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EXISTENCE OF PARETO EQUILIBRIA FOR MULTIOBJECTIVE GAMES WITHOUT COMPACTNESS

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Abstract. In this paper, we investigate the existence of Pareto and weak Pareto equilibria for multiobjective games without compactness. By employing an existence theorem of Pareto equilibria due to Yu and Yuan([10]), several existence theorems of Pareto and weak Pareto equilibria for the multiobjective games are established in a similar way to Flores-Bázan([4]).

1. Introduction

The concept of an equilibrium point for an n-person game was first introduced by Nash, who established the existence of the equilibrium point under certain assumptions. Since that time, the Nash equilibrium problem for n-person games have been intensively studied and extended by many authors.

Recently, the study of the existence of Pareto equilibria in game theory with vector payoffs has been focused by many authors, for example, see [3], [11]. As one of the reasons is that multicriteria models can be better applied to real-world situations. The motivation for the study of multicriteria models can be found in [2], [12]. The existence of Pareto equilibria is one of the basic problems in the game theory. In order to guarantee the existence of Pareto equilibria of the multiobjective games, some sufficient conditions have been given by several authors. Almost of such sufficient conditions are closely related to compactness.

In this paper, we investigate the existence of Pareto and weak Pareto equilibria for multiobjective games without compactness. By employing an existence theorem of Pareto equilibria due to Yu and Yuan([10]), several existence theorems of Pareto and weak Pareto equilibria for the multiobjective games without compactness are established in a similar way to Flores-Bázan([4]).

2. Notation and Preliminaries

In this paper, we shall consider a finite-players game with multicriteria in its strategic form $G := (X_i, F^i)_{i\in N}$, where $N := \{1, 2, \ldots, n\}$. For each $i \in N$, X_i is the set of strategies in $\mathbb{R}^{k_i}$ for the player i, and each F^i is mapping from

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$X := \Pi_{i\in N} X_i$ into $\mathbb{R}^{k_i}$, which is called the payoff function of the i's player, here k_i is a positive integer. If a strategy combination $x := (x^1, x^2, \dots, x^n) \in X$ is played, each player i gets his/her payoff $F^i(x) = (f_1^i(x), f_2^i(x), \dots, f_{k_i}^i(x))$, which consists of noncommensurable outcomes. Also, we assume that this game is a noncooperative game, that is, absolutely no preplay communication is permitted between the players, and so the players act as free agents, each wanting only to maximize his/her own payoff according to his/her preference.

For the games with vector payoff functions, in general, there does not exist a strategy combination $\bar{x} \in X$ to minimize all f_j^is for each player (see [9]). Therefore we need to define solution concepts for multicriteria games in the same way [8]. For each given $m \in \mathbb{N}$, we denote by $\mathbb{R}_+^m$ the nonnegative orthant of $\mathbb{R}^m$, that is,

$$\mathbb{R}_+^m = \{u = (u^1, u^2, \dots, u^m) \in \mathbb{R}^m \mid u^1, u^2, \dots, u^m \geq 0\},$$

also the interior of the nonnegative orthant $\mathbb{R}^m$ is a nonempty with the usual topology, that is,

$$\mathrm{int}\mathbb{R}_+^m = \{u = (u^1, u^2, \dots, u^m) \in \mathbb{R}^m \mid u^1, u^2, \dots, u^m > 0\}.$$

For each $i \in N$, denote

$$X_{\hat{i}} := \Pi_{j\in N\setminus\{i\}} X_j.$$

If $x = (x^1, x^2, \dots, x^n) \in X$, we write

$$x^{\hat{i}} := (x^1, x^2, \dots, x^{i-1}, x^{i+1}, \dots, x^n) \in X_{\hat{i}}.$$

If $x^i \in X_i$, we use $(x^{\hat{i}}, x^i)$ to denote $y = (y^1, y^2, \dots, y^n) \in X$ such that $y^i = x^i$ and $y^{\hat{i}} = x^{\hat{i}}$. Now we give the following solution concepts.

Definition 2.1. ([8]) A strategy combination $\bar{x} \in X$ is said to be a Pareto equilibrium (respectively, a weak Pareto equilibrium) of the game $G = (X_i, F^i)_{i\in N}$ if for each player i, there is no strategy $x^i \in X_i$ such that

$$F^i(\bar{x}) - F^i(\bar{x}^{\hat{i}}, x^i) \in \mathbb{R}_+^{k_i} \setminus \{0\} \text{ (respectively, } F^i(\bar{x}) - F^i(\bar{x}^{\hat{i}}, x^i) \in \mathrm{int}\mathbb{R}_+^{k_i}).$$

It is clear that any Pareto equilibrium is a weak Pareto equilibrium, however, the converse is not true.

Definition 2.2. ([8]) A strategy combination $\bar{x} \in X$ is said to be a weight Nash-equilibrium respect to weight vector $W = (W^1, W^2, \dots, W^n)$ of the game $G = (X_i, F^i)_{i\in N}$ if for each player $i \in N$,

(1) $W^i \in \mathbb{R}_+^{k_i} \setminus \{0\}$, and
(2) $\langle W^i, F^i(\bar{x})\rangle \leq \left\langle W^i, F^i(\bar{x}^{\hat{i}}, x^i)\right\rangle$ for each $x_i \in X_i$.

Furthermore, we prepare several concepts concerned with our main results. For any closed set K in $\mathbb{R}^m$, we define the recession cone K^∞ of K as the closed set

$$K^\infty = \{x \in \mathbb{R}^m \mid \exists t_n \downarrow 0, \exists x_n \in K \text{ s.t. } t_n x_n \to x\}.$$

If K is convex, it is well known(see [7]) that for any $x_0 \in K$,

$$K^\infty = \{x \in \mathbb{R}^m \mid x_0 + tx \in K, \forall t > 0\}.$$

Next, we define quasiconvexity and semi-strictly quasiconvexity of functions.

Definition 2.3. Let $C \subseteq \mathbb{R}^n$ be a convex set. A function $f : C \to \mathbb{R}$ is said to be

(1) quasiconvex if $f(tx + (1-t)y) \leq \max\{f(x), f(y)\}$ for all x, $y \in C$ and all $t \in [0, 1]$,

(2) semi-strictly quasiconvex if given any u, $v \in C$, $f(u) \neq f(v)$, one has $f(z) < \max\{f(u), f(v)\}$ for all $z \in (u, v)$.

All semi-strictly quasiconvex functions are not quasiconvex. But Karamardian([5]) showed Proposition 2.4.

Proposition 2.4. *([5]) Let $C \subseteq \mathbb{R}^n$ be a convex set and a function $f : C \to \mathbb{R}$ is semi-strictly quasiconvex and lower semicontinuous. Then, f is quasiconvex.*

3. Main results

In this section, we give some results of Pareto and weak Pareto equilibria for a multiobjective game.

Lemma 3.1. *([8]) Let $G = (X_i, F^i)_{i\in N}$ be a multiobjective game. Then, each weight Nash-equilibrium $\bar{x} \in X$ with a weight*

$$W = (W^1, W^2, \ldots, W^n) \in (\mathbb{R}^{k_1}_+ \setminus \{0\}) \times (\mathbb{R}^{k_2}_+ \setminus \{0\}) \times \cdots \times (\mathbb{R}^{k_n}_+ \setminus \{0\})$$

is a weak Pareto equilibrium of the game G. Also each weight Nash-equilibrium $\bar{x} \in X$ with a weight

$$W = (W^1, W^2, \ldots, W^n) \in \mathrm{int}\mathbb{R}^{k_1}_+ \times \mathrm{int}\mathbb{R}^{k_2}_+ \times \cdots \times \mathrm{int}\mathbb{R}^{k_n}_+$$

is a Pareto equilibrium of the game G.

Using Lemma 3.1, Yu and Yuan([10]) showed Theorem 3.2.

Theorem 3.2. *([10]) Let $G = (X_i, F^i)_{i\in N}$ be a multiobjective game where each X_i is a nonempty compact and convex subset of a Hausdorff topological vector space E_i. If there is a weight combination $W = (W^1, W^2, \ldots, W^n)$ with $W^i \in \mathbb{R}^{k_i}_+ \setminus \{0\}$ such that the following are satisfied: for each $i \in N$,*

(1) the function $(x, y) \mapsto \langle W^i, F^i(\bar{x}^{\hat{i}}, y^i)\rangle$ is lower semicontinuous,

(2) for each fixed $y \in X$, the mapping $x \mapsto \langle W^i, F^i(\bar{x}^{\hat{i}}, y^i)\rangle$ is upper semicontinuous, and

(3) for each fixed $x \in X$, the mapping $y \mapsto \langle W^i, F^i(\bar{x}^{\hat{i}}, y^i)\rangle$ is quasiconvex.

Then G has at least one weak Pareto equilibrium. Furthermore, if $W^i \in \mathrm{int}\mathbb{R}^{k_i}_+$ for all $i \in N$, then G has at least one Pareto equilibrium.

Now, we consider the existence of Pareto and weak Pareto equilibria for multiobjective games without compactness of X in a similar way to Flores-Bázan([4]). Henceforth $\|\cdot\|$ will denote any norm in $\Pi_{j\in N}\mathbb{R}^{k_j}$. We will need the following non-coercive condition $(*)$ to be satisfied by the payoff functions F^i.

$(*)$ for any sequence $\{x_n\}$ in X satisfying:

(i) $\|x_n\| \to +\infty$, and

(ii) for all $y \in X$, there exists $n_y \in \mathbb{N}$ such that $F^i(x_n) - F^i(x_n^{\hat{i}}, y^i) \notin \mathrm{int}\mathbb{R}^{k_i}_+$ for all $n \geq n_y$ and for all $i \in N$, there exist $u \in X$ and $\bar{n} \in \mathbb{N}$ such that $\|u\| < \|x_{\bar{n}}\|$ and $F^i(x_{\bar{n}}) - F^i(x_{\bar{n}}^{\hat{\imath}}, u^i) \in \mathbb{R}_+^{k_i}$ for all $i \in N$.

Theorem 3.3. *Let $G = (X_i, F^i)_{i \in N}$ be a multiobjective game where each X_i is a nonempty closed and convex subset of $\mathbb{R}^{k_i}$. Assume that the following conditions are satisfied for a weight combination $W = (W^1, W^2, \ldots, W^n)$ with $W^i \in \mathbb{R}_+^{k_i} \setminus \{0\}$: for each $i \in N$,*

1. *the function $(x, y) \mapsto \left\langle W^i, F^i(x^{\hat{\imath}}, y^i) \right\rangle$ is lower semicontinuous,*
2. *for each fixed $y \in X$, the mapping $x \mapsto \left\langle W^i, F^i(x^{\hat{\imath}}, y^i) \right\rangle$ is upper semicontinuous, and*
3. *for each fixed $x \in X$, the mapping $y \mapsto \left\langle W^i, F^i(x^{\hat{\imath}}, y^i) \right\rangle$ is semi-strictly quasiconvex.*

If, in addition F^i satisfies $(*)$, then G has at least one weak Pareto equilibrium. Moreover, if $W^i \in \mathrm{int}\mathbb{R}_+^{k_i}$ for all $i \in N$, then G has at least one Pareto equilibrium.

Proof. For each $i \in N$, define a mapping $g^i : X \times X \to \mathbb{R}$ by

$$g^i(x, y) = \left\langle W^i, F^i(x^{\hat{\imath}}, y^i) \right\rangle, (x, y) \in X \times X.$$

For every $n \in \mathbb{N}$, put $A_n := \{x \in X \mid \|x\| \leq n\}$. We may suppose, without loss of generality, that $A_n \neq \emptyset$ for all $n \in \mathbb{N}$. Let us consider the problem:

(P_n) find a weight Nash-equilibrium on A_n respect to W of the game G.

For each $i \in N$, we can check the restriction $g^i|_{A_n}$ satisfies (1), (2), and (3). Also $g^i|_{A_n}(x, \cdot)$ is also quasiconvex from (1), (3), and Proposition 2.4. By using Theorem 3.2, problem (P_n) has a weight Nash-equilibrium on A_n respect to the weight vector W of the game G, say $x_n \in A_n$ for all $n \in \mathbb{N}$. If $\|x_n\| < n$ for some $n \in \mathbb{N}$, then x_n is also weight Nash-equilibrium respect to the weight vector W of the game G. If not, there exist $y \in X$ and $i \in N$ such that $g^i(x_n, x_n) > g^i(x_n, y)$. Obviously $\|y\| > n$. Then we can find $\alpha \in (0, 1)$ satisfying $\|\alpha x_n - (1 - \alpha)y\| < n$ and put $z = \alpha x_n - (1 - \alpha)y$. Clearly $g^i(x_n, x_n) \leq g^i(x_n, z)$. Since $g^i(x_n, \cdot)$ is semi-strictly quasiconvex, we have

$$g^i(x_n, z) = g^i(x_n, \alpha x_n + (1 - \alpha)y) < g^i(x_n, x_n).$$

This is a contradiction. Hence, by Lemma 3.1, x_n is a weak Pareto equilibrium of the game G.

Now we may assume $\|x_n\| = n$ for all $n \in \mathbb{N}$. For each $y \in X$, choose $n_y \in \mathbb{N}$ satisfying $n_y > \|y\|$. For each $n \in \mathbb{N}$ with $n \geq n_y$ and $i \in N$, we have $\langle W^i, F^i(x_n) \rangle \leq \left\langle W^i, F^i(x_n^{\hat{\imath}}, y^i) \right\rangle$ since x_n is a solution of (P_n), therefore $F^i(x_n) - F^i(x_n^{\hat{\imath}}, y^i) \notin \mathrm{int}\mathbb{R}_+^{k_i}$. This shows that the sequence $\{x_n\}$ satisfies conditions (i) and (ii) of $(*)$. By the assumption, there exist $u \in X$ and $\bar{n} \in \mathbb{N}$ such that $\|u\| < \|x_{\bar{n}}\|$ and $F^i(x_{\bar{n}}) - F^i(x_{\bar{n}}^{\hat{\imath}}, u^i) \in \mathbb{R}_+^{k_i}$ for all $i \in N$. We have that $x_{\bar{n}}$ is also weight Nash-equilibrium respect to the weight vector W of the game G. If not, there exist $y \in X$ and $i \in N$ such that $g^i(x_{\bar{n}}, x_{\bar{n}}) > g^i(x_{\bar{n}}, y)$. Obviously $\|y\| > \bar{n}$, and so $\|u\| < \|x_{\bar{n}}\| = \bar{n}$. Then we can find $\alpha \in (0, 1)$ satisfying $\|\alpha u - (1 - \alpha)y\| < \bar{n}$, put $z = \alpha u - (1 - \alpha)y$. Clearly $g^i(x_{\bar{n}}, x_{\bar{n}}) \leq g^i(x_{\bar{n}}, z)$. Since $F^i(x_{\bar{n}}) - F^i(x_{\bar{n}}^{\hat{\imath}}, u^i) \in \mathbb{R}_+^{k_i}$, we have $g^i(x_{\bar{n}}, x_{\bar{n}}) \geq g^i(x_{\bar{n}}, u)$. Now, we check two cases:

$g^i(x_{\bar{n}}, x_{\bar{n}}) = g^i(x_{\bar{n}}, u)$, and $g^i(x_{\bar{n}}, x_{\bar{n}}) > g^i(x_{\bar{n}}, u)$. In the former case, $g^i(x_{\bar{n}}, \cdot)$ is semi-strictly quasiconvex, we have

$$\begin{aligned} g^i(x_{\bar{n}}, z) &= g^i(x_{\bar{n}}, \alpha u + (1-\alpha)y) \\ &< g^i(x_{\bar{n}}, u) \\ &= g^i(x_{\bar{n}}, x_{\bar{n}}). \end{aligned}$$

This is a contradiction. In the latter case, $g^i(x_{\bar{n}}, \cdot)$ is also quasiconvex, we have

$$\begin{aligned} g^i(x_{\bar{n}}, z) &= g^i(x_{\bar{n}}, \alpha u + (1-\alpha)y) \\ &\leq \max\{g^i(x_{\bar{n}}, u), g^i(x_{\bar{n}}, y)\} \\ &< g^i(x_{\bar{n}}, x_{\bar{n}}). \end{aligned}$$

This is also a contradiction. Furthermore, using Lemma 3.1, $x_{\bar{n}}$ is a weak Pareto equilibrium of the game G. When $W^i \in \text{int}\mathbb{R}_+^{k_i}$ for each $i \in N$, x_n and $x_{\bar{n}}$ in the proof above must be Pareto equilibria of the game G. We complete the proof. □

Next, we assume the following hypothesis (H_1):

(H_1) there exist functions $\bar{F}^1, \bar{F}^2, \ldots, \bar{F}^n : X \to \mathbb{R}$ such that for each $i \in N$, $F^i(x_n^{\hat{i}}, y_n^i) \to \bar{F}^i(y)$ whenever $\|x_n\| \to +\infty$ and $y_n \to y$.

Let

$$R = \bigcap_{y \in X} \left\{ v \in X^\infty \mid \forall \lambda > 0, \forall i \in N, \left\langle W^i, \bar{F}^i(\lambda v + y) \right\rangle \leq \left\langle W^i, \bar{F}^i(y) \right\rangle \right\},$$

and we give the following weaken non-coercive condition $(**)$:

$(**)$ for any sequence $\{x_n\}$ in X satisfying:

(i) $\|x_n\| \to +\infty$, $\dfrac{x_n}{\|x_n\|} \to v$ for some $v \in R$, and

(ii) for all $y \in X$, there exists $n_y \in \mathbb{N}$ such that $F^i(x_n) - F^i(x_n^{\hat{i}}, y^i) \notin \text{int}\mathbb{R}_+^{k_i}$ for all $n \geq n_y$ and for all $i \in N$,

there exist $u \in X$ and $\bar{n} \in \mathbb{N}$ such that $\|u\| < \|x_{\bar{n}}\|$ and $F^i(x_{\bar{n}}) - F^i(x_{\bar{n}}^{\hat{i}}, u^i) \in \mathbb{R}_+^{k_i}$ for all $i \in N$.

Theorem 3.4. *Let $G = (X_i, F^i)_{i \in N}$ be a multiobjective game where each X_i is a nonempty closed and convex subset of $\mathbb{R}^{k_i}$. Assume that the following conditions are satisfied for a weight combination $W = (W^1, W^2, \ldots, W^n)$ with $W^i \in \mathbb{R}_+^{k_i} \setminus \{0\}$: for each $i \in N$,*

1. *the function $(x, y) \mapsto \left\langle W^i, F^i(x^{\hat{i}}, y^i) \right\rangle$ is lower semicontinuous,*
2. *for each fixed $y \in X$, the mapping $x \mapsto \left\langle W^i, F^i(x^{\hat{i}}, y^i) \right\rangle$ is upper semicontinuous, and*
3. *for each fixed $x \in X$, the mapping $y \mapsto \left\langle W^i, F^i(x^{\hat{i}}, y^i) \right\rangle$ is semi-strictly quasiconvex.*

In addition, F^i satisfies (H_1) and $(**)$. Then G has at least one weak Pareto equilibrium. Moreover, if $W^i \in \text{int}\mathbb{R}_+^{k_i}$ for all $i \in N$, then G has at least one Pareto equilibrium.

Proof. In the same way to the proof of Theorem 3.3, we have $\langle W^i, F^i(x_n)\rangle \leq \left\langle W^i, F^i(x_n^{\hat{i}}, y^i)\right\rangle$ for each $i \in N$. We may suppose, without loss of generality, that $\frac{x_n}{\|x_n\|} \to v$, $v \neq 0$. Then $v \in X^\infty$. For each $y \in X$, $\lambda > 0$, $i \in N$, and $n \in \mathbb{N}$ sufficient large, since $g^i(x_n, \cdot)$ is also quasiconvex, we have

$$\begin{aligned} g^i\left(x_n, \frac{\lambda}{\|x_n\|}x_n + \left(1 - \frac{\lambda}{\|x_n\|}\right)y\right) &\leq \max\{g^i(x_n, x_n), g^i(x_n, y)\} \\ &\leq g^i(x_n, y), \end{aligned}$$

or

$$\left\langle W^i, F^i\left(x_n, \frac{\lambda}{\|x_n\|}x_n + \left(1 - \frac{\lambda}{\|x_n\|}\right)y\right)\right\rangle \leq \left\langle W^i, F^i(x_n^{\hat{i}}, y^i)\right\rangle.$$

¿From (H_1), as $n \to +\infty$, we have

$$\left\langle W^i, \bar{F}^i(\lambda v + y)\right\rangle \leq \left\langle W^i, \bar{F}^i(y)\right\rangle,$$

for each $i \in N$, and then $v \in R$. The rest is same to the proof of Theorem 3.3. □

Moreover, we can replace (H_1) to more general hypothesis (H_2):

(H_2) there exist functions $\bar{F}^1, \bar{F}^2, \ldots, \bar{F}^n : X \to \mathbb{R}$ and $\varphi^1, \varphi^2, \ldots, \varphi^n : X \to (0, +\infty)$ such that

$$\text{for each } i \in N,\ \frac{F^i(x_n^{\hat{i}}, y_n^i)}{\varphi^i(x_n)} \to \bar{F}^i(y) \text{ whenever } \|x_n\| \to +\infty \text{ and } y_n \to y,$$

and redefine R in the same way, that is,

$$R = \bigcap_{y \in X} \left\{v \in X^\infty \mid \forall\lambda > 0, \forall i \in N, \left\langle W^i, \bar{F}^i(\lambda v + y)\right\rangle \leq \left\langle W^i, \bar{F}^i(y)\right\rangle\right\}.$$

Theorem 3.5. *Let $G = (X_i, F^i)_{i \in N}$ be a multiobjective game where each X_i is a nonempty closed and convex subset of $\mathbb{R}^{k_i}$. Assume that the following conditions are satisfied for a weight combination $W = (W^1, W^2, \ldots, W^n)$ with $W^i \in \mathbb{R}_+^{k_i} \setminus \{0\}$: for each $i \in N$,*

1. *the function $(x, y) \mapsto \left\langle W^i, F^i(x^{\hat{i}}, y^i)\right\rangle$ is lower semicontinuous,*
2. *for each fixed $y \in X$, the mapping $x \mapsto \left\langle W^i, F^i(x^{\hat{i}}, y^i)\right\rangle$ is upper semicontinuous, and*
3. *for each fixed $x \in X$, the mapping $y \mapsto \left\langle W^i, F^i(x^{\hat{i}}, y^i)\right\rangle$ is semi-strictly quasiconvex.*

In addition, F^i satisfies (H_2) and $(**)$. Then G has at least one weak Pareto equilibrium. Moreover, if $W^i \in \mathrm{int}\mathbb{R}_+^{k_i}$ for all $i \in N$, then G has at least one Pareto equilibrium.

The proof is similar to Theorem 3.4, and omitted.

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