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地表水体に生じる輸送現象：数理，数値，応用の観点から

吉岡秀和

Transport phenomena in surface water bodies: from mathematical, numerical, application point of views

Hidekazu YOSHIOKA

Abstract Transport phenomena, such as fluid flows, solute dispersion, and migration of aquatic species, are ubiquitous in surface water bodies. These transport phenomena are inherently nonlinear and stochastic, and analyzing their dynamics requires using mathematically and physically sophisticated knowledge. Numerical analysis assisted by firm mathematics can serve as an effective tool for comprehending the transport phenomena; however, there exist only a limited number of such kinds of approaches in regional environment science and related research areas. This paper provides a brief summary of our previous researches on mathematical and numerical modelling of the transport phenomena for promoting the use of the modelling techniques. Their applicability to the problems specific to the Hii River system, Japan, is also discussed in this paper.

Keywords : transport phenomena, water bodies, fluid flows, solute dispersion, aquatic species

抄録 地表水体では，流体の流動，溶質の分散，水棲生物の移動などの輸送現象が生じる．こうした輸送現象は非線型的かつ確率論的であり，その予測や評価には数物的知見の活用が効果的である．とりわけ，数学的に裏付けられた数理モデルと数値計算手法の適用が威力を発揮する．しかしながら，地域環境科学や関連研究分野におけるそのような手法の普及率は著しく低い．本報は，著者や共同研究者らによる輸送現象に対する数理・数値的な接近手法を紹介する．また，それらの斐伊川水系における実問題への適用性を論じる．

Introduction

Transport phenomena, such as flows of water and mud, dispersion of water quality indices, and migration of fishes and planktons, are ubiquitous in surface water bodies. Transport phenomena in surface water bodies are inherently nonlinear and stochastic, and analyzing their dynamics can effectively reduce to solving appropriate differential equations based on continuum and discrete mechanics. Fluid flows in water bodies have been described with partial differential equations（PDEs）governing spatio-temporal dynamics of mass and momentum, such as the Navier-Stokes equations equipped with turbulent models（Liu and Jiang 2013; Sinha *et al.* 2012）and their hydrostatic counterparts, which are referred to as the shallow water equations（SWEs）（Delestre *et al.* 2013; Szymkiewicz 2010）. Transport phenomena of water quality indices, such as dissolved oxygen and heat, have effectively been handled with the advection-dispersion equations, which are the PDEs based on the mass conservation principles considering turbulent nature of the fluid flows (Chatwin and Allen 1985; Yoshioka *et al.* 2012). Transport phenomena of aquatic species have also been handled with the advection-dispersion equations; however, the dynamics to be considered is far more complicated than that of the passive solute because biological, ecological, and hydrodynamic factors would complexly intertwine (Goodwin *et al.* 2006; Kohannim and Iwasaki 2013).

Studying the transport phenomena requires using mathematically and physically sophisticated knowledge. Numerical analysis assisted by firm mathematics can serve as an effective tool for comprehending the transport phenomena; however, there exist only a limited number of such kinds of approaches in regional environment science and related research areas.

The purpose of this paper is to provide a brief summary of our previous researches on mathematical and numerical modelling of transport phenomena in surface water bodies for promoting the use of the modelling techniques. Their applicability to the problems in Hii River system, Japan is also discussed.

Mathematical models

Pivotal mathematical models serving as a foundation of analytically assessing transport phenomena in surface water bodies are introduced in this section. Due to limitations of pages, their derivation procedures and detailed mathematical expressions are not discussed in this paper, which are found in the published papers by the author and his coworkers provided in the reference list. Only their basic concepts and applicability of the mathematical models to real problems are provided in this section.

Fluid flows

The horizontally 2-D and the longitudinally 1-D SWEs are the most widely used mathematical models governing fluid flows in surface water bodies. The SWEs describe mass and momentum dynamics of incompressible and hydrostatic fluids. They are nonlinear PDEs whose analytical resolution is possible only under certain simplified conditions, such as the problems in straight open channels without friction (Szymkiewicz 2010).

Practical problems encountered in regional environmental science can often be reduced to solving the 1-D SWEs in open channel networks, such as river and canal networks having a number of junctions and bends. A key in reasonably simulating such water flows using the 1-D SWEs is mathematical and physical treatment of junctions and bends. Yoshioka *et al.* (2015b) presented a consistent mathematical formulation of the 1-D SWEs defined in open channel networks, which is an extension of the conventional counterparts that can be applicable only to the flows in single open channels. Their equation implicitly satisfies the mass conservation law at junctions and bends. Yoshioka *et al.* (2014a, 2014b) demonstrated that different physical assumptions on the momentum losses of the flows at junctions and bends lead to apparently different flow profiles, some of which significantly deviate from the experimental results (Ishida *et al.* 2011; Unami and Alam 2012).

There exist physically more sophisticated PDEs for describing fluid flows in surface water bodies, such as the Navier-Stokes equations (Sinha *et al.* 2012) and the Boussinesq equations (Madsen and Sørensen 1992), both of which are strongly nonlinear. These equations are free from the hydrostatic pressure assumption and can potentially more accurately capture dynamics of surface water flows; however, solving these equations are computationally far more demanding than solving the 1-D and 2-D SWEs, which would become practically possible in the near future.

Solute dispersion

The advection-dispersion equations have widely been used in practical analysis of solute dispersion (Cox 2003). Mathematical models used in the conventional researches are based mainly on some deterministic conservation laws of mass and the Fick's laws representing analogies to the gradient-type laws of molecular diffusion and heat condition; however, stochasticity inherent in the transport processes is not properly considered in most of such researches. On the other hand, some researchers including the author have been investigating transport problems by employing essentially different methods. Approaches that consider stochasticity contained in transport mechanisms using stochastic differential equations, referred to as the SDEs, have demonstrated their effectiveness. An SDE can be regarded as a time-dependent ordinary differential equation governing a stochastic process as explained in Øksendal (2000). The Kolmogorov's forward equation (KFE) and Kolmogorov's backward equation (KBE), which are the PDEs that associate to the SDEs, have also been shown to be useful mathematical tools for analyzing randomized phenomena in water environments as reviewed in Bodo *et al.* (1987) and Su (2004).

The authors recently found that the conventional Fick's laws are not necessary for the advection-dispersion phenomena in turbulent water bodies, which can alternatively be performed with a probabilistic mass conservation law and the linearity of the KFEs associated with the SDEs governing Lagrangian particle movements in the flows (Yoshioka *et al.* 2012; Yoshioka and Unami 2013). Another advantage of using the SDEs as the governing equations of the advection-dispersion phenomena is that the KBEs can quantify the statistical natures of the solute particle dynamics, which cannot be performed with the conventional deterministic models. Spatio-temporal statistical analyses on the transport phenomena of solute particles in vegetated open channels and freshwater lagoons have already been performed (Takagi *et al.* 2014; Yoshioka *et al.* 2014c; Yoshioka *et al.* 2015e).

Migration of aquatic species

Migration of aquatic species in surface water bodies are subject to inherent stochasticity due to the turbulence of the fluid flows and environmental and ecological disturbances that are often beyond our knowledge. A large number of researches discussed hydraulic and hydrological processes in surface water bodies; however, significantly smaller number of them focused on migrations of aquatic species due to difficulties to find their reasonable mathematical expressions.

A significant difference between the transport phenomena of solute particles and those of aquatic species are their drift mechanisms; the former passively move in the flows but the latter in general do not. The latter, in particular some migratory fishes, have been reported to adaptively swim in the flows based on a physiological energy consumption principle during their migration processes (Brodersen *et al.* 2008). One possible way to develop a mathematical model that can reasonably represent the adaptive swimming strategy considering hydraulic, biological, and ecological stochasticity is formulating the problem in the context of stochastic optimal control problem suing SDEs (Øksendal 2000). Yoshioka *et al.* (2015c-d) presented a dynamic energy minimization principle of migration of individual fishes based on SDEs. They derived a nonlinear PDE that governs optimal migration velocity of fishes, which is referred to as the Hamilton-Jacobi-Bellman equation (HJBE). Unfortunately, the HJBE is strongly nonlinear and does not have analytical solutions except for simplified cases.

Numerical methods

The governing equations of the transport phenomena, such as the SWEs, KFEs, KBEs, and HJBEs have to be numerically solved in real applications. This is because their coefficients, initial, terminal, and boundary conditions can be highly irregular and do not admit analytical expressions in general. We have developed numerical methods for accurately solving these governing equations in open channel network domains, which possibly possesses a number of junctions, bends, and loops. The developed numerical methods are based on appropriately defining the governing equations with local integrals, which allow handling the problems in the weak sense where the solutions may not be defined in the classical sense. Such numerical methods are broadly categorized into the two classes, which are referred to as the finite volume methods (FVMs) and the finite element methods (FEMs).

The FVMs are suitable for solving the PDEs in the conservative forms, such as the SWEs and the advection-dispersion equations, which describe some physical and mathematical conservation laws. The FVMs, which are referred to as the Dual-FVMs, have been presented and validated through test and real cases, demonstrating their sufficiently high accuracy (Yoshioka and Unami 2013; Yoshioka *et al.* 2015b).

On the other hand, the FEMs are effective for solving the PDEs in the non-conservative form, such as the KBEs and the HJBEs, which do not describe conservation laws. The FEMs, which are referred to as the Conforming Petrov-Galerkin FEMs, were developed and their accuracy and stability have been verified from both numerical and theoretical point of views (Yoshioka *et al.* 2013, 2014d). The theoretical analysis is based on the concept of discrete Green's function and functional analytic techniques (Miller *et al.* 2012), which turned out to be powerful mathematical tools for estimating the computational errors in particular. The horizontally 2-D numerical models, which can potentially be coupled with the 1-D counterparts, are being developed (Takagi *et al.* 2014; Yoshioka *et al.* 2014e).

The problems in the Hii River system

This section makes discussions on applicability of the above-presented mathematical models and numerical methods for specific problems of the Hii River system, Japan. The Hii River system contains the main stream of Hii River, its branches, and Lakes Shinji and Nakaumi, both of which are brackish lakes. This river system is suffering from serious environmental and ecological problems, which possibly are causing decrease of the total amount of fish catches of resident and migratory fishes, such as *Yamame* (*Tribolodon hakonensis*) and *Ayu* (*Plecoglossus altivelis*).

The mainstream of Hii River has a number of fishways, most of which associate with some weirs. **Figs. 1** and **2** show relatively larger fishways installed at the river, which are referred to as the Yoshii (Pool and nature-like combined type, repaired in 2013) and Hinobori fishways (Vertical slot type, completed in 1999), respectively. Heights of the Yoshii and Hinobori weirs are 3 (m) and 11 (m), respectively, indicating that the fishes cannot ascend up the weirs without using the fishways.

The previous Yoshii fishway consisted solely of a pool type fishway. The downstream part of this fishway suffered from severe depositions of fine soil particles, which might be due to its geometry that induced recirculating flows strongly trapping sediment particles. These depositions were considered to be a cause of degradation of fluid transport capacity of the fishway, which might have further lead to degradation of its attraction ability and ascending efficiency. The repair of the Yoshii fishway was intended to improve its ascending efficiency by additionally installing a nature-like type fishway. According to Mr. Yoshii of Hii-river fisheries cooperative, it is unclear whether the repair improved ascending efficiency of the Yoshii fishway for resident

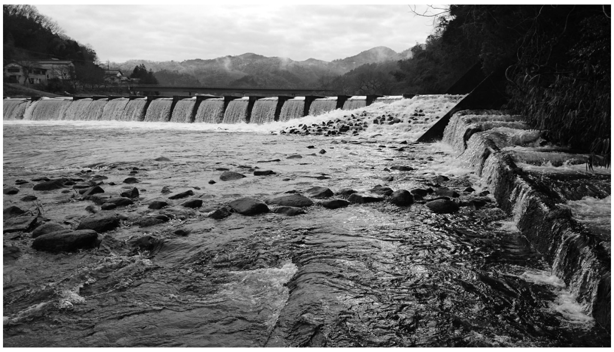

Figure 1 Yoshii fishway (taken from the downstream)

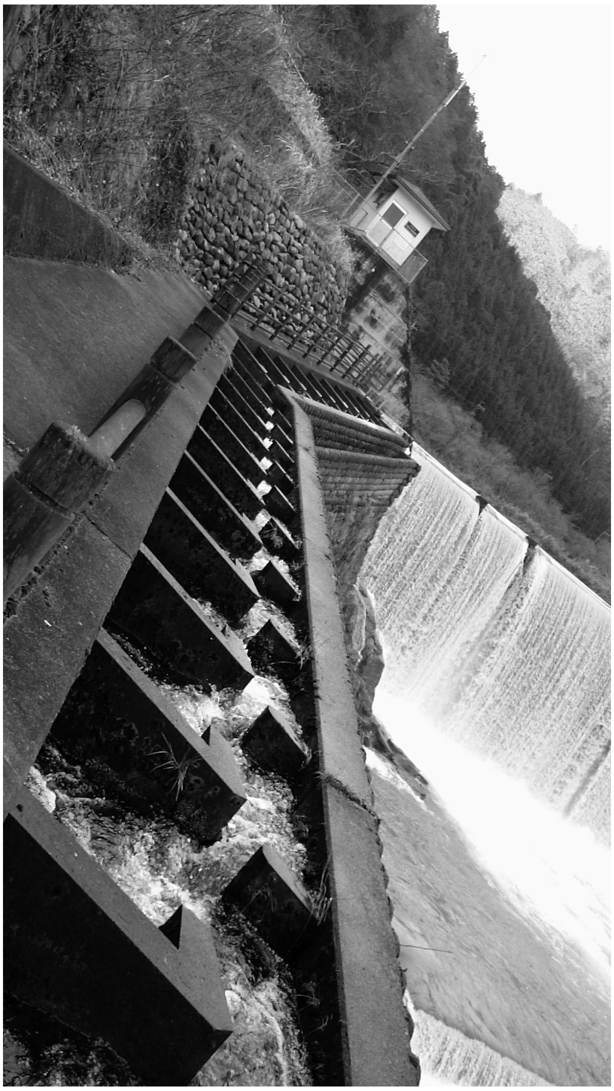

Figure 2 Hinobori fishway (taken from the downstream of a turning pool)

and migratory fishes in the river. Assessment of the ascending efficiency of Hinobori fishway, which has been considered to be effectively working as a passage through the weir, has also not been performed so far. One of the most urgent research topics to be addressed is therefore assessment of ascending efficiency of the two fishways, which would be potentially serving as physical barriers of fish migrations in Hii River.

The mathematical and numerical models introduced in the previous sections can be used as effective tools for assessment of the ascending efficiency of the fishways with multiple spatio-temporal scales. Local ascending behaviour of the fishways can be assessed using the 2-D models or their 3-D counterparts with appropriately imposing initial and boundary conditions. On the other hand, global ascending efficiency of the mainstream of the Hii River can be assessed utilizing the 1-D models. In these models, fishways and weirs are not explicitly considered but specified as implicit internal boundaries where values and gradients of hydraulic quantities and behaviour of fishes are parameterized.

Conclusions

Mathematical and numerical approaches previously developed by the author and his coworkers for analyzing transport phenomena in surface water bodies were briefly summarized and their applicability to the problems in the Hii River system, Japan was discussed.

The presented mathematical approach, which is based mainly on the PDEs and SDEs, is at a germinating stage and has to be validated through real applications more in detail. Many mathematical theories for rigorously analyzing properties of mathematical models, which are necessary for their effective operations, are available and will continue to progress for addressing unresolved problems. The mathematical theories on the weak solutions to PDEs, such as the theories on shock and rarefaction waves（Li and Wang 2009）and viscosity solutions（Fleming and Soner 2006）, would in particular serve as indispensable tools for analyzing the transport phenomena. The author is currently working with mathematical and numerical analyses on the HJBEs of fish migration. A part of the results has already been presented in Yoshioka *et al.*（2015a-b）.

The author hopes strongly that as many people as possible will be interested in and promote the presented research topics.

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