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Note on the Hyperfine Mass-Splittings of Baryonium States

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ABSTRACT : Under the assumption that the chromomagnetic interaction between quarks is dominated by the one gluon exchange, the hyperfine mass-splittings of baryonium states are discussed in the framework of quantum chromodynamics.

At present the most favorite candidate for the theory describing the strong interaction physics may be the theory of qnantum chromodynamics. Here the forces among the colored quarks are generated by the exchange colored vector gluons coupled to quarks in a gauge-invariant manner. By this QCD picture of hadrons many features observed in the hadron mass spectrum can be understood, at least qualitatively. For example, those are the facts that the S-state $J^{PC}=1^{--}$ vector mesons (ρ, ω, ϕ, ψ,....) are heavier than their pseudoscalar partners (π, η, η', ...), that the $=3/2$ baryon resonances (Σ, Δ,...) are heavier than the $J=1/2$ baryons (p, n,...), and that $M(\Lambda)<M(\Sigma)$. Those mass differences are usualy called as "hyperfine splittings" and can be understood as the effects due to chromomagnetic forces.

Recently narrow resonances called baryoniums are discovered in the $p\bar{p}$ total cross-sections and produced in backward $\pi^- p$ scattering. There are also indications for several narrow baryonium resonances below $p\bar{p}$ threshold. All these states are summarized in Table I [1]. The quark model is very convincing in the classification of hadrons and in describing their general features. The baryoniums give a possibility of studying a new type of family of quark states $qq\bar{q}\bar{q}$.

In this note we will examine the hyperfine mass-splittings of S-wave $qq\bar{q}\bar{q}$ states in QCD with the assumption that the chromomagnetic interaction between quarks is dominated by the one gluon exchange. Narrowness of the width of the $qq\bar{q}\bar{q}$ states will be also discussed briefly.

II. BARYOMIUM STATES IN QCD

The observed narrow baryonium states are not yet proven to be $qq\bar{q}\bar{q}$ states rather than, for example, $B\bar{B}$ bound states. Although not so convincing, they are currently regarded as the candidates for the long-sought diquark-antidiquark systems.

Consider then a system with two quarks and two antiquarks bound together. Each quark is a triplet in color $SU(3)_c$ and a doublet in spin $SU(2)_s$, which we denote by (3, 2). The diquark system can thus be $3\times3=\bar{3}+6$ in color, and $2\times2=1+3$ in spin. There are four types of diquark, namely,

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$(\bar{3}, 1), (\bar{3}, 3), (6, 1), (6, 3).$

Since these S-wave diquarks have the orbital wave functions symmetric under quark permutation, they must be totally-antisymmetric in (color, spin ; flavor). So that, when we take $SU(3)$ flavor symmmetry, totally antisymmetric diquark states represented as follows ;

$$(\bar{3}, 1, \bar{3}) = A_1,$$
$$(3, 3, 6) = A_3,$$
$$(6, 1, 6) = S_1,$$
$$(6, 3, \bar{3}) = S_3$$

Next we combine these diquarks and antiquarks to form a color singlet hadron. Then we obtain the following possible combinations :

$$M_{SS}(33) = (A_1 A_1)\ [(1+8\qquad)_f,\ (1\qquad)_s],$$
$$M_{ST}(33) = (A_1 A_3) = [(8+10\qquad)_f,\ (3\qquad)_s],$$
$$M_{TS}(33) = (A_3 A_1) = [(8+10\qquad)_f,\ (3\qquad)_s],$$
$$M_{TT}(33) = (A_3 A_3) = [(1+8+27)_f,\ (1+3+5)_s],$$

and

$$M_{SS}(66) = (S_1 S_1) = [(1+8+27)_f,\ (1\qquad)_s],$$
$$M_{ST}(66) = (S_1 S_3) = [(8+10\quad)_f,\ (3\qquad)_s],$$
$$M_{ST}(66) = (S_3 S_1) = [(8+10\quad)_f,\ (3\qquad)_s],$$
$$M_{TT}(66) = (S_3 S_3) = [(1+8\qquad)_f,\ (1+3+5)_s].$$

The cofining potential does not distinguish between these states and leave them degenerate. They will, however, be split by the chromomagnetic force. Before showing the fact, we should construct the color state vector of the baryonium. The color state vector of the (33) -baryonium and the (66) -bary-onium are

$$|\ (3\bar{3})\text{ -Baryonium} > \sum \frac{1}{\sqrt{12}} (\delta_{ac}\delta_{bd} - \delta_{ad}\delta_{bc})\ |\ q_a q_b \bar{q}_c \bar{q}_d >$$

$$|\ (6\bar{6})\text{ -Baryonium} > \sum \frac{1}{\sqrt{24}} (\delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc})\ |\ q_a q_b \bar{q}_c \bar{q}_d > ,$$

where a, b, c, and d are color indecies.

MASS(GEV)	DECAY WIDTH (MEV)
2.95	<15
2.6 (S = ±1)	<18
2.2	~16
2.02	~24
1.939	≤ 4
1.694	<20
1.661	<20
1.457	<35

TABLE I
Masses and decay widths of the observed baryoniums.

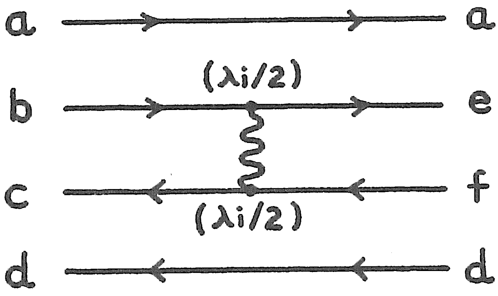

Fig. 1 : The interaction between quark and antiquark via a colored gluon exchange in a baryonium.

III. CHROMOMAGNETIC INTERECTION

Here we will discuss the hyperfine splittings of the lowest baryonium states. In the S-wave state only the chromomagnetic Fermi interaction contributes, and we can write the mass of the states composed of the same set of quarks as

$$M = M_0 + B < \sum \lambda_i \lambda_j \, s_i \cdot s_j \kappa_{ij} >$$

where B is a positive parameter, depending on the space parts of the wave functions, λ_i is the chromomagnetic moment of the quark, and κ_{ij} is the color factor. In the baryonium state κ_{if} is not a universal one and takes different values between qq and $q\bar{q}$ interactions. For example, let us calculate the effective coupling of $q\bar{q}$ interaction in the $(3\bar{3})$-baryonium.

$$g^2 = g_0^2 \sum \frac{1}{\sqrt{12}} (\delta_{ac}\delta_{bd} - \delta_{ad}\delta_{bc}) \left(\frac{\lambda_i}{2}\right)_{be} \left(\frac{\lambda_i^*}{2}\right)_{cf} \frac{1}{\sqrt{12}} (\delta_{af}\delta_{cd} - \delta_{ad}\delta_{fc})$$

$$= g_0^2 \sum \frac{1}{12} (\delta_{cf}\delta_{be} + \delta_{bc}\delta_{ef}) \frac{1}{2} (\delta_{bc}\delta_{ef} - \frac{1}{3} \delta_{be}\delta_{cf})$$

$$= \frac{1}{3} g_0^2 \qquad \text{(See Fig. 1).}$$

so that $\kappa_{q\bar{q}} = 1/3$, and one can easily obtain $\kappa_{qq} = 2/3$. And $\kappa_{q\bar{q}} = -1/3$, $\kappa_{qq} = 5/6$ for the $(6\bar{6})$ $-$baryonium. Note that the force between the quarks in $(6\bar{6})$ $-$baryonium is repulsive one. Next we consider the baryoniums composed of u and d quarks only, and require isospin symmetry $\lambda_u = \lambda_d$, for simplicity. The hyperfine splitting terms for the $(3\bar{3})$ $-$and $(6\bar{6})$$-$baryoniums are obtained as follows ;

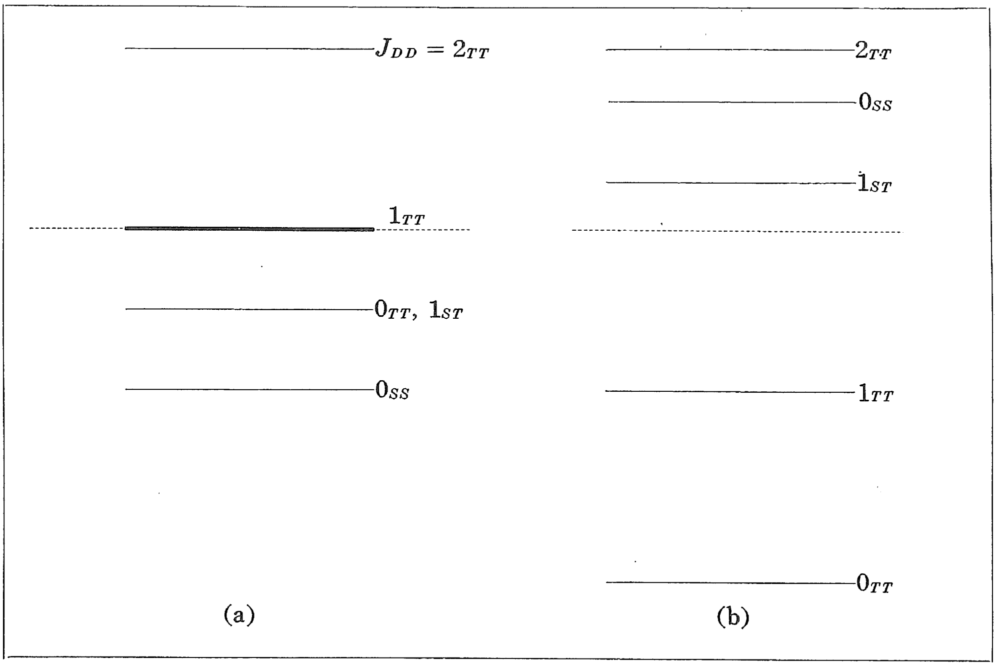

Fig. 2 : The level splitting patterns caused by the chromomagnetic interaction : (a) $(3\bar{3})$ $-$baryonium (b) $(6\bar{6})$ $-$baryonium. Dotted line is unperturbed level.

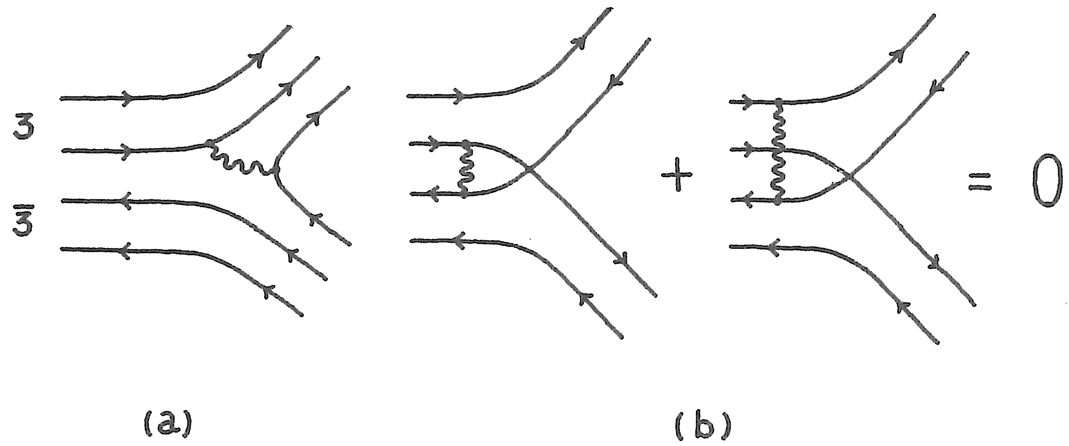

Fig. 3 : (a) $(3\overline{3})$ —baryonim decay into baryon and antibaryon.
(b) $(3\overline{3})$ —baryonium decay into mesons.

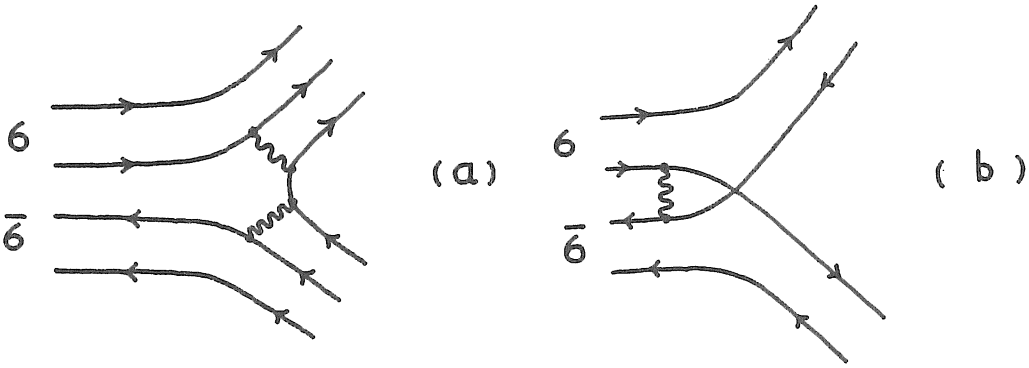

Fig. 4 : (a) $(6\overline{6})$ —baryonium decay into baryon and antibaryon.
(b) $(6\overline{6})$ —baryonium decay into mesons.

$$M(3\overline{3}) = \frac{B\lambda^2}{6}[S^2_{\mathrm{Total}} + s^2_{12} + s^2_{34} - 6]$$

$$M(6\overline{6}) = \frac{B\lambda^2}{12}[5S^2_{\mathrm{Total}} - 7s^2_{12} - 7s^2_{34} + 6],$$

where $\vec{s}_{12}$ is the diquark spin and $\vec{s}_{34}$ is the antidiquark spin. Consequently we find

$$\Delta M_{SS}(3\overline{3})_0 = (-6)a \qquad \Delta M_{SS}(6\overline{6})_0 = (3)a$$
$$\Delta M_{ST}(3\overline{3})_1 = (-2)a \qquad \Delta M_{ST}(6\overline{6})_1 = (1)a$$
$$\Delta M_{TT}(3\overline{3})_0 = (-2)a \qquad \Delta M_{TT}(6\overline{6})_0 = (-11)a$$
$$\Delta M_{TT}(3\overline{3})_1 = (0)a \qquad \Delta M_{TT}(6\overline{6})_1 = (-6)a$$
$$\Delta M_{TT}(3\overline{3})_2 = (4)a \qquad \Delta M_{TT}(6\overline{6})_2 = (4)a$$

where a $= B\lambda^2/6$ Fig. 2 shows the mass breaking patterns of the $(3\overline{3})$ —and (66) —baryoniums caused by the above

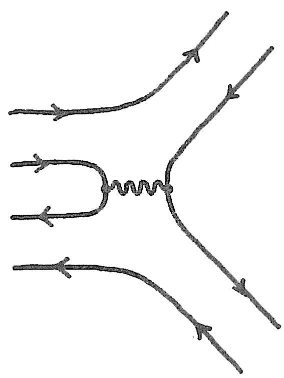

Fig. 5 : Possible decay mechanism of the baryonium into mesons.

chromomagnetic interactions. In this manner the baryoniums take an interesting level-splitting pattern because of the difference of the color factor between qq—and $q\bar{q}$—interactions. Whether or not the predictions meet the future observation is very interesting.

IV. DECAY OF THE BARYONIUM

The observed baryoniums have narrow decay width. Here we consider the baryonium decays. Let us take the following assumption : Decays can proceed either by colored gluon exchange or gluon materialization into $q\bar{q}$-pair. The $(3\bar{3})$—baryonium decays into baryons and into mesons are illustrated in Fig. 3. Its decay into baryons proceeds by single gluon materialization into $q\bar{q}$—pair, and its decay into mesons by single gluon exchange. The later case, however, has a cancellation mechanism in color factor, as illustrated in Fig. 3 (b), so that the mesonic decay is suppressed. Thus the $(3\bar{3})$ —baryonium prefers the decay into baryons rather than into mesons. We can expect that the $(3\bar{3})$ —baryoniums have narrow decay width. Consider next the $(6\bar{6})$ —baryonium decays. Its decays into baryons and into mesons are illustrated in Fig. 4 (a) and Fig. 4 (b), respectively. One can easily find that the $(6\bar{6})$ —baryonium decay into baryons can not proceed by single gluon materialization into $q\bar{q}$-pair : The color factor of the decay amplitude via single gluon materialization is zero ;

$$\sum \frac{1}{\sqrt{24}}(\delta_{ac}\delta_{bd}+\delta_{ad}\delta_{bc})\left(\frac{\lambda^i_{be}}{2}\right)\left(\frac{\lambda^i_{gf}}{2}\right)\frac{1}{\sqrt{6}}\varepsilon_{aef}\frac{1}{\sqrt{6}}\varepsilon_{gcd}=0.$$

So that its decay into baryons can proceed by double gluon materialization into a $q\bar{q}$-pair in the lowest order, and the decay is suppressed relative to the $(3\bar{3})$ —baryonium decay. The $(6\bar{6})$ —baryonium decay into mesons can proceed by single gluon exchange, and, in this case, there exists no cancellation mechanism. Thus the $(6\bar{6})$ baryonium prefer the mesonic decays rather than the baryonic decay. We can expect that the $(6\bar{6})$ —baryoniums are rather broad and unstable compared with the $(3\bar{3})$ —baryoniums. In this way the observed narrow width of the $(3\bar{3})$ —baryonium seems to be explained. However there exist another possible decay mechanism. It is illustrated in Fig. 5, the decay via qq-annihilation and subsequent gluon materialization into $q\bar{q}$-pair. This diagram can not be cancelled in either case of the $(3\bar{3})$ —and $(6\bar{6})$ —baryoniums. Additional assumption seems to be needed to prevent the annihilation to explain the narrow decay width of the baryonium : The diquark and the anti-diquark are spatially separated to prevent the annihilation ? The diquark is a tightly bound state to prevent the single quark — single antiquark annihilation ? The other possibility could also be considered. For example, Rossi and Veneziano argued that strings representing gluon exachange could have a junction and that baryons should have Y-shape in the string model. They introduce a new OZI rule for a junction conservation which would restrict the decay width of a diquark-diantiquark system [2]. However the correlation between their model and pure field-theoretical hadron view mensioned above have not yet clear. More detail disscusion will be presented elsewhere.

In this note we discussed the hyperfine mass splittings of the baryonium. The baryoniums have various spin configurations so that present good tests for the chromomagnetic interactions which have been very successful for classifying the old hadrons. The narrow decay width of the baryoniums was also discussed. It is still an open problem. It was pointed out that an additional assumption in order to prevent the $q\bar{q}$-annihilation inside the baryonium seems to be needed. This note is the first approximation for the future complete, and more detailed will be published elsewhere.

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