_aP}{m_a+m_b}+p, \quad p_2=\frac{m_bP}{m_a+m_b}-p. \tag{15}$$

Here the relations $G(\{\Pi(P,-k)\}^2)_K=G(\{\Pi(P,k)\}^2)_K$ and $f((-k)^2)=f(k^2)$ are taken into account.

For the Dirac matrices we use $\gamma^0=\gamma_0=\beta$, $\gamma^k=\beta\alpha^k=-\gamma_k (k=1, 2, 3)$ with

$$\beta=\begin{pmatrix} I & 0\\ 0 & -I\end{pmatrix}, \quad \alpha^k=\begin{pmatrix} 0 & \sigma^k\\ \sigma^k & 0\end{pmatrix}$$

and $\gamma_5(\equiv\gamma^5)\equiv i\gamma^0\gamma^1\gamma^2\gamma^3$, where I and σ^k are the 2×2 unit and Pauli matrices respectively.

The eigenvalue and eigenfunction of Eq. (14) with a $g\Lambda$ are obtained from

$$\int G(\{\Pi(P,p')\}^2)_K f((p-p')^2)\phi_P(p)d^4pd^4p'$$
$$=-\frac{ig}{(2\pi)^8}\int G(\{\Pi(P,p)\}^2)_K G(\{\Pi(P,p')\}^2)_K f((p-p')^2)$$
$$\times\frac{\not p_1+m_a}{p_1^2-m_a^2+i\delta}\frac{\not p_2+m_b}{p_2^2-m_b^2+i\delta}d^4pd^4p'$$
$$\times\left[\int G(\{\Pi(P,k')\}^2)_K f((k-k')^2)\Lambda\phi_P(k)d^4kd^4k'\right]. \tag{16}$$

The ortho-normalization condition for each of bound-state solutions is

$$\int_{-\infty}^{\infty}d\boldsymbol{p}\int_{-\infty}^{\infty}dX\left\{\int_{-\infty}^{\infty}\phi_{P'}^{+}(p)dp^0\right\}\left\{\int_{-\infty}^{\infty}\phi_P(p)dp^0\right\}e^{-i(P'-P)X}=(2\pi)^3(P^0/M)\delta^3(\boldsymbol{P}'-\boldsymbol{P}). \tag{17}$$

For the system of a spin-1/2 particle and a spin-1/2 anti-particle with combinations of the Fermi-type interactions and a resort for convergence, we have the equations which correspond to Eqs. (14), (16) and (17).

We assume

$$ig_I\Lambda_I=\frac{ig_I}{4}(1-\gamma_5{}^a\gamma_5{}^b-\gamma_\mu{}^a\gamma^{\mu b}/2+\gamma_5{}^a\gamma_\mu{}^a\gamma_5{}^b\gamma^{\mu b}/2)$$
$$=\frac{ig_I}{4}(1-\gamma_5{}^a\gamma_5{}^b)(1-\gamma_\mu{}^a\gamma^{\mu b}/2) \tag{18a}$$

or

$$ig_{II}\Lambda_{II}=\frac{ig_{II}}{16}(1+\gamma_5{}^a\gamma_5{}^b+\gamma_\mu{}^a\gamma^{\mu b}+\gamma_5{}^a\gamma_\mu{}^a\gamma_5{}^b\gamma^{\mu b}-\sigma_{\mu\nu}{}^a\sigma^{\mu\nu b}/2) \tag{18b}$$

for $ig\Lambda$ in the system of two spin-1/2 particles, and

$$ig_I'\Lambda_I'=-\frac{ig_I'}{4}(1-\gamma_5{}^a\gamma_5{}^b+\gamma_\mu{}^b\gamma^{\mu b}/2-\gamma_5{}^a\gamma_\mu{}^a\gamma_5{}^b\gamma^{\mu b}/2)$$
$$=-\frac{ig_I'}{4}(1-\gamma_5{}^a\gamma_5{}^b)(1+\gamma_\mu{}^a\gamma^{\mu b}/2) \tag{18c}$$

or

$$ig_{II}'\Lambda_{II}'=-\frac{ig_{II}'}{16}(1+\gamma_5{}^a\gamma_5{}^b-\gamma_\mu{}^a\gamma^{\mu b}-\gamma_5{}^a\gamma_\mu{}^a\gamma_5{}^b\gamma^{\mu b}-\sigma_{\mu\nu}{}^a\sigma^{\mu\nu b}/2) \tag{18d}$$

for $ig'\Lambda'$ in the system of a spin-1/2 particle and a spin-1/2 anti-particle.

Under $ig_I\Lambda_I$ or $ig_{II}\Lambda_{II}$, one has a $J^P=0^+$ particle-particle bound-state solution respectively, and under $ig_I'\Lambda_I'$ or $ig_{II}'\Lambda_{II}'$, one has a $J^P=0^-$ particle-anti-particle bound-state solution respectively.

It turns out that the bound-state solutions under $ig_I\Lambda_I$ and $ig_I'\Lambda_I'$ are able to be normalized and have the eigenvalues even in the case where the cut-off momentum is taken to be infinity, because the factor for convergence works.

The explicit expressions for the bound-state solutions under $ig_I\Lambda_I$ and $ig_I'\Lambda_I'$ are given elsewhere, together with a method where the wave function and eigenvalue under the interaction motivated by QCD (composed of a confinin potential and a one-gluon-exchange interaction) are found compatibly wit the present study of the short-range interactions including the spin-spin interaction.

References

1) H. Katsumori, Prog. Theor. Phys. **11** (1954), 505.
2) P. Kristensen and C. Møoller, Dan. Mat. Fys. Medd., **27** (1952), no. 7.