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Measurement of the plural elastic moduli of wood by static bending test

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Abstract

In this study, static bending tests were made with the specimens on which strain gages were bonded, and the plural elastic constants; Young's modulus, shear modulus, and Poisson's ratio, were obtained.

Material used for the experiment was yellow popular (*Liriodendron tulipfera* L.). Triaxial strain gages were bonded at the centers of the longitudinal-radial (LR) planes of the specimen, and three-point static bending tests were made. The Young's modulus and the Poisson's ratio were obtained by loading on the LR-plane, whereas the shear modulus was obtained by loading on the longitudinal-tangential (LT) plane. These bending tests were made under the various span/depth ratios, and the influence of the span/depth ratio on the elastic constants was examined. On the other hand, the Young's modulus and the Poisson's ratio were measured by the uniaxial tension and compression tests, and the shear modulus was obtained by the torsion test independently. The obtained elastic constants were compared with each other, and the validity of the measurement method was examined.

The elastic constants obtained from the bending tests showed rather good agreement with those obtained from the other conventional methods. Therefore, it was concluded the elastic constants can be obtained effectively by the static bending used here.

1. INTRODUCTION

There are several methods for measuring the elastic constants of wood; Young's modulus, shear modulus, and Poisson's ratio. The Young's modulus and Poisson's ratio usually are measured by uniaxial tension or compression tests,[1] and the shear modulus by the uniaxial loading tests of the specimen with the grain inclination of 45 degrees or torsion test of a bar.[1-3]

In bending a bar-shaped specimen, combined stress condition occurs in the specimen, and the plural elastic constants mentioned above are thought to be measured by use of this combined stress condition easily. However there are few examples measuring the plural elastic constants of wood by means of the combined stress condition in bending. Here, the static bending test were examined whether it can be used for the measurement of the plural elastic constants of wood.

2. THEORIES

It is supposed that the beam whose depth and width are h and w, respectively, is subjected to three-point bending. The long axis of the specimen is defined as x-axis, whereas the depth and width directions are defined as y- and z-axes, respectively. The deflection curve of the neutral axis in the range of $x \leq l/2$ can be written as the following equation when the load P is imposed at the center of the beam with the span of l according to the simple bending theory:[4)]

$$y=\frac{Px}{E_x wh^3}\left(\frac{3l^2}{4}-x^2\right), \tag{1}$$

where E_x is the Young's modulus in the long axis. Thus, the curvature of neutral plane, $1/r$, is represented as follows:

$$\frac{1}{r}=-\frac{d^2y}{dx^2}=\frac{6P}{E_x wh^3}x. \tag{2}$$

Thus, the strain in the long axis at the distance of y from the neutral plane, ε_x, is:

$$\varepsilon_x=\frac{y}{r}=\frac{6P}{E_x wh^3}xy. \tag{3}$$

As pointed out by Uemura, the stress condition near the loading point is seriously distorted, and the strains should be measured in the region which is free from the stress concentration.[5)] In this paper, hence, the strains were measured in the midpoint between the loading point and a support ($x=l/4$), and the normal strain components at the tension- and compression-side surfaces at the point of $x=l/4$ are defined as ε_x^+ and ε_x^-, respectively. These ε_x^+ and ε_x^- are given by substituting $h/2$ and $-h/2$, respectively, into y of Eq. (3) as:

$$\begin{cases} \varepsilon_x^+=\dfrac{3Pl}{4E_x wh^2} \\ \varepsilon_x^-=-\dfrac{3Pl}{4E_x wh^2}. \end{cases} \tag{4}$$

Hence, the Young's moduli measured at the tension and compression surfaces, E_x^+ and E_x^-, are given by the following equation:

$$\begin{cases} E_x^+=\dfrac{3l}{4wh^2}\cdot\dfrac{dP}{d\varepsilon_x^+} \\ E_x^-=-\dfrac{3l}{4wh^2}\cdot\dfrac{dP}{d\varepsilon_x^-}. \end{cases} \tag{5}$$

Of course, the Young's moduli E_x^+ and E_x^- should be theoretically equal to each other.

The Poisson's ratio can be obtained by measuring the strains in the direction parallel and perpendicular to the long axis. When the normal strain components in the direction perpendicular to the long axis are defined as ε_y^+ and ε_y^-, the Poisson's ratios at the top and bottom of the specimen, ν_{xy}^+ and ν_{xy}^-, can be given as the following equation:

$$\begin{cases} \nu_{xy}^+ = -\dfrac{d\varepsilon_y^+}{d\varepsilon_x^+} \\ \nu_{xy}^- = -\dfrac{d\varepsilon_y^-}{d\varepsilon_x^-}, \end{cases} \tag{6}$$

These Poisson's ratios should be also equal to each other.

The maximum shear stress at the zx-plane, $(\tau_{zx})_{max}$, occurs at the neutral plane of the beam, and is represented as follows:

$$(\tau_{zx})_{max} = \frac{3P}{4wh}. \tag{7}$$

The shear modulus in the zx-plane can be obtained by measuring the shear strain at the neutral axis. When the shear strain components in the zx-plane of the specimen are defined as γ_{xy}, the shear modulus in the zx-plane, G_{zx}, can be given by the following equation:

$$G_{zx} = \frac{d(\tau_{zx})_{max}}{d\gamma_{zx}} = \frac{3}{4wh} \cdot \frac{dP}{d\gamma_{zx}}. \tag{8}$$

3. EXPERIMENT

3.1 Specimens

Yellow popular (*Liriodendron tulipfera* L.) was used for the specimens. Specimens were conditioned at 20°C and 65% relative humidity before and during the tests.

3.2 Static bending tests

In this experiment, the Young's modulus in the longitudinal (L) direction, and the shear modulus and the Poisson's ratio on the longitudinal-radial (LR) plane were measured by three-point bending tests according to the following procedure.

Initially, the beam specimens were cut with the dimensions of 500 mm (longitudinal direction) × 30 mm (radial direction) × 20 mm (tangential direction). Triaxial strain gages (FRA–C1–11, Tokyo Sokki Co., Ltd.) were bonded at the centers of the LR-planes. The gage axes were determined as Fig. 1, and the strains in the longitudinal, radial, and 45-degree inclined directions were defined as ε_L, ε_R, and ε_{45}, respectively. The radius of loading nose used was 15 mm.

Firstly, the vertical load (P) whose velocity was 2 mm/min was applied to the center of the LR-surface as Fig. 2(a), and the Young's moduli E_L^+ and E_L^-, and the Poisson's ratio ν_{LR}^+ and ν_{LR}^- were obtained from the linear parts of P–ε_L and P–ε_R relationships. The side faces subjected by the tensile and compressive strains were defined as Face A and Face B, respectively. Then, the vertical load with the same velocity was applied to the center of the longitudinal-tangential (LT-) surface as Fig. 2(b), and the load-strains relationships were obtained.

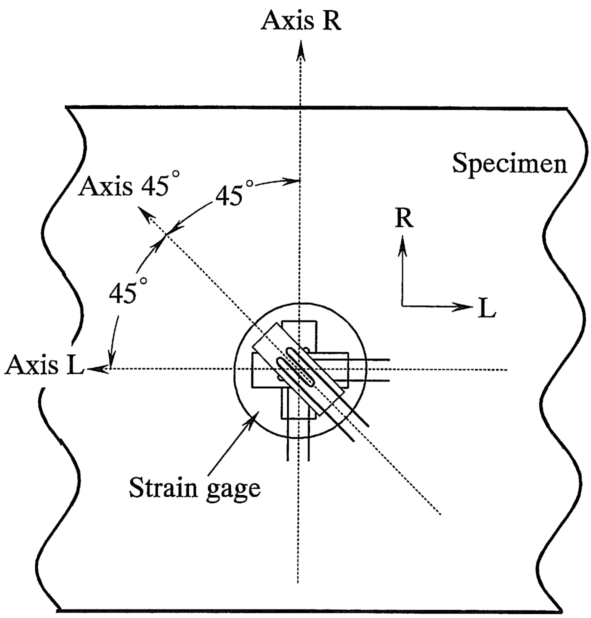

Fig. 1. Triaxial strain gage arrangement.
Notes: L, R represent the longitudinal and radial directions, respectively.

The shear strain γ_{LR} was calculated from the strain components as follows:

$$\gamma_{LR} = 2\varepsilon_{45} - \varepsilon_L - \varepsilon_R. \quad (9)$$

From the linear parts of the load-shear strain relationships, the shear moduli G_{LR} on both LR-planes (Face A and Face B) were obtained. After then, the specimen was cut and the bending test of a shorter span was made. As shown in Fig. 2, the specimen was settled so that the strain gages were located at the midpoint between the loading nose and a support. The spans varied from 480 to 180 mm at the interval of 100 mm, and the influence of span/depth ratio on the measurement of elastic constants was examined. Five specimens were used in the experiment.

3.3 Uniaxial tension, uniaxial compression, and torsion tests

After the bending tests, the specimens were cut for using the tension, compression, and torsion tests.

Tension and compression tests were made for the measurements of Young's moduli and the Poisson's ratios. The dimensions of the tension-testing specimen were 100(L) × 10(R) × 2(T) mm, whereas those of the compression-testing specimen were 40(L) × 20(R) × 20(T). Biaxial strain gages (FCA–C1–11, Tokyo Sokki Co., Ltd.) were bonded on the LR-planes. The load was applied along the longitudinal axis of the specimen at the loading velocity of 1 mm/min, and the Young's modulus and Poisson's ratio were obtained. Here, the Young's modulus

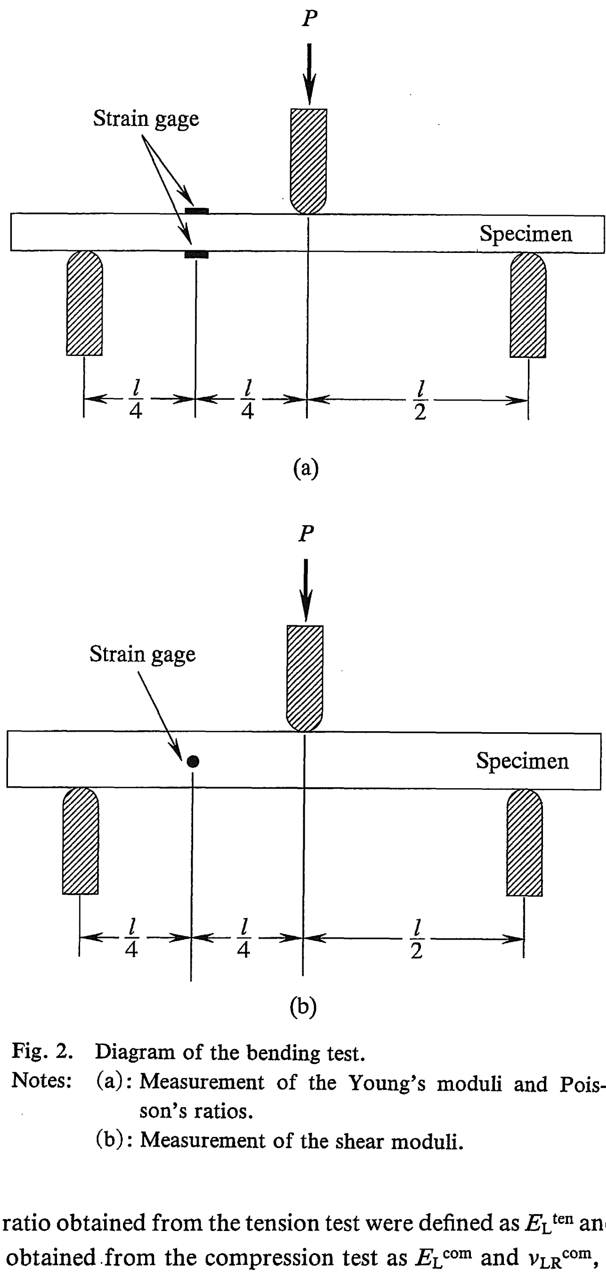

Fig. 2. Diagram of the bending test.
Notes: (a): Measurement of the Young's moduli and Poisson's ratios.
(b): Measurement of the shear moduli.

and the Poisson's ratio obtained from the tension test were defined as E_L^{ten} and ν_{LR}^{ten}, respectively, whereas those obtained from the compression test as E_L^{com} and ν_{LR}^{com}, respectively.

Torsion tests were made for measuring the shear moduli. The dimensions of the torsion-testing specimen were 230(L) × 30(R) × 12(T) mm. Biaxial strain gages (same ones used in the uniaxial-compression tests) were bonded on the centers of the LR- and LT-planes to measure the shear strains. This specimen was twisted by a manual torsion test equipment, and the torsional moment (M)/shear strains (γ_{LR} and γ_{LT}) relationships were obtained. The shear moduli on the LR- and LT-planes were calculated by the following equation:

$$\begin{cases} G_{LR}=\dfrac{k_{LR}}{a^2bk}\left[-2\left(\dfrac{2}{\pi}\right)^2\sqrt{\dfrac{G_{LR}}{G_{LT}}}\displaystyle\sum_{n=1}^{\infty}\dfrac{(-1)^{n-1}}{(2n-1)^2}\tanh\dfrac{(2n-1)\pi b}{2a}\sqrt{\dfrac{G_{LR}}{G_{LT}}}\right] \\ G_{LT}=\dfrac{k_{LT}}{a^2bk}\left[1-2\left(\dfrac{2}{\pi}\right)^2\displaystyle\sum_{n=1}^{\infty}\dfrac{1}{(2n-1)^2}\left\{\cosh\dfrac{(2n-1)\pi b}{2a}\sqrt{\dfrac{G_{LR}}{G_{LT}}}\right\}^{-1}\right], \end{cases} \tag{10}$$

where a and b are the lengths in the R- and T-directions, respectively, k_{LR} and k_{LT} are the initial inclination of M–γ_{LR} and M–γ_{LT} relationships, and k is written as follows:

$$k=\frac{1}{3}-\frac{2a}{b}\sqrt{\frac{G_{LT}}{G_{LR}}}\left(\frac{2}{\pi}\right)^5\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}\tanh\frac{(2n-1)\pi b}{2a}\sqrt{\frac{G_{LR}}{G_{LT}}}. \tag{11}$$

The shear modulus obtained from the torsion test was defined as $G_{LR}{}^{tor}$, and the values of $G_{LR}{}^{tor}$ were compared with those given by the bending tests, G_{LR}.

4. RESULTS AND DISCUSSION

Table 1 shows the means and the standard deviations of the elastic constants obtained from the uniaxial tension, uniaxial compression, and torsion tests, whereas the results of static bending tests are shown in Fig. 3. The Young's moduli and the Poisson's ratios obtained by the bending tests were closer to those obtained by the tension tests than those by the compression tests. These discrepancies are because of the restriction effect at the ends of the compression test specimen. In compression tests, we usually use the short column specimen which does not buckle during the test. In the compression of short column, however, the restriction caused by the frictional forces at the ends of the specimen distorts the uniform stress condition. The tension test is free from the frictional force when the distance between the grips is long enough. Also, the effect of the frictional force is small enough in the bending tests. On the contrary, the shear moduli obtained by the bending tests were larger than those by the torsion tests. This phenomenon cannot be described well here.

Fig. 4 shows the Young's moduli and the Poisson's ratios independently measured on the tension and compression sides of the specimens. Before the tests, it was presumed that the

Table 1 Elastic moduli obtained by the tension, compression, and torsion tests.

	E_L^{ten}	E_L^{com}	ν_{LR}^{ten}	ν_{LR}^{com}	G_{LR}^{tor}
Average	14.4	12.1	0.40	0.46	0.93
S.D.	0.4	1.2	0.02	0.03	0.09

Units: Young's modulus and sheat modulus=GPa.
Note: S.D.=Standard deviation.
Suffixes ten, com, and tor represent the uniaxial tension, uniaxial compression and torsion tests, respectively, and suffixes L and R represent the longitudinal and radial directions, respectively.

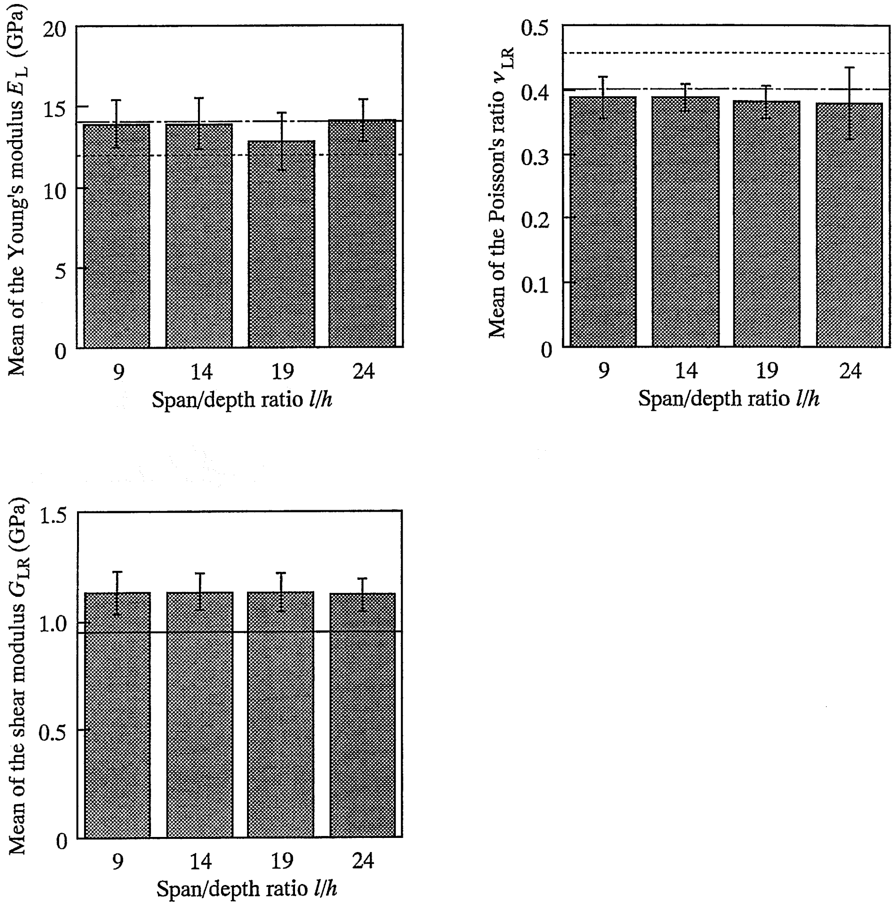

Fig. 3. Means of the elastic moduli corresponding to the span/depth ratios obtained by the static bending tests.

Legend: Semi solid, dashed, and solid lines represent the means of corresponding constants obtained by tension, compression, and torsion tests, respectively. Horizontal bar: Standard deviation.

Young's modulus and the Poisson's ratio would depend on the span/depth ratio because the stress concentration around the loading point would has different influences on the specimens with the different span/depth ratios. However, there was not remarkable dependence on the span/depth ratio examined here. As for the elastic moduli obtained from the tension and compression sides, there were small differences between the Young's moduli, whereas the Poisson's ratios on the tension sides were larger than those on the compression sides. The difference of the Poisson's ratio was caused by the transverse strain ε_R because of the small differences in the longitudinal strains ε_L. In these tests, however, the reason why the transverse strain components were differently measured in the tension and compression sides could not be found. Although there were differences of the Poisson's ratios between the tension and compression sides, the obtained moduli were appropriate enough. Therefore, the static bending test by bonding the

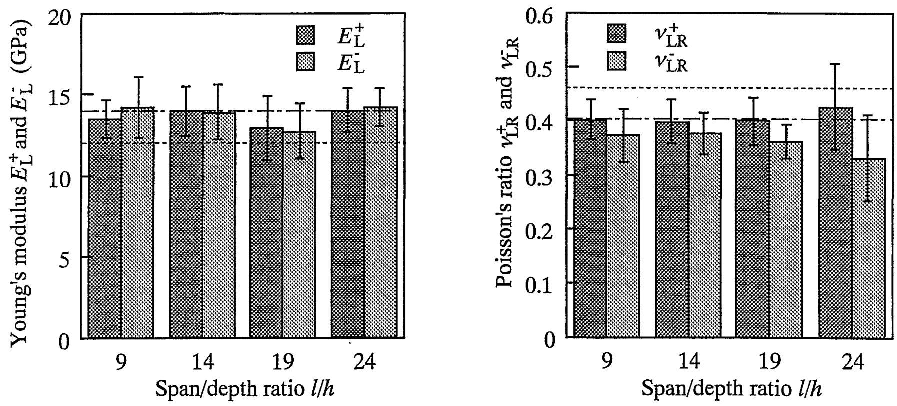

Fig. 4. Comparisons of the Young's moduli and the Poisson's ratios obtained from the tension and compression sides.
Legend: Semi solid and dashed lines: same as in Fig. 3.
Notes: Suffixes + and − represent the tension and compression, respectively.

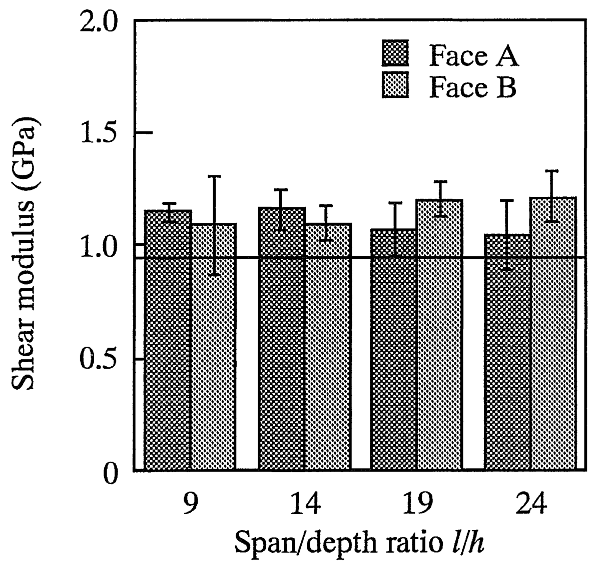

Fig. 5. Comparisons of the shear moduli obtained from the different side faces.
Legend: Solid line: Same as in Fig. 3.

strain gages is effective in measuring the Young's modulus and the Poisson's ratio.

Fig. 5 shows the comparisons of the shear moduli obtained from the different side faces. There were small differences between the shear moduli on the different faces. In a previous paper, it was suggested that the stress concentration around the loading point has a serious influence on the stress distribution in the specimen with a small span/depth ratio, and that this stress concentration should be taken into account carefully in obtaining the shear modulus from the load-deflection diagram of static bending.[7] As in this experiment, however, the load-shear strain relationship is free from the stress distortion around the loading point when the shear strain is measured at the point far enough from the loading point, and the shear modulus can be measured even when the specimen has a small span/depth ratio. Conventionally, the shear modulus were measured by the uniaxial-compression tests of 45 degree off-axis specimens or torsion tests of rectangular bars. Nevertheless, these methods have the following drawbacks. The uniaxial-compression test has a difficulty in cutting the test specimens. In the torsion tests, there is a complexity in separating the two shear moduli on the side planes from each other. In addition, it is difficult to make the torsion tests with a universal testing machine usually used for the tension, compression, and bending tests, and a special equipment should be needed. Comparing these conventional methods, the bending tests is much easier. As previously mentioned, the obtained moduli were tend to be larger than those given by the torsion tests. However, the difference between the moduli given by these methods was almost similar to that between the Young's moduli obtained from the bending tests and compression tests. Thus, the shear modulus obtained by the bending test is appropriate enough in using.

Of course, this method can be applicable only for the materials which is regarded homogenous, and the elastic constants of inhomogenous materials such as plywood and LVL should be measured by other methods.

5. CONCLUSION

The plural elastic constants of wood were measured by three-point static bending tests, and verified its validity when the strain components were measured at the midpoint between the loading point and a span.

The elastic constants obtained from the bending tests showed rather good agreement with those obtained from the other conventional methods. Therefore, it was concluded that the static bending used here is appropriate in measuring the plural elastic constants.

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