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Higgs pair production at the LHC and ILC from a general potential

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Higgs cubic coupling plays a crucial role in probing an origin of electroweak symmetry breaking. It is expected that the cubic coupling is measured by Higgs pair production at the LHC and ILC, and the deviations from the standard model can be extracted from the Higgs pair production process, and those can give us a hint of new physics beyond the standard model. We consider a general potential that achieves the suitable electroweak symmetry breaking. As one of the interesting models, we suggest a nonperturbative Higgs model in which a runaway type of potential is used. In the model, the cross sections of pair production at the LHC are enlarged compared to the standard model. We also study the Higgs pair production induced by a noncanonical kinetic term of Higgs fields which will be important for searching the pair production at the ILC.

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I. INTRODUCTION

In July 2012, the CMS/ATLAS Collaborations at CERN's Large Hadron Collider (LHC) reported that they had discovered a boson, which is consistent with the Higgs boson in the standard model (SM) [1,2], and further in 2013, they confirmed the evidence that it is most likely the long sought Higgs boson of the SM [3–6]. The Higgs boson is the last piece of the SM, and its discovery at the LHC would complete the particle content of the theory. All the interactions of the Higgs boson have to be investigated to see whether the Higgs boson has the properties expected from the SM. While the gauge interactions among the particles have been confirmed successfully, the other interactions in which the Higgs boson participates have not been fully explored experimentally. This situation will change dramatically as the LHC Run II starts and even more so with the ILC experiment.

The Yukawa interactions and the Higgs self-interaction are responsible for describing the generation of fermion masses and for inducing the electroweak symmetry breaking (EWSB), respectively. The experimental data of the single Higgs production and its decay to fermions and vector bosons at the LHC are largely consistent with the SM prediction. The other Higgs interactions have to be probed experimentally to reveal how the EWSB occurs and whether the fermions acquire their masses as described by the SM.

The Higgs self-coupling is one of the key parameters for investigating how the EWSB occurs [7]. In the SM, only the quartic Higgs coupling is allowed by the electroweak gauge symmetry within the renormalizable Lagrangian. The origin of the interaction has been the subject of great debate in last few decades and an inspiration to many theories. For most of these theories, it is expected that the quartic coupling is described by a fundamental physics which precedes the SM at a higher energy scale. For instance, in models with supersymmetry (SUSY), the Higgs quartic coupling is induced by the D-term potential. Therefore, the quartic coupling originates from the electroweak gauge interaction, and it is a function of the gauge couplings, which nicely accommodates the range of the (SM-like) light Higgs mass [8–10]. In addition to this questions regarding the origin of the self-coupling, in the SM, it is not clear whether the negative sign of the Higgs squared mass parameter, which triggers the EWSB, has a dynamical reason and why its size remains separated from the Planck scale.

Indeed, these questions have led to the expectations and thereby many attempts to describe the EWSB as the result of radiatively or dynamically induced mechanisms. For instance, in the SUSY extension of the SM, it is well known that the symmetry breaking can be induced radiatively due to the large top quark Yukawa coupling even when the high scale initial value for the Higgs mass parameter is positive. On the other hand, in a model with dynamically induced symmetry breaking, the Higgs self-interaction can be quite different from the SM. In other possibilities, a new physics related to the EWSB appears at TeV scale where the Higgs self-interaction is modified from its SM value. In such a sense, it is important to probe the Higgs self-interaction, which governs the essence of how the Higgs boson acquires a vacuum expectation value (VEV).

After expanding the Higgs potential around the Higgs VEV, the physical Higgs particle acquires a cubic self-coupling. It is expected that the cubic coupling constant, while challenging, can be measured by Higgs pair

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production at the LHC and ILC [11–16]. The measurement of the cubic Higgs coupling provides important hints for the Higgs self-interaction which stabilizes the Higgs potential [17–22]. The deviation of the Higgs cubic coupling from the SM can be parametrized in a model-independent way by considering a general potential as in Refs. [23–25]. Such a general potential may be either generated by a loop level or can be constructed from a nonperturbative model. At the LHC, if there is a negative contribution to the Higgs cubic coupling from these considerations, the Higgs pair production rate always tends to be enlarged. In that case the deviations are easier to be detected. Therefore, it is interesting to investigate a model which can induce a negative contribution to the cubic coupling.

It is expected that more precise measurements of the cubic coupling can be done from the processes of Higgs pair production at the ILC, compared to the LHC [16]. The process in which the cubic coupling is probed receives contributions not only from the diagrams with the cubic coupling but also from diagrams with the gauge couplings. In order to measure the cubic coupling, one has to therefore know the dependency of the total amplitudes on the individual couplings. Indeed, in models where the couplings differ from the SM, one has to guarantee that the hVV coupling (V stands for a massive gauge boson) remains the same as the one in the SM. Although the experimental data of the single Higgs production support that the hVV coupling is consistent with the SM, $hhVV$ coupling has no such constraint at the moment and can deviate from its SM value. This happens whenever the kinetic term of the Higgs boson is given by higher dimensional effective operators. If the $hhVV$ coupling deviates from the SM, so does the pair Higgs production cross section even if the cubic coupling remains the same. This shows that it is important to investigate how the cross section depends on both the $hhVV$ coupling and the cubic Higgs coupling.

In this paper, we start from a general Higgs potential and investigate how the cubic Higgs coupling can be modified in general. We show that the negative contribution from the SM to the cubic coupling enhances the cross section of the pair Higgs production via gluon fusion at the LHC. From the analysis of the general Higgs potential, we find a type of potential that can induce a sizable negative contribution to the cubic coupling, if the potential contains a piece of repulsive effect from the origin of Higgs configuration. Such a type of potential (a so-called runaway-type potential) can be constructed in nonperturbative models. We also investigate the correction from the general kinetic term of the Higgs boson. We learn how the deviation from the SM couplings are parametrized, and we investigate the parametric dependency of the cross sections of the pair Higgs productions at the ILC and LHC. We also construct a nonperturbative model with SUSY to induce the negative contribution to the cubic Higgs coupling and enhance the pair Higgs production cross section at the ILC. The modification of the $hhVV$ coupling in the model is also investigated.

This paper is organized as follows: In Sec. II, we formulate the cubic Higgs coupling from general Higgs potential. In Sec. III, we show the calculation of the cross section of the Higgs pair production at the LHC; and the negative contribution to the cubic Higgs coupling can enlarge the cross section. In Sec. IV, we study the modification of the $hhVV$ coupling from the noncanonical kinetic term of the Higgs boson and how it affects the pair production of the Higgs bosons at the LHC and ILC. In Sec. V, we construct a nonperturbative model by SUSY QCD, in which a negative contribution is induced in the cubic coupling of the physical Higgs field. Section VI is devoted to the summary and conclusions of this paper.

II. THE CUBIC HIGGS COUPLING FROM THE GENERAL POTENTIAL

It is important to investigate the interaction of Higgs to the other particles and to know what dynamics makes the Higgs boson have a VEV. In the SM, the tree-level Higgs potential is given as

$$V = m_H^2|H|^2 + \lambda|H|^4. \tag{2.1}$$

It is necessary that the squared mass m_H^2 is negative, and in combination with the quartic self-interaction it forces the Higgs field to acquire the VEV. The Yukawa couplings to fermions (especially to top quarks) and the gauge couplings are also important for the loop corrections of the Higgs potential.

Let us describe the Higgs potential in terms of a general function:

$$V = V(|H|^2). \tag{2.2}$$

The function $V(x)$ can contain any effects from loop corrections, or any nonperturbative effects. Surely, due to the gauge invariance, it has to be a function of $|H|^2$ (if there is only one Higgs doublet). In unitary gauge, $|H|^2$ is expressed as

$$|H|^2 = \frac{v^2}{2} + vh + \frac{h^2}{2}, \tag{2.3}$$

where h is a physical Higgs mode and v denotes the Higgs VEV [$H^0 = (v+h)/\sqrt{2}$]. Expanding the function $V(x)$ around the VEV, we obtain

$$V = V\left(\frac{v^2}{2}\right) + V'\left(\frac{v^2}{2}\right)\left(vh+\frac{h^2}{2}\right) + \frac{1}{2}V''\left(\frac{v^2}{2}\right)\left(vh+\frac{h^2}{2}\right)^2 + \frac{1}{6}V'''\left(\frac{v^2}{2}\right)\left(vh+\frac{h^2}{2}\right)^3 + \cdots. \tag{2.4}$$

The stationary condition (vanishing the linear term of h) is $V'(v^2/2) = 0$. The mass of the physical Higgs is obtained as

$$m_h^2 = v^2 V''\left(\frac{v^2}{2}\right). \tag{2.5}$$

In order to obtain the 126 GeV Higgs mass, one requires $V''(v^2/2) = m_h^2/v^2 = 0.26$. In the standard model, for instance, the function $V(x)$ is $V(x) = m^2 x + \lambda x^2$ and one obtains $m_h^2 = 2\lambda v^2$. In this expression of the Higgs mass, it is not necessary to solve the stationary condition $V' = 0$ since we use $v = 246$ GeV as an input.

The cubic interaction of the physical Higgs can be also obtained as

$$-\mathcal{L}_{hhh} = \frac{1}{2}\left(V'' + \frac{1}{3}v^2 V'''\right) v h^3 = \frac{m_h^2}{2v}\left(1 + \frac{1}{3}v^2 \frac{V'''}{V''}\right) h^3. \tag{2.6}$$

The tree-level Higgs potential in the SM gives $V''' = 0$, and therefore, the modification from the tree-level SM Higgs potential can be parametrized by

$$C_h = \frac{1}{3} v^2 \frac{V'''}{V''}, \tag{2.7}$$

and the ratio of the cubic coupling is expressed as

$$\frac{\lambda_{hhh}}{\lambda_{hhh}^{\rm SM}} = 1 + C_h. \tag{2.8}$$

Precisely speaking, in this formulation, C_h parametrizes the deviation from the tree-level cubic Higgs coupling in the SM: $\lambda_{hhh}^{\rm SM} = 3m_h^2/v$. As mentioned before, the general function $V(x)$ can contain loop effects. One can easily evaluate the contribution from the top quark 1-loop effective potential:

$$V(x) = m^2 x + \lambda x^2 - \frac{3}{16\pi^2} y_t^4 x^2 \left(\ln\left(\frac{y_t^2 x}{Q^2}\right) - \frac{3}{2}\right), \tag{2.9}$$

where y_t is the top quark Yukawa coupling ($m_t = y_t v/\sqrt{2}$) and Q is the renormalization scale. Because $V'''(v^2/2) = -3y_t^4/(4\pi^2 v^2)$, we obtain

$$C_h = -\frac{m_t^4}{\pi^2 v^2 m_h^2} \simeq -0.1, \tag{2.10}$$

for the loop correction in the SM.[1]

[1] W, Z and Higgs contributions are subdominant compared to the top quark contribution. Since they have the opposite signs to the top-loop one, they contribute destructively. The correction to the cubic coupling including $W/Z/H$-loop contributions is estimated as $C_h = (9m_h^4 + 8(-4m_t^4 + 2M_W^4 + M_Z^4))/(32\pi^2 v^2 m_h^2) \sim -0.08$.

Let us consider the following Higgs potential as a toy example:

$$V = m_H^2 |H|^2 + \Lambda^{4-2a}(|H|^2)^a, \tag{2.11}$$

where Λ is a dimensional parameter. The minimization condition is

$$m_H^2 + a\Lambda^{4-2a} x^{a-1} = 0, \tag{2.12}$$

where $x = v^2/2$. Therefore, if $a < 0$ (namely, the potential for $m_H \to 0$ has a runaway kind of behavior), m_H^2 is positive. The Higgs mass is obtained as

$$m_h^2 = 2a(a-1)\Lambda^{4-2a} x^{a-1} = 2(1-a) m_H^2. \tag{2.13}$$

One can calculate

$$C_h = \frac{2}{3}\frac{xV'''}{V''} = \frac{2}{3}(a-2), \tag{2.14}$$

and the correction from the standard model is specified only by the exponent a. We note that the correction C_h is negative for the runaway-type Higgs potential ($a < 0$).

As one can find from the above expression, the pair Higgs production from the general scalar potential can be parametrized by a single parameter C_h. The expansion of the scalar potential is described in unitary gauge. Here, we comment on the case of the 't Hooft-Feynman gauge:

$$H = \begin{pmatrix} \chi^+ \\ \frac{v+h+i\chi}{\sqrt{2}} \end{pmatrix}. \tag{2.15}$$

In this case, $|H|^2 = v^2/2 + vh + h^2/2 + \chi^2/2 + \chi^+\chi^-$. Expanding the potential $V(|H|^2)$, we obtain that the Nambu-Goldstone (NG) bosons χ and $\chi^\pm$ are massless under the stationary condition $V'(v^2/2) = 0$, and they will be eaten by the gauge bosons. The interactions between the physical Higgs h and the NG bosons are

$$-\mathcal{L} = \frac{m_h^2}{v} h\left(\frac{\chi^2}{2} + \chi^+\chi^-\right) + \frac{m_h^2}{2v^2}(1 + 3C_h) h^2 \left(\frac{\chi^2}{2} + \chi^+\chi^-\right). \tag{2.16}$$

Therefore, the single Higgs production is same as the one in the SM, but for the pair Higgs production via longitudinal vector boson fusion, the scattering amplitude is modified by the C_h parameter from the SM. The scattering amplitude of $\chi^+\chi^- \to hh$ is obtained as

$$\mathcal{M}(\chi^+\chi^- \to hh) = \frac{m_h^2}{v^2}\left(1 + 3C_h + \frac{3(1+C_h)m_h^2}{s - m_h^2} + \frac{m_h^2}{t - M_W^2} + \frac{m_h^2}{u - M_W^2}\right). \tag{2.17}$$

The equivalence theorem [26] tells us that this scattering amplitude is same as the longitudinal WW scattering amplitude up to the $O(M_W^2/s)$ correction (namely, neglecting gauge coupling in $M_W^2 = g^2v^2/4$). One can easily verify this equivalence by calculating the amplitude in the unitary gauge. However, since the 126 GeV Higgs is not very heavy compared to the gauge boson masses, we should calculate in the unitary gauge without neglecting the gauge couplings for the numerical evaluation of the cross sections.[2]

The general scalar potential can be also specified to the two-Higgs doublet model (2HDM). The scalar potential in terms of H_1 and H_2 (whose hypercharges are $-1/2$ and $+1/2$, respectively) is a function of $|H_1|^2$, $|H_2|^2$ and $H_1 \cdot H_2 (\equiv \epsilon_{ab} H_1^a H_2^b)$. The cubic Higgs coupling can be written, in general, similarly to the one-Higgs case. We exhibit the relevant expressions in Appendix B.

III. HIGGS PAIR PRODUCTION VIA GLUON FUSION AT THE LHC

The Higgs cubic coupling can be probed by pair production of the Higgs boson. At the LHC, the dominant contribution of the pair Higgs production is the gluon fusion process. There are two diagrams for the pair Higgs production via the gluon fusion: (i) $gg \to h \to hh$, (ii) $gg \to hh$ via a box diagram. The $gg \to h$ and $gg \to hh$ couplings are generated by triangle and quadrangle top quark loop diagrams, respectively. The effective coupling (neglecting the top quark momentum) can be obtained by [27]

$$\begin{aligned} \mathcal{L}_{\text{eff}} &= \frac{\alpha_s}{12\pi} (\log H) G^a_{\mu\nu} G^{a\mu\nu} \\ &= \frac{\alpha_s}{12\pi} \left(\frac{h}{v} - \frac{h^2}{2v^2} + \cdots \right) G^a_{\mu\nu} G^{a\mu\nu}. \end{aligned} \tag{3.1}$$

Because of the opposite signs of the effective couplings (in addition to a kinematical reason), the cross section of the pair Higgs production at the LHC is very small at the order of $O(10^{-3})$ compared to the single Higgs production. Inversely speaking, this fact makes the process sensitive to any additional contributions and a good probe of new physics beyond SM.

The cross section of $pp \to hh$ can be obtained by

$$\sigma(pp \to hh) = \int_{4m_h^2/s}^{1} d\tau \frac{d\mathcal{L}^{gg}}{d\tau} \hat{\sigma}(gg \to hh; \hat{s} = \tau s), \tag{3.2}$$

and the parton-level amplitude of $gg \to hh$ (using the effective coupling) is

$$\mathcal{M}(gg \to hh) = \frac{\alpha_s}{3\pi v^2} \left(-1 + \frac{3m_h^2(1 + C_h)}{\hat{s} - m_h^2} \right). \tag{3.3}$$

The amplitude vanishes at $\hat{s} = (4 + 3C_h)m_h^2$. From the kinematics, we integrate the parton cross section from $\hat{s} = 4m_h^2$ to s. One can find that the cross section of $pp \to hh$ is enhanced for $C_h < 0$ as a result. Although positive C_h can also provide a large cross section, a significant enhancement requires $C_h \gtrsim 4$. Therefore, in the negative C_h case the cross section can be enhanced relatively easily compared to the positive case, and thus, the former is more feasible to be realized in a model.[3] Models which give a negative C_h contribution are interesting since its implication at the LHC and ILC becomes potentially more pronounced for the Higgs pair productions and, therefore, can be scrutinized in these experiments. We note that the runaway-type potential provides an example of $C_h < 0$, as mentioned before.

In Fig. 1, we show the ratio of cross sections between the C_h-dependent $gg \to hh$ cross section and the SM one. The left and right figures represent 8 TeV and 14 TeV collisions at the LHC, respectively. The cross sections at 8 TeV and 14 TeV at the next-to-leading order calculation are 5–11 fb and 25–45 fb, respectively [11,12].[4] The numerical numbers in the plots are given at the leading order calculation. It is expected that the factor in the next-to-leading order/leading order calculation is canceled in the C_h dependence, and thus, we show the ratio of the cross sections. We utilized FormCalc/LoopTools[28] to evaluate the cross sections employing MRST2006nnlo [29] and CTEQ6.1 [30] Parton Distribution Function (PDF) sets. The renormalization and factorization scales are set to be equally μ_F, and we take $\mu_F = M_{hh}$ where M_{hh} is the invariant mass of the Higgs pair. As a characteristic feature of the amplitude (3.3), one can find that negative C_h enhances the production cross section compared to positive C_h in the figure.[5] Note that $C_h \sim 1.5$ gives the minimum value for the cross section.

We comment on the Higgsstrahlung process $q\bar{q} \to V^* \to Vhh$ and WW fusion process. At the LHC, these processes are subdominant and the cross sections are an order of magnitude smaller than the gluon fusion process in the SM, where both processes give the cross section $\sigma(pp \to hhjj) = 1.6$ fb at 14 TeV LHC. However, if the Higgs cubic coupling is modified, these processes should be affected. Figure 2 shows the ratios of cross sections denoted by R, where $R = \sigma(pp \to hhjj)/\sigma(pp \to hhjj)_{\text{SM}}$ for the

[2] Note that the scattering amplitudes of $t\bar{t}(b\bar{b}) \to hh$ are also affected by C_h, and if we specify a type of Yukawa interactions in 2HDM, the amplitudes depend on $\tan\beta$ as well.

[3] If we only observe the $gg \to hh$ process, the cross sections for $C_h \sim 4$ and $C_h \sim -1$ are degenerate. However, the degeneracy can be distinguished if we can observe $pp \to hhjj$ at the LHC, or pair production at ILC.

[4] The next-to-next-to-leading order calculation is given in Ref. [31].

[5] The discovery potential for pair Higgs production at the LHC is studied in Ref. [11]. Promising channels at a large luminosity phase of the LHC are $hh \to b\bar{b}W^-W^+$, $b\bar{b}\gamma\gamma$ and $b\bar{b}\tau^+\tau^-$.

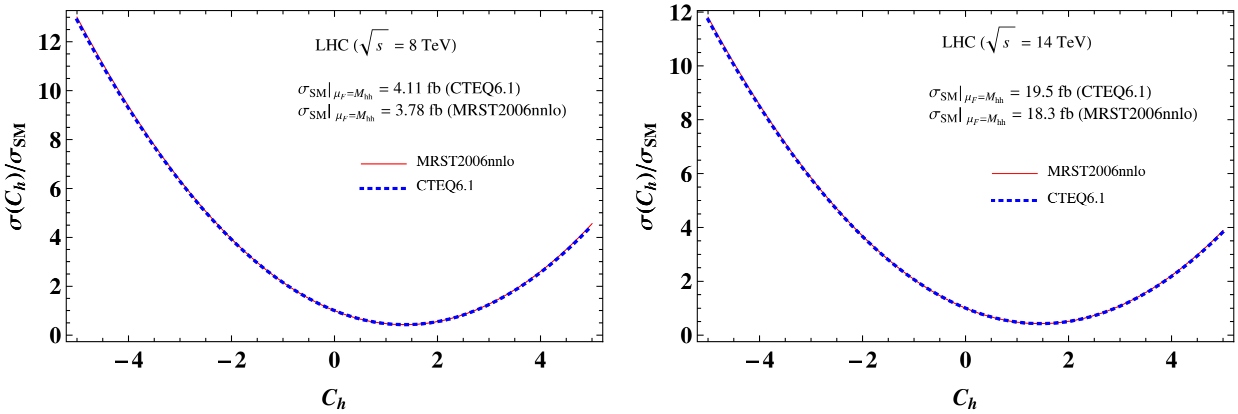

FIG. 1 (color online). The ratio of cross sections $\sigma(C_h)/\sigma_{SM}$ for $gg \to hh$ at LHC, where the left and the right figures show $\sqrt{s} = 8$ TeV and $\sqrt{s} = 14$ TeV collisions, respectively. MRST2006nnlo and CTEQ6.1 PDF sets are used to calculate the cross section, which is represented by the solid (red) and the dotted (blue) lines, respectively.

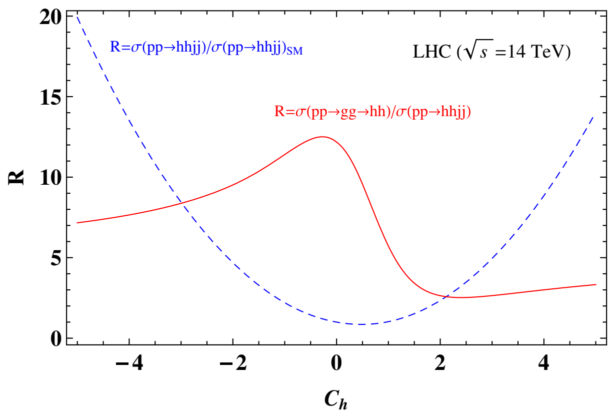

FIG. 2 (color online). The ratios of cross sections R in which $R = \sigma(pp \to hhjj)/\sigma(pp \to hhjj)_{SM}$ and $R = \sigma(pp \to gg \to hh)/\sigma(pp \to hhjj)$ are shown by dashed (blue) and solid (red) lines, respectively. The cross section of $pp \to hhjj$ in the SM is given by $\sigma(pp \to hhjj)_{SM} = 1.6$ fb at 14 TeV LHC.

$pp \to hhjj$ process denoted by a dashed (blue) line. In the figure, we can see that $C_h \neq 0$ can enhance the cross section. On the other hand, when we take $R = \sigma(pp \to gg \to hh)/\sigma(pp \to hhjj)$ for various C_h, which is denoted by a solid (red) line in the figure, one can find that the Higgsstrahlung and vector boson fusion processes are subdominant compared to the gluon fusion process even if $C_h \neq 0$.

IV. CONTRIBUTION FROM THE NONCANONICAL KINETIC TERM OF THE HIGGS BOSON

At the ILC, the Higgsstrahlung process $e^+e^- \to Z^* \to Zhh$ and the WW fusion process $e^+e^- \to WW^*\nu\bar{\nu} \to hh\nu\bar{\nu}$ are expected to be important in probing the cubic coupling. In particular, the Higgs cubic coupling can be measured using the WW fusion process [7,16].

These processes receive the contributions not just from the cubic coupling but also from the hVV and $hhVV$ couplings due to the gauge interactions. Therefore, for more general consideration, we study cases wherein either or all of these couplings are modified from their SM values. If the results for these processes at the ILC differ from the SM expectations, it is important to understand which one of these modifications is responsible, since those modified Higgs-gauge boson couplings obscure the measurement of the cubic coupling.

The modification to the Higgs-gauge interactions due to the following noncanonical kinetic term has been considered[6] in Ref. [24]:

$$\mathcal{L}_{\rm kin} = F\left(\frac{2|H|^2}{v^2}\right) D_\mu H^\dagger D^\mu H, \tag{4.2}$$

where D_μ is the covariant derivative for the Higgs field. For the convenience of the kinetic normalization, the function F is defined as $F(1) = 1$ [otherwise, the kinetic normalized field is $\sqrt{F(1)}H$]. Expanding the general kinetic function $G(x) \equiv xF(x)$, we obtain the W/Z boson masses and coupling to the physical Higgs as

$$\left(M_W^2 W_\mu^+ W^{-\mu} + \frac{M_Z^2}{2} Z_\mu Z^\mu\right)\left(1 + G'(1)\frac{2h}{v} + (G'(1) + 2G''(1))\frac{h^2}{v^2}\right). \tag{4.3}$$

[6]In general, there can be a different type of operator,

$$(HD_\mu H^\dagger)(H^\dagger D^\mu H), \tag{4.1}$$

which causes the different Higgs couplings to W and Z bosons. However, it also modifies the ρ parameter. Here we do not consider such an operator for simplicity. General dimension-six operators are enumerated in Ref. [32].

In SM, $F(x) = 1$, and obviously, $G' = 1$ and $G'' = 0$. We denote these shifts for the couplings hVV and $hhVV$ from the SM by the parameters C_1 and C_2, respectively:

$$1 + C_1 = G'(1), \qquad 1 + C_2 = G'(1) + 2G''(1). \quad (4.4)$$

Both hVV and $hhVV$ couplings are important for the Higgsstrahlung and vector boson fusion processes. As for the hVV coupling, it is expected to be measured accurately by means of the parameters relating to the single Higgs production and its decay [33] before the pair Higgs production can be observed. In addition to this chronological reason, the hVV coupling is restricted by oblique corrections for the precise electroweak measurements [25], while the $hhVV$ coupling is not. We, therefore, fix the hVV coupling in our analysis to its SM value $C_1 = 0$. We comment that even if the single Higgs production is fully consistent with the SM prediction, it is possible that $G'(1) = 1 + C_1 = -1$. In that case, however, one can redefine $h \to -h$, and the cubic h coupling changes its signature, which affects the Higgs pair production.

We note that perturbative partial-wave unitarity of $WW \to hh$ scattering [26] is violated unless $(1 + C_1)^2 = 1 + C_2$ is satisfied. Since we choose $C_1 = 0$, the perturbative unitarity is violated for C_2. In fact, the model is described as an effective theory, and we expect that new particles appear at around the TeV scale.

The corrections C_1 and C_2 can be generated by the noncanonical kinetic term in Eq. (4.2), as given in Eq. (4.4) As a simple perturbative toy example one can consider

$$F(x) = 1 + a \ln x, \quad (4.5)$$

where the coefficient a contains the appropriate loop factor in the model. In this case one obtains $C_1 = a$ and $C_2 = 3a$. On the other hand, the noncanonical kinetic function $F(x)$ may have a powerlike behavior if it is generated by a strong dynamics. We will give later an explicit model. Let us consider the following power function:

$$F(x) = x^n. \quad (4.6)$$

In this case, one can obtain $C_1 = n$ and $C_2 = n(2n + 3)$. If $n = -2$, we have $1 + C_1 = -1$. It is obvious that the single Higgs production is consistent with the SM if $1 + C_1 = -1$. It can be understood by the (unphysical) redefinition $h \to -h$. However, upon this change, the cubic h coupling flips its sign and $C_2 = 2$, and therefore, the cross section of pair Higgs production is modified. This toy example can be obtained if the Kähler potential of the Higgs fields is given as

$$K = (H_1^\dagger H_1)^3 + (H_2^\dagger H_2)^3 \quad (4.7)$$

in a SUSY model.

A. LHC

As mentioned before, the pair Higgs production via vector boson fusions and Higgsstrahlung are subdominant compared to the gluon fusion process at the LHC. This situation may change if the $hhVV$ couplings are modified ($C_2 \neq 0$), so that these subdominant processes are enhanced. The vector boson fusion processes can be calculated by the so-called effective vector boson approximation [34], which can be obtained by using the amplitude of the longitudinal vector boson scattering to pair Higgs bosons, as we mentioned in the previous section. While the approximation is illustrative and easier to derive than the exact treatment, it is not particularly good due to the fact that the self-coupling of the 126 GeV Higgs boson is not so strong and gauge couplings cannot be neglected. Therefore, we use MadGraph 5 [35,36] for our numerical calculation which is essentially equivalent to the exact treatment. The disagreements we have obtained agrees well with the comparative study reported in Ref. [23].

We have scanned the cross section for the process $pp \to hhjj$ by the parameters C_h and C_2. The results are shown in Fig. 3. As we see deviations from the SM can be quite large. In the case for the canonical kinetic term, i.e., $C_2 = 0$, the enhancements are appreciable only at very large deviations at $C_h = -3$ or 4. On the other hand, the rate is more sensitive to the changes in C_2 as relatively smaller values for the parameter C_2 lead to much more enhanced deviations compared to C_h.

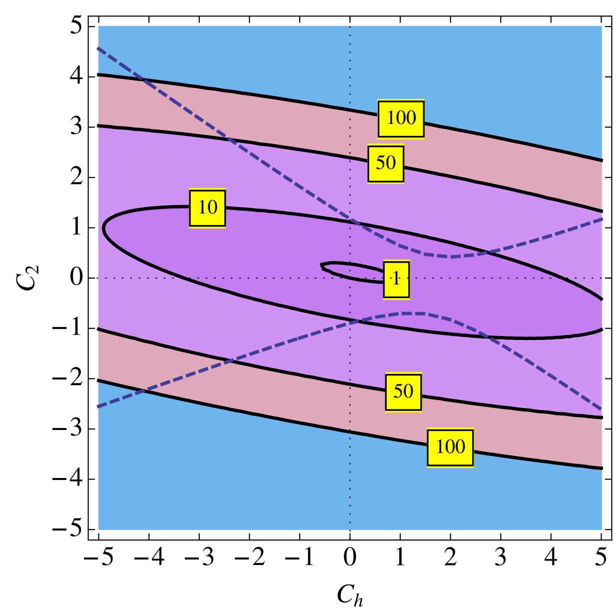

FIG. 3 (color online). The Higgs pair production $pp \to hhjj$ enhancements are shown as a contour plot in the C_h-C_2 plane for the 14 TeV run at the LHC. The enhancement factors are shown as numerical labels. The dashed line shows when the process becomes equal to the leading Higgs pair production via the gluon fusion.

Here we briefly note on the process $q\bar{q} \to hh$ which is induced at loop level. As for the SM, the rate is subleading compared to the leading gluon fusion process due to the fact that it is induced by weak interactions. We expect this to remain the same even when the vertices hWW and $hhWW$ are modified. In the SM the unitarity for the process $WW \to hh$ is granted by the cancellation among the s-channel diagrams where hWW and $hhWW$ are related. This is lost in the presence of nonzero C_2 indicating that a new physics is nearby in the TeV range as we have mentioned. Therefore, one should treat this as an effective operator of the form $|H|^2\bar{q}\partial q$ which has a corresponding counter term. At large values of C_2 the effect may become important. In this work we do not attempt a thorough analysis for this operator and ignore its effect.

The enhancements in the Higgs pair production at the LHC due to the changes in C_2 and C_h couplings may be as large as a factor of 50 and it is very challenging to detect them as they are still more than the order of magnitude below the single Higgs production. Therefore these deviations still require very high luminosity.

B. ILC

There are two processes for the pair Higgs production at the ILC, $e^+e^- \to Z^* \to Zhh$ (double Higgsstrahlung) and $e^+e^- \to hh\nu\bar{\nu}$ (WW fusion process) [7]. For the 126 GeV SM Higgs boson, the cross section of the double Higgsstrahlung is dominant for the pair Higgs production below $\sqrt{s} \simeq 1$ TeV [$\sigma(e^+e^- \to Zhh) = 0.15$ fb at $\sqrt{s} =$ 500 GeV]. The cross section of double Higgsstrahlung is maximized at around $\sqrt{s} = 600$ GeV, and it dumps for larger $\sqrt{s}$. The WW fusion process, on the other hand, grows with larger $\sqrt{s}$, and its cross section is comparable to the Higgsstrahlung at around $\sqrt{s} = 1.2$ TeV. The primary goal at the ILC is to refine the details of the Higgs interactions and it is expected that the Higgs cubic coupling, while challenging, can be measured. In addition to the diagram whose contribution to the amplitude is proportional to the cubic coupling ($e^+e^- \to Zh \to Zhh$), there are diagrams which interfere with it. Therefore, the accuracy of the measurements of the cubic coupling using the two processes does not directly depend on the cross sections. In fact, the measurement of the cubic Higgs coupling is obscured by the C_2 contribution. Therefore, it is important to investigate the C_2 and C_h contribution to the pair Higgs production at the ILC.

Similar to the LHC case, the cross section of $e^+e^- \to hh\nu\bar{\nu}$ via the WW fusion process is calculated by using MadGraph 5 as a function of C_2 and C_h. In Fig. 4, the results are shown for $\sqrt{s} = 500$ GeV (right) and 1 TeV (left) at the ILC compared to the SM expectation. In the first case, the enhancement is of the order of 1, or higher is possible in large values of C_2 and C_h with both having the same signs. On the other hand, for the latter case, the effect of C_2 can be dramatic with an enhancement at the level of $\sim$50 starting from $C_2 \simeq -2$ even at $C_h \simeq 0$.

In Fig. 5, the rates of the Higgsstrahlung process is plotted relative to the SM result. The effect is milder compared to the WW fusion for both center of mass energies. This does not mean that it is more important to consider the former since their simultaneous measurements complement each other in entangling the interference which obscures the Higgs self-coupling determination.

The merit of the ILC compared to the LHC is that the center of mass energy of e^+e^- is fixed. The energy

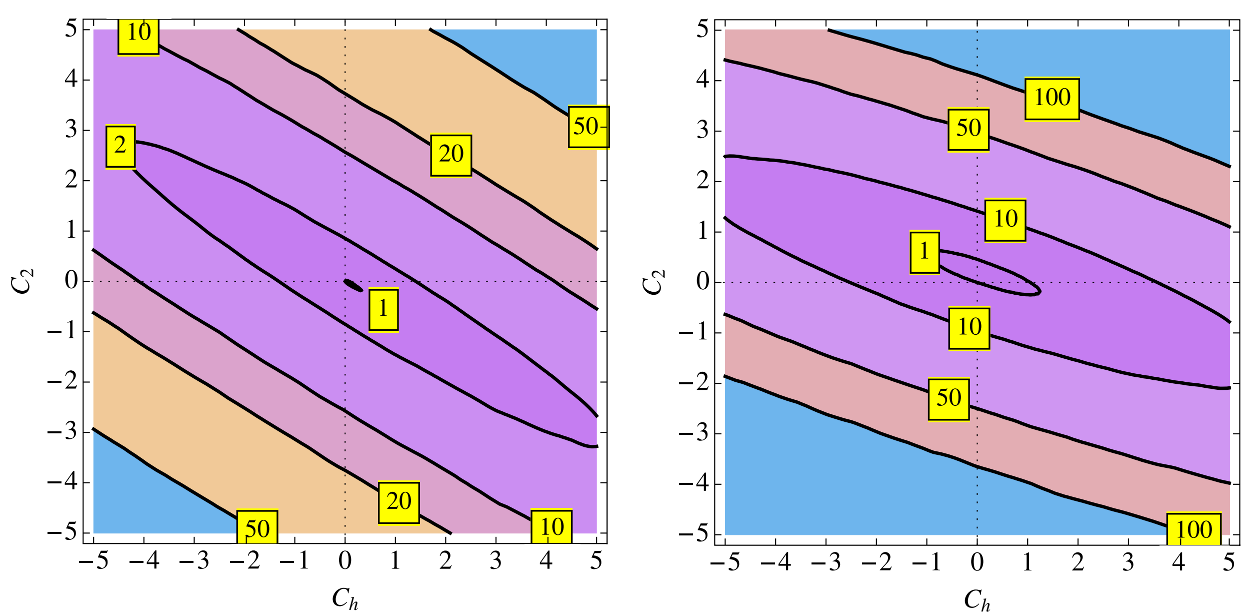

FIG. 4 (color online). The contour plots of the ratio of the cross section, $\sigma(C_h, C_2)/\sigma(C_h = C_2 = 0)$ of $e^+e^- \to hh\nu\bar{\nu}$. Left ($\sqrt{s} = 500$ GeV), and right ($\sqrt{s} = 1$ TeV).

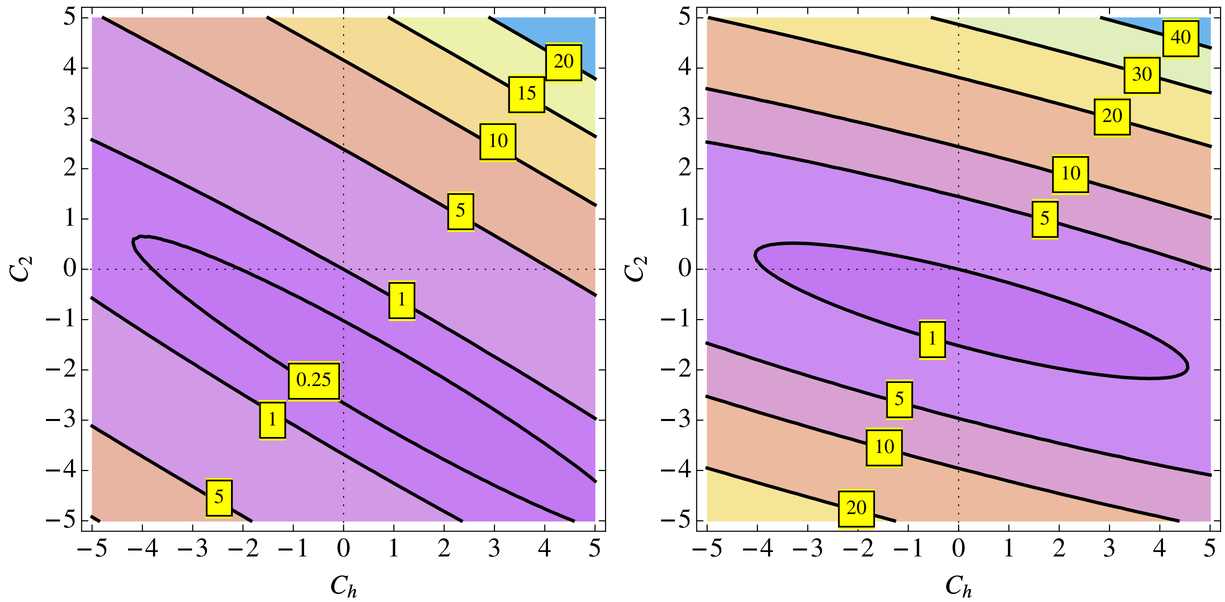

FIG. 5 (color online). The countour plots of the ratio of the cross section, $\sigma(C_h, C_2)/\sigma(C_h = C_2 = 0)$ of $e^+e^- \to Zhh$. Left ($\sqrt{s} = 500$ GeV), and right ($\sqrt{s} = 1$ TeV).

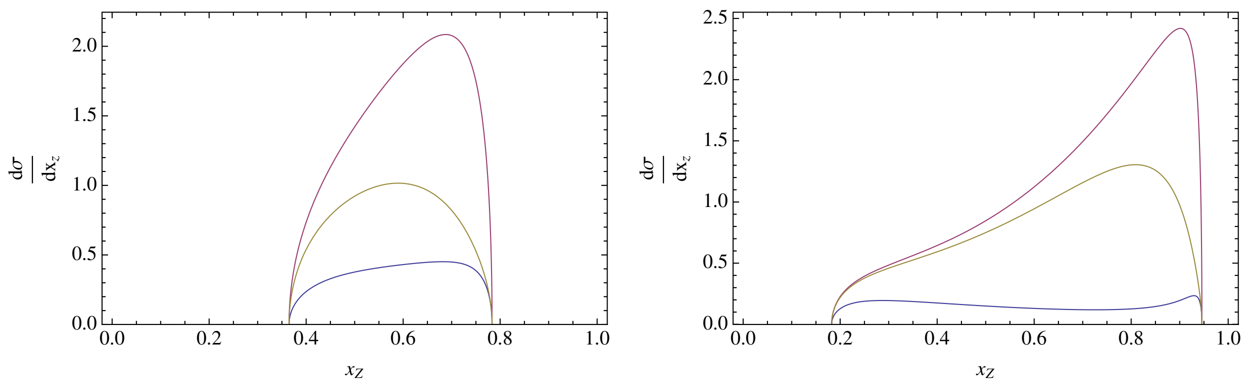

FIG. 6 (color online). The differential cross section (in fb) of $e^+e^- \to Zhh$. Left ($\sqrt{s} = 500$ GeV), and right ($\sqrt{s} = 1$ TeV). x_Z is a scaled energy of the Z boson in the final state: $x_Z = 2E_Z/\sqrt{s}$. $(C_h, C_2) = (0, 0)$, $(-2, 2)$, $(0, 2)$ from below to the top in each graph.

distribution of the final states can be used as a clear signal to probe the model parameters. In fact, in addition to the cross section, the shape of the energy distribution of the Z boson is sensitive to the parameters C_h and C_2. The explicit form of the differential cross section of the double Higgsstrahlung is given in Ref. [7]. In Fig. 6, we show the energy distribution of the Z boson in the $e^+e^- \to Zhh$ process for $\sqrt{s} = 500$ GeV and 1 TeV. The scaled energy of the Z boson x_Z is defined as $x_Z \equiv 2E_Z/\sqrt{s}$. As seen from the figure, the nonzero C_2 not only enhance the total cross section, but also change the shape of the energy distribution. We expect that C_2 and C_h can be measured at the ILC if there are non-SM effects in them.

At the ILC, unlike at the LHC, the environment is much cleaner, which makes even mild enhancements detectable for the Higgs pair productions relatively easy. Therefore both processes are essential for determining what kind of deviations from the SM are present.

V. A MODEL BUILDING

In the previous sections, we have considered the deviation from the SM and have parametrized them as A general extension. Therefore, it can be applied to any models (perturbative, effective theories, or nonperturbative models). As described, the pair Higgs production can be described by three parameters C_h, C_1, and C_2 (if there is only one Higgs doublet):

$$-\mathcal{L} \supset \frac{m_h^2}{2v}(1 + C_h)h^3 + \left(M_W^2 W^+W^- + \frac{M_Z^2}{2}ZZ\right) \times \left((1 + C_1)\frac{2h}{v} + (1 + C_2)\frac{h^2}{v^2}\right). \tag{5.1}$$

At the tree level in the SM, we have $C_h = C_1 = C_2 = 0$ which are modified by loop corrections. If there are new particles, the quantities can become nonzero in effective theories by integrating the heavy fields.

As we have explained in Sec. III, the pair Higgs production at the LHC is enhanced if $C_h < 0$. Therefore, it is interesting to build a model in which the cubic Higgs coupling has negative contribution compared with the SM. Such a situation can be realized if the potential is runaway-type nonperturbative behavior. Indeed, the instanton effects can induce the runaway potential in SUSY $SU(N)$ QCD with the N_f flavor model for $N > N_f$ [37]. In the model, thus, the symmetry breaking occurs due to the nonperturbative effects of SUSY gauge theories [38].

The symmetry of the SUSY QCD is $SU(N) \times SU(N_f)_L \times SU(N_f)_R \times U(1)_B$ with chiral fields representations:

$$Q\colon\ (\mathbf{N}, \mathbf{N_f}, 1), \qquad \bar{Q}\colon (\bar{\mathbf{N}}, 1, \mathbf{N_f}). \tag{5.2}$$

The nonperturbative superpotential is generated by instanton effects [37]:

$$W_{\rm np} = \frac{\Lambda_0^{3+\frac{2N_f}{N-N_f}}}{(\det \bar{Q}Q)^{\frac{1}{N-N_f}}}, \tag{5.3}$$

where Λ_0 is a nonperturbative scale. We consider a case where $N_f = 2$. Suppose that $SU(N_f)_L$ is the weak gauge symmetry, and the $U(1)$ subgroup of $SU(N_f)_R \times U(1)_B$ is the hypercharge symmetry. [Then N has to be an even number to eliminate $SU(2)_L$ anomaly.]

The composite field $\bar{Q}Q$, which is a moduli field of the SUSY QCD, can be identified as a Higgs bidoublet.

$$\bar{\Lambda} H_1^a = \bar{Q}_1 Q^a, \qquad \bar{\Lambda} H_2^a = \bar{Q}_2 Q^a, \tag{5.4}$$

where $\bar{\Lambda}$ is a composite scale. Since $\det \bar{Q}Q = \bar{\Lambda}^2 H_1 \cdot H_2$, the nonperturbative superpotential can be written as [39,40]

$$W_{\rm np} = \frac{\Lambda^{3+2\alpha}}{(H_1 \cdot H_2)^\kappa}, \tag{5.5}$$

where $\kappa = 1/(N-2)$, and $\Lambda^{3+2\kappa} = \Lambda_0^{3+2\kappa} (\Lambda_0/\bar{\Lambda})^{2\kappa}$.

The Kähler potential in terms of the Higgs fields is obtained from the canonical form

$$\mathcal{L} = \int d^4\theta (Q^\dagger e^V Q + \bar{Q} e^{-V} \bar{Q}^\dagger), \tag{5.6}$$

by integrating out the heavy gauge multiplet V ($e^V QQ^\dagger = \bar{Q}^\dagger \bar{Q} e^{-V}$), or using D-flat condition $D^a = Q^\dagger T^a Q - \bar{Q} T^a \bar{Q}^\dagger = 0$ [using $QQ^\dagger = \bar{Q}^\dagger \bar{Q}$ (for $N_f < N_c$), we obtain $(Q^\dagger Q)^2 = Q^\dagger \bar{Q}^\dagger \bar{Q} Q = H^\dagger H$] [37]:

$$K = 2\bar{\Lambda} \mathrm{tr} \sqrt{H^\dagger H}, \tag{5.7}$$

where H is a 2×2 matrix (H_i^a), which contains two $SU(2)_L$ doublets. Rewriting the Kähler potential in terms of H_1 and H_2, we obtain[7]

$$K = 2\bar{\Lambda} \sqrt{H_1^\dagger H_1 + H_2^\dagger H_2 + 2\sqrt{(H_1 \cdot H_2)^\dagger (H_1 \cdot H_2)}}. \tag{5.10}$$

The Kähler metric from the Kähler potential is given in Appendix A.

Using the nonperturbative potential and the Kähler potential, the scalar potential can be calculated as

$$V_{\rm np} = 2\kappa^2 \frac{\Lambda^{6+4\kappa}}{\bar{\Lambda}} \frac{\sqrt{|H_1|^2 + |H_2|^2 + 2\sqrt{|H_1 \cdot H_2|^2}}}{(|H_1 \cdot H_2|^2)^{\kappa+\frac{1}{2}}}. \tag{5.11}$$

This potential is given in the case where the nonperturbative potential is exact (in the SUSY limit) and the classical Kähler potential is assumed. Just for an interest, assuming that $V_{\rm np}$ is the only piece of the runaway potential, we can obtain the correction of C_h for the cubic Higgs coupling:

$$C_h = -\frac{5}{3} - \frac{4}{3}\kappa. \tag{5.12}$$

Because the SUSY breaking will disturb the scalar potential, we do not insist that this potential gives the numerical quantities of C_h for the cubic Higgs coupling. However, we expect that the instanton effects induce the runaway behavior to the potential, and it adds a negative contribution to the cubic Higgs coupling.

The kinetic term from the Kähler potential can be calculated as

$$\begin{aligned}\mathcal{L}_{\rm kin} = {} & \frac{K}{2} \partial_\mu H^* \partial^\mu H \\ & + \frac{2}{K}((H\partial_\mu H^*)(H^* \partial^\mu H) - (H \cdot \partial_\mu H)(H^* \cdot \partial^\mu H^*)),\end{aligned} \tag{5.13}$$

[7]Formally, $H = H_i^a$, and $H^\dagger H$ is a positive definite Hermite 2×2 matrix. The trace of a square root Hermite matrix A is

$$\mathrm{Tr}\sqrt{A} = \sum_i \sqrt{a_i}, \tag{5.8}$$

where a_i are eigenvalues of A. The eigenvalues of $A^\dagger A$ for 2×2 matrix A is

$$\begin{aligned}&\frac{\mathrm{Tr}A^\dagger A \pm \sqrt{(\mathrm{Tr}A^\dagger A)^2 - 4(\det A^\dagger A)}}{2} \\ &\quad = \left(\frac{\sqrt{\mathrm{Tr}A^\dagger A + 2\sqrt{\det A^\dagger A}} \pm \sqrt{\mathrm{Tr}A^\dagger A - 2\sqrt{\det A^\dagger A}}}{2}\right)^2.\end{aligned} \tag{5.9}$$

where the contractions of H are given as

$$\partial H^* \partial H = \partial H^{*a}_i \partial H^a_i, \qquad H \cdot \partial H = \epsilon_{ij}\epsilon_{ab} H^a_i \partial H^b_j. \tag{5.14}$$

Denoting $\langle H_1^0 \rangle = \bar{v}_1$ and $\langle H_2^0 \rangle = \bar{v}_2$ ($\bar{v}_1$ and $\bar{v}_2$ are real and positive), we obtain

$$\frac{\langle K \rangle}{2} = \bar{v}_1 + \bar{v}_2. \tag{5.15}$$

The kinetic term of the neutral components is obtained as

$$\mathcal{L}_{\rm kin}^{\rm neutral} = 2(\bar{v}_1 \partial_\mu H_1^{0*} \partial^\mu H_1^0 + \bar{v}_2 \partial_\mu H_2^{0*} \partial^\mu H_2^0). \tag{5.16}$$

The kinetic normalized fields (h and H) are defined as

$$\sqrt{2\bar{v}_1} \mathrm{Re} H_1^0 = v_1 + \frac{1}{\sqrt{2}}(-h \sin\alpha + H \cos\alpha), \tag{5.17}$$

$$\sqrt{2\bar{v}_2} \mathrm{Re} H_2^0 = v_2 + \frac{1}{\sqrt{2}}(h \cos\alpha + H \sin\alpha). \tag{5.18}$$

From these definitions, we obtain

$$\bar{v}_1^3 = \frac{v_1^2}{2}, \qquad \bar{v}_2^3 = \frac{v_2^2}{2}. \tag{5.19}$$

The gauge boson mass term is obtained by replacing the derivative to covariant derivative in Eq. (5.13). We note that the last term in Eq. (5.13) does not contribute to the gauge boson mass due to $H \cdot \partial H = H_1 \cdot \partial H_2 - H_2 \cdot \partial H_1 = \partial(H_1 \cdot H_2)$. In order to extract the interaction between the physical Higgs h and gauge bosons, we pick up the real part of H^0 which generates the gauge boson masses:

$$\mathcal{L}_V = \frac{g^2}{2} W_\mu^+ W^{-\mu} \frac{K}{2}((H_1^0)^2 + (H_2^0)^2) \tag{5.20}$$

$$+ \frac{g^2 + g'^2}{4} Z_\mu Z^\mu \left(\frac{K}{2}((H_1^0)^2 + (H_2^0)^2) + \frac{2}{K}((H_1^0)^2 - (H_2^0)^2)^2 \right). \tag{5.21}$$

If $\langle H_1 \rangle \neq \langle H_2 \rangle$, the ρ parameter $\rho = M_W^2/(M_Z^2 \cos^2\theta_W)$ shifts from 1. Beware of the fact that VEVs of the kinetic normalized fields satisfy $M_Z^2 = \frac{g^2+g'^2}{2}(v_1^2 + v_2^2)$. The Z boson mass terms and the interaction to h terms can be obtained as

$$\mathcal{L}_Z = \frac{g^2 + g'^2}{2}((H_1^0)^3 + (H_2^0)^3) Z_\mu Z^\mu = \frac{M_Z^2}{2} Z_\mu Z^\mu \left(1 + 3\frac{h}{v}\sin(\beta - \alpha) + 3\frac{h^2}{v^2} + \cdots\right), \tag{5.22}$$

where $\tan\beta = v_2/v_1$[8]. It is interesting to compare this result with the two-Higgs doublet model:

$$\mathcal{L}_Z = \frac{g^2 + g'^2}{2}((H_1^0)^2 + (H_2^0)^2) Z_\mu Z^\mu = \frac{M_Z^2}{2} Z_\mu Z^\mu \left(1 + 2\frac{h}{v}\sin(\beta - \alpha) + \frac{h^2}{v^2}\right). \tag{5.23}$$

For the $hhZZ$ coupling, thus, we obtain $C_2^Z = 2$. For the h and W boson interaction terms, the expression is complicated to show for general $\tan\beta$, and thus we show the case $\tan\beta = 1$ in which the ρ parameter is 1:

$$\mathcal{L}_W = \frac{M_W^2}{2} W_\mu^+ W^{-\mu} \left(1 + 3\frac{h}{v}\sin\left(\frac{\pi}{4} - \alpha\right) + (2 - \sin 2\alpha)\frac{h^2}{v^2} + \cdots\right). \tag{5.24}$$

If we choose $C_1 = 0$ (to make the single Higgs production remain unchanged), we obtain $C_2^W = 8/9$.

VI. SUMMARY AND CONCLUSIONS

The discovery of the Higgs boson opens the new era of the particle physics. The experimental data support the prediction of the single Higgs production rate and decays to gauge bosons by the SM. The gluon fusion process is the dominant mechanism for the Higgs production at the LHC, while the vector boson fusion process is subdominant and starts to be observed in the latest analysis from both experiments. So, the gauge and Yukawa interactions for the single Higgs modes seem to be consistent with the SM. The decays to fermions (b and τ), while they have large errors, are consistent with the SM predictions. It is expected that the couplings for the single Higgs production can be measured more accurately for the LHC run after 2015. In addition to the single Higgs production, it is important to observe the pair Higgs production in order to reveal how the electroweak symmetry occurs by the Brout-Englert-Higgs mechanism.

The cross section of the Higgs pair production via gluon fusion in the SM is about 25–45 fb at the LHC. With such a low rate, it may be observed only after the measurement of couplings for single Higgs production, decays to gauge bosons and fermions become more accurate. If the pair Higgs production rate is enlarged compared to the SM, the process can be observed earlier. Thus, it is interesting to investigate the models, in which the pair production rate is enlarged. Indeed, the production rate is enlarged if there is a negative

[8]Contrary to the case of the minimal supersymmetric standard model, $\tan\beta = 1$ is allowed since the potential stabilization does not originate from the D-term potential.

contribution to the cubic Higgs coupling compared to the SM. More precise measurements of various couplings are expected at the ILC including the cubic coupling. If any deviation is observed, it is important to know what can disturb the measurement of the cubic Higgs coupling. In the case of only one Higgs doublet and the ρ parameter is fixed to be 1, all the deviations in the Higgs pair production from the SM are described by three parameters. One of the parameters is the cubic Higgs coupling, and other two are the hVV and $hhVV$ couplings. Even if the single Higgs production data turns out to be fully consistent with the SM prediction and therefore, the hVV coupling is fixed to comply with this fact, there is still enough room to modify the pair Higgs production rate substantially. The $hhVV$ coupling can be modified if the kinetic term is extended, and it can be related to the anomalous dimension of the Higgs field. Therefore, the importance is the character of the Higgs boson. For example, if the Higgs boson is a composite field, the $hhVV$ coupling is easily modified from the SM (but hVV can be also modified naively). We have investigated the dependency of the Higgs pair productions on the two parameters which describe deviations of the cubic Higgs coupling and $hhVV$ coupling from the SM values. These parameters are chosen in the case where the hVV coupling is the same as the SM, keeping in mind that it will have been measured more accurately when the pair production starts to be observed. It is important to observe various processes (gluon fusion, vector boson fusion, and double Higgsstrahlung) at the LHC and ILC, in order to determine the three couplings. In this paper we have exhibited the parametric dependency of those processes on the three couplings.

The pair Higgs production rate at the LHC is enlarged if there is a negative contribution for the deviation from the SM in the cubic Higgs coupling. The negative contribution is naturally generated if the Higgs potential is the quadratic mass term plus a runaway potential, namely, a repulsive effect from the origin of the Higgs configuration. It is known that such behavior can be generated by instanton effects. Therefore, in such a system, the symmetry breaking happens by the nonperturbative effects in gauge theories. We construct a model in which the runaway piece exists in the Higgs potential, and thus the pair Higgs production rate is enlarged. We have also included the case where the kinetic term of the Higgs field is modified from the SM. In this case we have shown that the pair Higgs production can be enlarged at the LHC and ILC compared to the SM. This is especially important for ILC since a factor of few enhancements would be clearly measurable. Such modifications can be tested at the LHC and ILC by observing the various pair Higgs production processes and if observed may lead us to discover a mechanism behind the electroweak symmetry breaking.

ACKNOWLEDGEMENTS

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APPENDIX A: DERIVATION OF THE KÄHLER METRIC

In this appendix, we show the calculation of the scalar potential and kinetic term from the Kähler potential

$$K = 2\sqrt{Z + 2\sqrt{DD^*}}, \tag{A1}$$

where

$$Z = \sum_{i=1}^{4} |a_i|^2, \qquad D = a_1 a_4 - a_2 a_3. \tag{A2}$$

We will identify the Kähler coordinates as

$$\begin{pmatrix} a_1 & a_3 \\ a_2 & a_4 \end{pmatrix} = \begin{pmatrix} H_1^0 & H_2^+ \\ H_1^- & H_2^0 \end{pmatrix}. \tag{A3}$$

We obtain the Kähler metric as[9]

$$K_{ij^*} = \frac{1}{2K}(K^2)_{ij^*} - \frac{1}{4K^3}(K^2)_i (K^2)_{j^*}, \tag{A4}$$

where

$$(K^2)_i = 4\left(Z_i + D_i \sqrt{\frac{D^*}{D}}\right),$$
$$(K^2)_{ij^*} = 4\left(\delta_{ij^*} + \frac{D_i D^*_{j^*}}{2\sqrt{DD^*}}\right). \tag{A5}$$

As a formula, for a matrix,

$$M_{ij} = I_{ij} + X_i \bar{X}_j - Y_i \bar{Y}_j, \tag{A6}$$

[9]As a common notation to describe the Kähler geometry, we denote $K_i = \partial K / \partial a_i$, for example.

where I is an identity matrix, we obtain

$$\det M = 1 + X_i\bar{X}_i - Y_i\bar{Y}_i - (X_i\bar{X}_i)(Y_i\bar{Y}_i) + (X_i\bar{Y}_i)(\bar{X}_iY_i), \tag{A7}$$

$$\bar{X}_iM^{-1}_{ij}X_j = \frac{1}{\det M}(X_i\bar{X}_i - (X_i\bar{X}_i)(Y_i\bar{Y}_i) + (X_i\bar{Y}_i)(\bar{X}_iY_i)). \tag{A8}$$

Choosing

$$X_i = \frac{D_i}{\sqrt{2D}}, \qquad Y_i = \frac{\sqrt{2}}{K}\left(Z_i + D_i\frac{D^*}{D}\right), \tag{A9}$$

we obtain

$$X_i\bar{X}_i = \frac{Z}{2\sqrt{DD^*}}, \qquad Y_i\bar{Y}_i = 1, \qquad X_i\bar{Y}_i = \frac{K}{4\sqrt{D^*}}. \tag{A10}$$

Because $Y_i\bar{Y}_i = 1$, the formula obeys

$$\bar{X}_iM^{-1}_{ij}X_j = 1. \tag{A11}$$

Applying the formula to the Kähler metric[10], we obtain

$$D_iK^{ij^*}D^*_{j^*} = K\sqrt{DD^*}. \tag{A12}$$

When the superpotential is a function of D,

$$W = f(D), \tag{A13}$$

we obtain the scalar potential as

$$V = W_iK^{ij^*}W^*_{j^*} = K\sqrt{DD^*}f'(D)f'(D^*). \tag{A14}$$

The kinetic term can be obtained using the following formula:

$$\begin{aligned}(\det M)\bar{A}_iM^{-1}_{ij}A_j = {}& (\bar{A}A) + (\bar{A}A)(\bar{X}X) - (\bar{A}X)(\bar{X}A) \\ & - (\bar{A}A)(\bar{Y}Y) + (\bar{A}Y)(\bar{Y}A) \\ & -(\bar{A}A)(\bar{X}X)(\bar{Y}Y) + (\bar{A}A)(\bar{X}Y)(\bar{Y}X) \\ & +(\bar{A}X)(\bar{X}A)(\bar{Y}Y) - (\bar{A}X)(\bar{X}Y)(\bar{Y}A) \\ & + (\bar{A}Y)(\bar{X}X)(\bar{Y}A) - (\bar{A}Y)(\bar{X}A)(\bar{Y}X),\end{aligned} \tag{A15}$$

where $(\bar{A}A) = \bar{A}_iA_i$, for example.

[10] $K_{ij^*}K^{jk^*} = \delta_i^{k^*}$.

APPENDIX B: GENERAL POTENTIAL FOR TWO-HIGGS DOUBLETS

In this section, we describe the Higgs self-coupling from the general scalar potential in 2HDM. The general scalar potential is a function[11] of $|H_1|^2$, $|H_2|^2$ and $H_1 \cdot H_2$.

In order to make the following calculation simple, it is convenient to define linear combinations of the Higgs doublet:

$$\begin{aligned}\Phi_1 &= H_1\cos\beta + \hat{H}_2\sin\beta, \\ \Phi_2 &= -H_1\sin\beta + \hat{H}_2\cos\beta,\end{aligned} \tag{B2}$$

where $\hat{H} = i\sigma_2H^*$, so that the VEV of Φ_2^0 is zero by definition. We define

$$\begin{aligned}x &= |\Phi_1|^2, \qquad y = |\Phi_2|^2, \\ z &= \hat{\Phi}_2\cdot\Phi_1, \qquad \bar{z} = \hat{\Phi}_1\cdot\Phi_2,\end{aligned} \tag{B3}$$

and the general potential is a function $V(x,y,z,\bar{z})$. The stationary conditions are $V_x = V_z = V_{\bar{z}} = 0$, where V_x denotes a partial derivative by x for example. We denote

$$\Phi_1 = \begin{pmatrix}\frac{v+\phi_1+i\chi}{\sqrt{2}} \\ \chi^-\end{pmatrix}, \qquad \Phi_2 = \begin{pmatrix}\frac{\phi_2+iA}{\sqrt{2}} \\ H^-\end{pmatrix}. \tag{B4}$$

The would-be-NG bosons are χ and χ^-, and ϕ_1, ϕ_2, A and H^- are physical Higgs fields. The ϕ_1 and ϕ_2 fields are mixed in this basis. Expanding the potential around the VEV, $\langle x\rangle = v^2/2$, we obtain the mass term of the neutral Higgs bosons:

$$\frac{1}{2}(\phi_1 \quad \phi_2)\begin{pmatrix} v^2V_{xx} & \frac{v^2}{2}(V_{xz}+V_{x\bar{z}}) \\ \frac{v^2}{2}(V_{xz}+V_{x\bar{z}}) & V_y + \frac{1}{4}v^2(V_{zz}+V_{\bar{z}\bar{z}}+2V_{z\bar{z}})\end{pmatrix} \times \begin{pmatrix}\phi_1 \\ \phi_2\end{pmatrix}. \tag{B5}$$

The mixing angle of H_1^0 and H_2^0 is defined as α, and thus,

$$\begin{aligned}\begin{pmatrix}H \\ h\end{pmatrix} &= \begin{pmatrix}\cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha\end{pmatrix}\begin{pmatrix}\sqrt{2}(H_1^0 - v_1) \\ \sqrt{2}(H_2^0 - v_2)\end{pmatrix} \\ &= \begin{pmatrix}\cos(\beta-\alpha) & -\sin(\beta-\alpha) \\ \sin(\beta-\alpha) & \cos(\beta-\alpha)\end{pmatrix}\begin{pmatrix}\phi_1 \\ \phi_2\end{pmatrix}.\end{aligned} \tag{B6}$$

If V_y is large, $\beta-\alpha$ mixing is closed to $\pi/2$, and ϕ_1 is roughly the lightest Higgs boson h, and $m_h^2 \simeq v^2V_{xx}$.

[11] The other $SU(2)$ invariants are a function of $|H_1|^2$, $|H_2|^2$ and $H_1\cdot H_2$. For example,

$$H_1^aH_2^b(H_1^*)_b(H_2^*)_a = |H_1|^2 + |H_2|^2 - |H_1\cdot H_2|^2. \tag{B1}$$

The mass of CP odd Higgs boson A and the charged Higgs mass $m_{H^+}^2$ can be also obtained:

$$m_A^2 = V_y + \frac{1}{4}v^2(-V_{zz} - V_{\bar{z}\bar{z}} + 2V_{z\bar{z}}), \tag{B7}$$

$$m_{H^+}^2 = V_y. \tag{B8}$$

The following expressions are useful to calculate the mass spectrum from the general potential:

$$|H_1|^2 = x\cos^2\beta + y\sin^2\beta - \frac{1}{2}(z+\bar{z})\sin 2\beta, \tag{B9}$$

$$|H_2|^2 = x\sin^2\beta + y\cos^2\beta + \frac{1}{2}(z+\bar{z})\sin 2\beta, \tag{B10}$$

$$H_1 \cdot H_2 = \frac{1}{2}(x-y)\sin 2\beta + z\cos^2\beta - \bar{z}\sin^2\beta. \tag{B11}$$

In the two-Higgs doublet model, the cubic coupling can be modified from m_h^2/v^2 if $\cos(\beta-\alpha) \neq 0$ even in the renormalizable model. Surely, the lightest Higgs and vector bosons hVV coupling is proportional to $\sin(\beta-\alpha)$ and a sizable value of $\cos(\beta-\alpha) \neq 0$ can modify $h \to WW$ and $h \to ZZ$ decays. If we neglect the $\cos(\beta-\alpha)$ contribution, the modification from the cubic coupling and $hhVV$ coupling is given by V_{xxx}.

The physical mass parameters are related to the second derivatives of V as follows:

$$v^2 V_{xx} = s^2 m_h^2 + c^2 m_H^2, \tag{B12}$$

$$v^2(V_{xz} + V_{x\bar{z}}) = 2sc(m_h^2 - m_H^2), \tag{B13}$$

$$\frac{v^2}{4}(V_{zz} + V_{\bar{z}\bar{z}} + 2V_{z\bar{z}}) = c^2 m_h^2 + s^2 m_H^2 - m_{H^+}^2, \tag{B14}$$

where $s = \sin(\beta-\alpha)$, and $c = \cos(\beta-\alpha)$.

The cubic hhh coupling is written as

$$\begin{aligned}\lambda_{hhh} = {}& \frac{s(1+c^2)}{2v}m_h^2 - \frac{c^2 s}{v}m_{H^+}^2 + \frac{v}{4}c^3(V_{yz} + V_{y\bar{z}}) \\ & + \frac{v}{2}c^2 s V_{xy} + \frac{v^3}{6}s^3 V_{xxx} + \frac{v^3}{4}cs^2(V_{xxz} + V_{xx\bar{z}}) \\ & + \frac{v^3}{8}c^2 s(V_{xzz} + V_{x\bar{z}\bar{z}} + 2V_{xz\bar{z}}) \\ & + \frac{v^3}{48}c^3(V_{zzz} + 3V_{zz\bar{z}} + 3V_{z\bar{z}\bar{z}} + V_{\bar{z}\bar{z}\bar{z}}).\end{aligned} \tag{B15}$$

The hhH coupling also affects the pair Higgs production if $\cos(\beta-\alpha)$ is not small and H is not very heavy. The hhH coupling is

$$\begin{aligned}\lambda_{hhH} = {}& \frac{c^3}{v}m_h^2 - \frac{cs^2}{2v}m_H^2 - \frac{c(c^2-2s^2)}{v}m_{H^+}^2 \\ & - \frac{3v}{4}c^2 s(V_{yz} + V_{y\bar{z}}) + \frac{v}{2}c(c^2-2s^2)V_{xy} \\ & + \frac{v^3}{2}cs^2 V_{xxx} + \frac{v^3}{4}s(2c^2-s^2)(V_{xxz} + V_{xx\bar{z}}) \\ & + \frac{v^3}{8}c(c^2-2s^2)(V_{xzz} + V_{x\bar{z}\bar{z}} + 2V_{xz\bar{z}}) \\ & - \frac{v^3}{16}c^2 s(V_{zzz} + 3V_{zz\bar{z}} + 3V_{z\bar{z}\bar{z}} + V_{\bar{z}\bar{z}\bar{z}}).\end{aligned} \tag{B16}$$

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