Sloshing Damping Control in a Cylindrical Container on a Wheeled Mobile Robot Using Dual-Swing Active-Vibration Reduction

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***Abstract*— This paper proposes a damping control of sloshing in a cylindrical container on a wheeled mobile robot. The container can be independently tilted in the running direction and the orthogonal direction by a dual swing-type active vibration reducer. The mobile robot runs along a curved path on a slope. Sloshing generated by the action of the mobile robot is damped using the vibration reducer. In addition, the vibration reducer can make the container level on the slope. The control system of the vibration reducer is an optimal servo controller with a Kalman filter. The usefulness of proposed damping control system is demonstrated through experiments.**

***Keywords*—Sloshing, Active vibration reducer, Wheeled mobile robot, Damping control**

I. INTRODUCTION

With the advancement of automation at various factories, liquid container transfers have become very important processes in production lines. For example, there are transfers of molten metals, transfers of molds after pouring as a part of material processing in steel and casting industries, or transfers of raw material solutions and mixed solutions in chemical plants without using pipes. In such processes, containers have generally no lid to speed up and simplify pouring works. In liquid container transfers, sloshing (liquid vibration) is generated by changes in the container's acceleration. Overflows and degradation of quality caused by sloshing in containers are problems that directly affect productivity. For instance, there is the degradation by the contamination of air and slag in transfers of molten metals. In addition, production processes are shifting to job shop type productions recently. It means that transfer paths are often changed to deal with the change of processes. Wheeled mobile robots (WMRs) are more useful than belt conveyer or rail systems in terms of the easiness on path changes.

Path planning and trace control for WMRs have been studied in [1], [2]. However, there are few studies on the liquid container transfer using WMR. Damping control of liquid container was studied by using a vehicle, a robot arm or a robot in [3]-[6]. We previously reported a damping path design for a cylindrical liquid container on WMR using an input shaping method in [7]. Reference [4]-[7] proposed methods to design damping paths, but these methods required to calculate the damping paths in advance. Therefore, it was difficult to change transfer paths immediately, and it was impossible to damp the sloshing generated by unexpected actions of avoiding obstacles. Carrying on a slope was not taken into consideration at all in [3] and [7].

This paper proposes a damping control of sloshing in a cylindrical container with a dual swing-type active vibration reducer on a WMR. The WMR runs along a straight and curved path on a horizontal plane or a slope. The control system of the active vibration reducer is an optimal servo controller with a Kalman filter. The usefulness of this method is demonstrated through experiments.

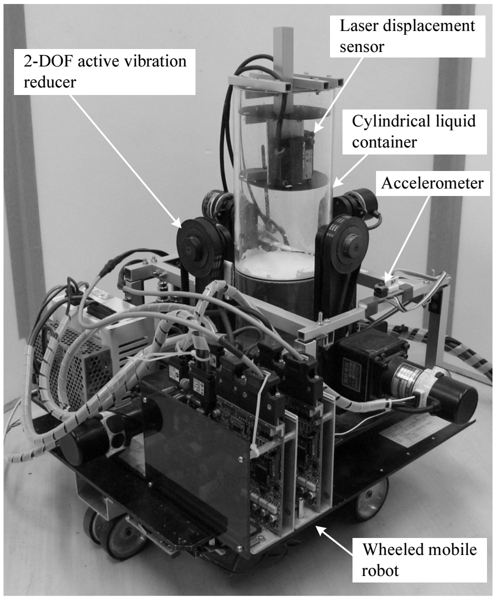

Fig. 1. Experimental equipment

II. EXPERIMENTAL EQUIPMENT

The liquid in the cylindrical container is transferred with the WMR as shown in Fig. 1. The WMR is the tricycle type which consists of two driving wheels and one steering wheel. These wheels are driven by DC motors. The rotational speed of the right and left driving wheels are measured by means of rotary encoders, and the steering angle is done by a potentiometer. The steering wheel is adjusted using a PID controller in order to work just as a caster. This steering system improves straightness and steering above those of common casters. The driving wheels are driven using a PID controller to achieve the reference speed.

The container is mounted on the dual swing-type active vibration reducer. The inner diameter of the container is 0.10 m, its height is 0.30 m. Water is chosen as the target liquid because of its simplicity in handling and low cost. Liquid level is 0.10 m. Two laser displacement sensors are used to observe the liquid level at the back measuring point and the right side one. The measuring points are at the point of 0.025 m from the center of the container. The water is colored with a blue paint and plastic powder is floated so that the laser sensors can easily detect the water surface.

The container can be independently tilted in the running direction and the orthogonal direction by the active vibration reducer. Two accelerometers are installed at both sides of the container. These accelerometers can measure the acceleration on three axis, X, Y and Z-axis. The running acceleration and the centripetal acceleration of the center of the liquid surface are calculated by means of outputs of the accelerometers on both sides. The running acceleration can be calculated by averaging both values of the accelerometers even if the WMR runs along a curved path. A low-pass filter is applied to the output of the accelerometers.

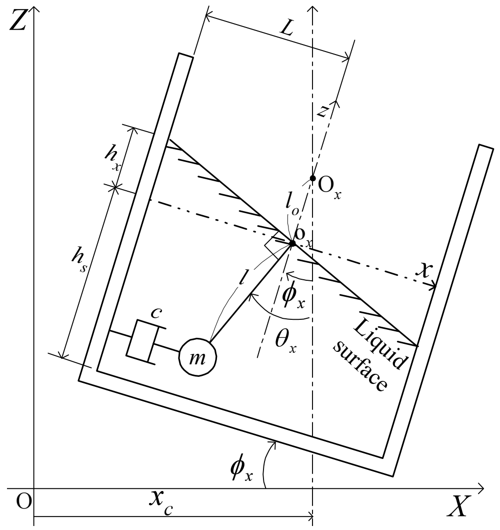

Fig. 2. Pendulum-type sloshing model in X-direction

III. SYSTEM EQUATION

The pendulum-type sloshing model in X-direction, the running direction, is shown in Fig. 2. This sloshing model can be obtained from the spherical pendulum-type sloshing model in [7], which approximately expresses (1, 1)-mode sloshing. The sloshing model in Y-direction, the orthogonal direction for the running direction, is similar to that in X-direction. The detail of the model in Y-direction is omitted here. These models are used to design a damping control system. When the angle of the spherical pendulum is small, the coupling effect between X-direction and Y-direction can be neglected. The linearized equation of the sloshing model in X-direction is obtained as follows:

$$\ddot{\theta}_x = -\frac{g}{l}\theta_x - \frac{c}{m}\dot{\theta}_x + \frac{c}{m}\dot{\phi}_x - \frac{l_o}{l}\ddot{\phi}_x + \frac{1}{l}\ddot{x}_c, \tag{1}$$

where θ_x is the angle of the pendulum in the Z-X plane, ϕ_x is the angle of the container in the Z-X plane, g is the gravitational acceleration, l is the equivalent length of the pendulum, l_o is the distance between the rotation axis of the container O_x and the center of the liquid surface o_x, m is the mass of the liquid, x_c is the position of the container in the X-axis, and c is the equivalent coefficients of viscosity for sloshing. The displacement of the liquid level h_x at the back measuring point is described as

$$h_x = L(\theta_x - \phi_x), \tag{2}$$

where L is the distance between the measuring point and the center of the container. The liquid surface is flat when both h_x and h_y are zero in (1, 1)-mode sloshing. In other words, the liquid surface has no sloshing as long as both h_x and h_y are kept to zero.

The transfer function $G_x(s)$ from the input voltage u_x of the DC motor to the angular velocity $\dot{\phi}_x$ of the container is a linear second-order system given as

$$G_x(s) = \frac{\dot{\phi}_x}{u_x} = \frac{k_x \omega_{nx}^2}{s^2 + 2\zeta_x \omega_{nx} s + \omega_{nx}^2}, \tag{3}$$

where ω_{nx} is the natural angular frequency, ζ_x is the damping ratio, and k_x is the gain.

The system equation in X-direction is described as follows:

$$\left.\begin{aligned} \dot{\boldsymbol{x}}_x &= \boldsymbol{A}_x \boldsymbol{x}_x + \boldsymbol{b}_x u_x + \boldsymbol{d}_x a_x, \\ \boldsymbol{y}_x &= \boldsymbol{C}_x \boldsymbol{x}_x, \end{aligned}\right\} \tag{4}$$

where

$$\left.\begin{aligned} \boldsymbol{A}_x &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -\frac{g}{l} & -\frac{c}{m} & 0 & \frac{c}{m} & -\frac{l_o}{l} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\omega_{nx}^2 & -2\zeta_x\omega_{nx} \end{bmatrix}, \\ \boldsymbol{b}_x &= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ k_x\omega_{nx}^2 \end{bmatrix}, \ \boldsymbol{C}_x = \begin{bmatrix} L & 0 & -L & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \\ \boldsymbol{d}_x &= \begin{bmatrix} 0 \\ \frac{1}{l} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \ \boldsymbol{x}_x = \begin{bmatrix} \theta_x \\ \dot{\theta}_x \\ \phi_x \\ \dot{\phi}_x \\ \ddot{\phi}_x \end{bmatrix}, \ \boldsymbol{y}_x = \begin{bmatrix} h_x \\ \phi_x \end{bmatrix}, \end{aligned}\right\} \tag{5}$$

and $a_x(= \ddot{x}_c)$ is the running acceleration. With respect to the system equation in Y-direction, a_x is replaced with the centripetal acceleration $a_y = v^2/r$, where v is the running velocity of the WMR and r is the turning radius. Actual values in the system equation are shown in Table I.

TABLE I
ACTUAL VALUES IN SYSTEM EQUATION

	X-direction	Y-direction
c [Ns/m]	1.27	
l [m]	0.0252	
m [kg]	0.7854	
l_o [m]	0.0500	
L [m]	0.0255	
k [rad/(sV)]	1.69	1.71
ω_n [rad/s]	6.82	3.64
ζ [-]	1.55	0.49

IV. DAMPING CONTROL SYSTEM

The damping control system in X-direction is shown in Fig. 3. This control system can damp the sloshing by means of tilting the container. The acceleration a_x which generates the sloshing is considered as a disturbance for this system. The Kalman filter is used to estimate the state $\boldsymbol{x}_x$ and to remove the observation noise in the output $\boldsymbol{y}_x$. This controller is called an optimal servo system in [8].

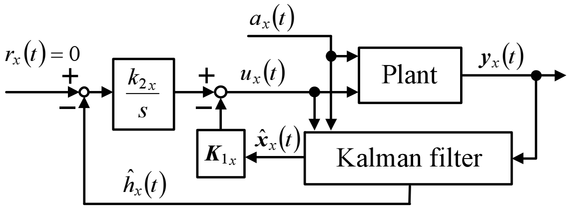

Fig. 3. Damping control system in X-direction

The control system is made discrete to implement itself on a computer. The discrete-type system equation to design the control system is shown as follows:

$$\left.\begin{aligned}\boldsymbol{x}_x(k+1) &= \boldsymbol{P}_x\boldsymbol{x}_x(k)+\boldsymbol{q}_x u_x(k),\\ \boldsymbol{y}_x(k) &= \boldsymbol{C}_x\boldsymbol{x}_x(k),\end{aligned}\right\} \tag{6}$$

where

$$\boldsymbol{P}_x = \exp(\boldsymbol{A}_x\Delta T), \quad \boldsymbol{q}_x = \int_0^{\Delta T} \exp(\boldsymbol{A}_x\tau)d\tau\boldsymbol{b}_x, \tag{7}$$

and ΔT is a sampling period, here ΔT=1.0[ms].

Kalman filter gain is calculated from a discrete-type Riccati equation in [8] as the dispersion of observation noise on h_x or h_y is set to 8.274×10^{-5} m^2, the dispersion of observation noise on ϕ_x or ϕ_y is set to 1.000×10^{-10} rad^2, and the system noise is assumed to be zero.

The damping control gains $\boldsymbol{K}_1$ and k_2 are obtained as follows. A feedback gain $\boldsymbol{K}^*$ of an optimal regulator is obtained by minimizing the discrete-type quadratic performance index in Eq. (8). Actually, the discrete-type Riccati equation is solved to obtain the gain as shown in [8].

$$J = \sum_{k=0}^{\infty}(\|\boldsymbol{x}(k)\|^2_{\boldsymbol{Q}_e} + \|u(k)\|^2_{r_e}), \tag{8}$$

where

$$\left.\begin{aligned}\boldsymbol{Q}_{ex} &= \boldsymbol{C}_x^T\begin{bmatrix}5.0\times10^5 & 0\\ 0 & 10\end{bmatrix}\boldsymbol{C}_x, \quad r_{ex}=1.0,\\ \boldsymbol{Q}_{ey} &= \boldsymbol{C}_y^T\begin{bmatrix}1.7\times10^5 & 0\\ 0 & 10\end{bmatrix}\boldsymbol{C}_y, \quad r_{ey}=1.0.\end{aligned}\right\} \tag{9}$$

The gains $\boldsymbol{K}_1$ and k_2 are calculated from Eq. (10).

$$[\boldsymbol{K}_1 \quad k_2] = [\boldsymbol{K}^*\boldsymbol{P} \quad \boldsymbol{K}^*\boldsymbol{q}+1]\begin{bmatrix}\boldsymbol{P}-\boldsymbol{I} & \boldsymbol{q}\\ \boldsymbol{c}_1 & 0\end{bmatrix}^{-1}, \tag{10}$$

where $\boldsymbol{I}$ denotes an identity matrix and $\boldsymbol{c}_1$ denotes the 1st row of $\boldsymbol{C}$.

When the weighting matrix $\boldsymbol{Q}_e$ is set to larger value, the sloshing in the container can be almost damped. However, the container keeps swinging by the vibration reducer. Therefore, the value of $\boldsymbol{Q}_e$ should be decided by trial and error through experiments.

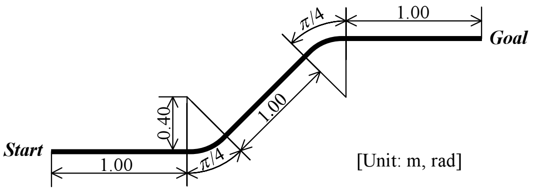

Fig. 4. Curved path on plane

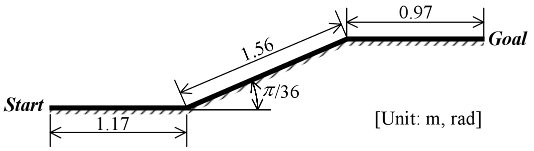

Fig. 5. Straight path on slope

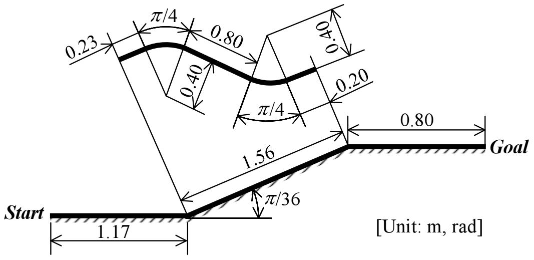

Fig. 6. Curved path on slope

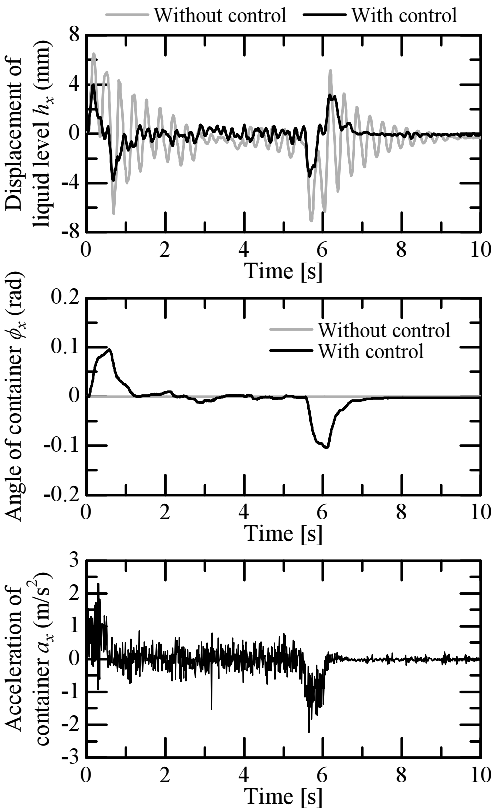

Fig. 7. Experimental result of damping control in Type (1)

V. EXPERIMENTAL RESULT OF DAMPING CONTROL

The experimental conditions are described as follows:

- The running velocity of the WMR is 0.5 m/s,
- The running acceleration/deceleration of the WMR is 1.0 m/s^2,
- There are four types of paths of the WMR;
  Type (1) the straight of 2.7 m on the horizontal plane,
  Type (2) the curve on the horizontal plane in Fig. 4,
  Type (3) the straight on the slope in Fig. 5,
  Type (4) the curve on the slope in Fig. 6.

The WMR runs from the start to the goal along the path.

Figure 7 shows the experimental result of the damping control in Type (1). The sloshing is generated by the running acceleration a_x of the container. The sloshing h_x is damped by tilting action of angle ϕ_x of the container. From ϕ_x in Fig. 7, the container is tilted backward in the acceleration section, and the container is done forward in the deceleration section.

Figure 8 shows the experimental result of the damping control in Type (2). The sloshings h_x and h_y are respectively generated by the running acceleration a_x and the centripetal acceleration a_y of the container. Both of the sloshings are damped by tilting actions of angles ϕ_x and ϕ_y of the container, respectively.

The experimental result of the damping control in Type (3) is shown in Fig. 9. The liquid level is kept around zero on the slope. The accelerometers detect the component of the gravitational acceleration, $g\sin(\pi/36) \approx 0.855$ [m/s^2]. To suppress the offset in the displacement of the liquid level occurred by this component, the container is tilted by the vibration reducer.

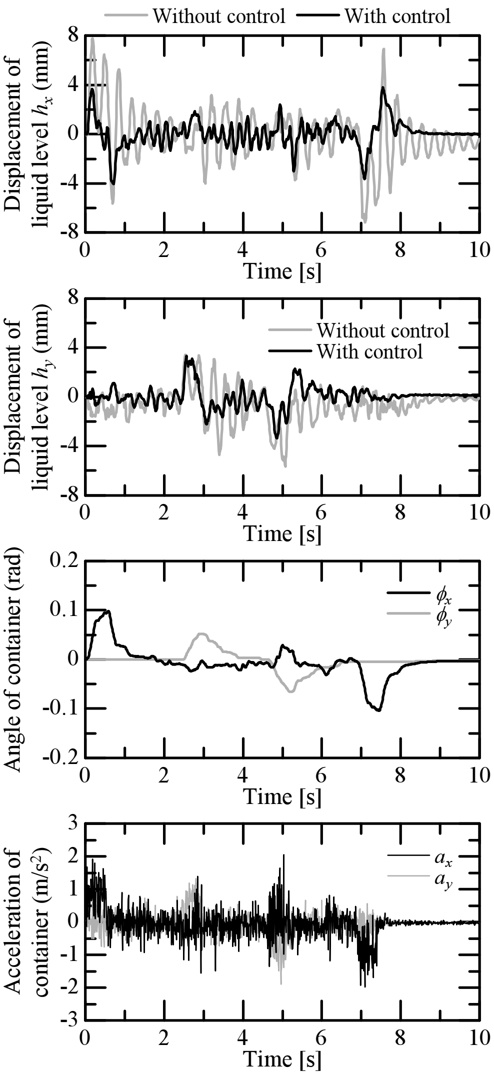

Fig. 8. Experimental result of damping control in Type (2)

The experimental result of the damping control in Type (4) is shown in Fig. 10. The sloshing h_x (h_y) is generated by the acceleration a_x (a_y), and the offset in h_x (h_y) is occurred by the slope angle. Both h_x and h_y can be damped with the vibration reducer. The acceleration of the container in Fig. 10 is a little more vibrational than that in Figs. 7-9. Although it is occurred, the good damping performance can be obtained by using the Kalman filter to estimate the state of the system model.

VI. CONCLUSION

A damping control system of a dual swing-type active vibration reducer for a liquid container carried by a WMR has been designed as an optimal servo system with a Kalman filter. It is notable that the proposed control system can damp the sloshing in the container on not only a horizontal plane

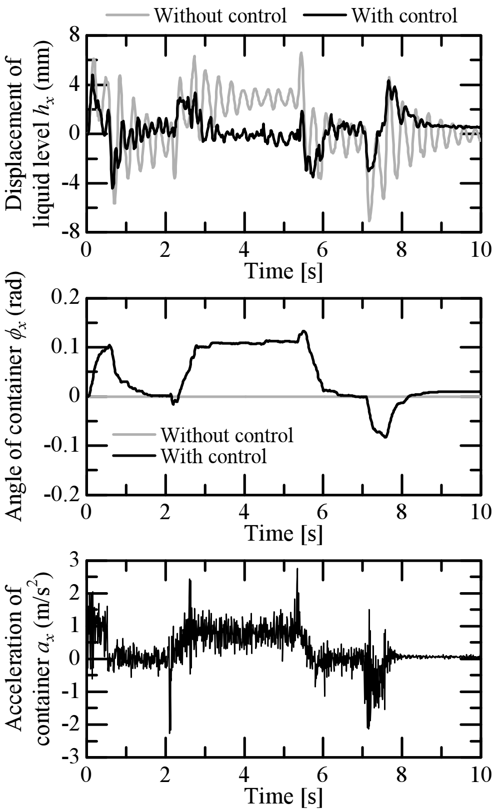

Fig. 9. Experimental result of damping control in Type (3)

but also a slope. The proposed system can be applied to a variety of transfer system.

When a damping path design and a velocity control are applied to the proposed system, damping performance must be more improved.

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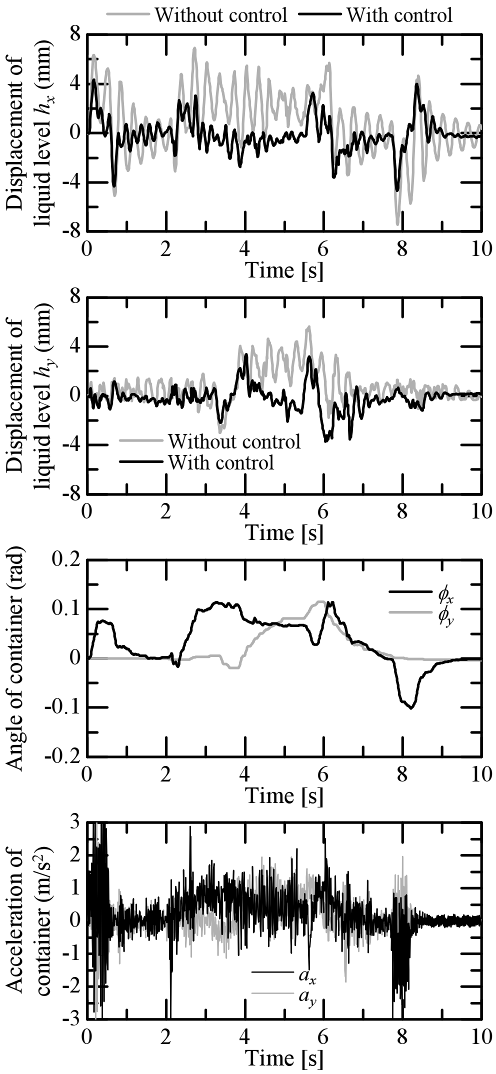

Fig. 10. Experimental result of damping control in Type (4)