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On the Evaluation of the Internal Friction of Anisotropic, Viscoelastic Bars in Warping Torsion Theory.

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異方性粘弾性はりの曲げねじり理論における
内部摩擦の評価について

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The warping torsion theory considering not only shear force but bending moment in torsion was expanded to be able to describe the vibration of anisotropic, viscoelastic body. The viscoelastic, warping torsion theory is useful to predict the internal friction values of torsionally vibrating beams at higher modes. However, the analytically estimated values are inconsistent at the bar ends and nodal points of the bars.

1. Introduction

It is well-known that the torsion of a bar is described by St. Venant theory in case that the cross section of the bar is small compared with the length. In our previous reports,[1] the torsional vibration of wooden bars was analyzed under a both ends free condition. According to the results, warping torsion theory consideringnot only shear force in the St. Venant theory but bending moment in torsion must be applied to the analysis of the vibration of the bars at higher modes.

In this paper, the warping torsion theory was expanded to be able to describe the vibration of anisotropic, viscoelastic body such as wood by the manner developed in recent years.[2-4]

2. Vibration analysis of a bar by viscoelastic warping torsion theory

When we consider the two different coefficients of viscous damping η_E and η_G associated with Young's modulus E and shear modulus G_{55},[2] the following vibration equation can be derived from the warping torsion theory[1]:

$$EI_W\frac{\partial^4\theta}{\partial z^4}-G_{55}K\frac{\partial^2\theta}{\partial z^2}+\eta_E I_W\frac{\partial^5\theta}{\partial z^4\partial t}-\eta_G K\frac{\partial^3\theta}{\partial z^2\partial t}+\rho J\frac{\partial^2\theta}{\partial t^2}=0 \qquad (1)$$

where θ, twist angle ; z, longitudinal distance from the center of a bar ; t, time ; I_W, warping torsion constant ; K, St. Venant's torsion factor ; ρ, density ; J, moment of inertia. For the recutangular cross section of $2b\times 2h$,

$J=4/3\cdot bh(b^2+h^2)$

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$$K=16/3\ bh^3\{1-192/\pi^5\cdot h/b\sqrt{G_{55}/G_{44}}\cdot\sum_{n=1,3,5}^{\infty}1/n^5\tanh(n\pi b/2h\sqrt{G_{44}/G_{55}})\}$$

and the warping torsion constant I_W is expressed as follows :

$$I_W=(4bh)^3/m$$

where m is the empirical constant.

Now we use the following solution :

$$\theta=X(z)\cdot Y(t)$$

where $X(z)$ is the normal function for a vibrating elastic bar and is expressed for antisymmetric modes as follows :

$$X(z)=A\cosh\mu_1 z+C\cos\mu_2 z$$

$$\mu_1^2,\ \mu_2^2=\sqrt{s^4+\gamma^4}\pm\beta^2,$$

$$2\beta^2=GK/(EI_W),\ \gamma^4=\rho J/(EI_W)\cdot\omega_n^2, \tag{2}$$

and ω_n is resonant angular frequency ($=2\pi f_n$, f_n being the resonant frequency). Substituting eq. (2) into eq. (1), we obtain

$$\omega_n^2 Y+\frac{EI_W}{\rho J}\left\{A\mu_1^2\left(\mu_1^2\frac{\eta_E}{E}-\frac{\eta_G K}{EI_W}\right)\cosh\mu_1 z\right.$$

$$\left.+C\mu_2^2\left(\mu_2^2\frac{\eta_E}{E}-\frac{\eta_G K}{EI_W}\right)\cos\mu_2 z\right\}\cdot\frac{1}{X(z)}\cdot\frac{\mathrm{d}Y}{\mathrm{d}t}+\frac{\mathrm{d}^2Y}{\mathrm{d}t^2}=0$$

$$\text{or}\quad \omega_n^2 Y+2\varepsilon\frac{\mathrm{d}Y}{dt}+\frac{\mathrm{d}^2Y}{\mathrm{d}t^2}=0 \tag{3}$$

By the definition of viscoelasticity, the internal frictions associated with E and G can be expressed as follows :

$$\tan\delta_E=\omega_n\eta_E/E,\ \tan\delta_G=\omega_n\eta_G/G$$

Analogous to the linear vibration system with one degree of freedom,[5] the equation to evaluate the internal friction of the bar is derived from eq. (3):

$$\tan\delta=2\varepsilon/\omega_n$$

$$=\{(A\mu_1^4\cosh\mu_1 z+C\mu_2^4\cos\mu_2 z)\tan\delta_E$$

$$-2\beta^2(A\mu_1^2\cosh\mu_1 z-C\mu_2^2\cos\mu_2 z)\tan\delta_G\}/(\gamma^4X(z)) \tag{4}$$

Also for symmetric modes, by using the following equation

$$X(z)=A\sinh\mu_1 z+C\sin\mu_2 z$$

the equation corresponding to eq. (4) is derived, that is,

$$\tan\delta=2\varepsilon/\omega_n$$

$$=\{(A\mu_1^4\sinh\mu_1 z+C\mu_2^4\sin\mu_2 z)\tan\delta_E$$

$$-2\beta^2(A\mu_1^2\sinh\mu_1 z-C\mu_2^2\sin\mu_2 z)\tan\delta_G\}/(\gamma^4X(z)) \tag{5}$$

The free-free edge condition is

$$EI_W\mathrm{d}^3\theta/\mathrm{d}z^3-G_{55}K\mathrm{d}\theta/\mathrm{d}z=0$$

$$\text{and}\ \mathrm{d}^2\theta/\mathrm{d}z^2=0\ \text{at}\ z=\pm\ell/2. \tag{6}$$

The following frequency equation can be obtained from the condition :

antisymmetric mode (odd number mode):

$$-\mu_2/\mu_1\cdot\tanh(\mu_1\ell/2)=(\mu_2^2+2\beta^2)/(\mu_1^2-2\beta^2)\cdot\tan(\mu_2\ell/2)$$

symmetric mode (even number mode):

$$\mu_2/\mu_1\cdot\tanh(\mu_1\ell/2)=(\mu_2^2-2\beta^2)/(\mu_1^2+2\beta^2)\cdot\tan(\mu_2\ell/2) \tag{7}$$

Furthermore, the following relations between coefficients A and C in eqs. (4) and (5) can be obtained :

antisymmetric mode :

$A/C=\{\mu_2{}^2\cos(\mu_2\ell/2)\}/\{\mu_1{}^2\cosh(\mu_1\ell/2)\}$

symmetric mode :

$A/C=\{\mu_2{}^2\sin(\mu_2\ell/2)\}/\{\mu_1{}^2\sinh(\mu_1\ell/2)\}$ (8)

We can obtain the values of ω, μ, β, and γ in eqs. (4) and (5) by solving the super equation (7) numerically. The values of internal friction of the bar are estimated from the values and the relations in eqs.(8).

3. Evaluation of internal friction of a bar by viscoelastic, warping torsion theory

The internal friction in eqs. (4) and (5) was calculated by using the referred values of mechanical properties of wood specimens.[1] The values are

$\ell=29.0$cm, $2b=3.92$cm, $2h=0.92$cm,

$\rho=0.43$, $E=12.4$ GPa, $G_{55}=0.823$ GPa,

$G_{55}/G_{44}=1.152$, $m=440$,

$\tan\delta_E=0.008$, and $\tan\delta_G=0.0145$,

and the resonant frequencies f_n from eqs. (7) and the above conditions were as follows :

1st mode : 1008.6 Hz,

2nd mode : 2069.5 Hz,

3rd mode : 3230.5 Hz,

4th mode : 4531.2 Hz,

5th mode : 6031.1 Hz,

and

6th mode : 7700.2 Hz.

The values of internal friction are shown in Fig. 1 for each mode and location on the bar. The values agree with the values of internal friction of Young's modulus E at the bar ends and the agreement is due to the end conditions in eqs. (6). The singular points appear at the nodal points of vibration modes besides the nodal point at the center of the bar.

In the cases of elementary St. Venant theory, $EI_W=0$ and $\tan\delta_E=0$, and isotropic viscoelastic theory, $\tan\delta_E=\tan\delta_G$, we easily find that eqs. (4) and (5) become

$\tan\delta=\tan\delta_G$.

Therefore, the results in Fig.1 are entirely peculiar to the anisotropic, viscoelastic body.

On the otherhand, the value of internal friction can approximately be calculated by the energy method. Namely, in eq. (7), the viscous frequency f_n' is calculated by replacing the elastic constants E and G to viscoelastic constants, $E\times\tan\delta_E$ and $G\times\tan\delta_G$, respectively, and then, the internal friction can be evaluated with the two different frequencies f_n and f_n' as follows :

$\tan\delta=(f_n'/f_n)^2$ (9)

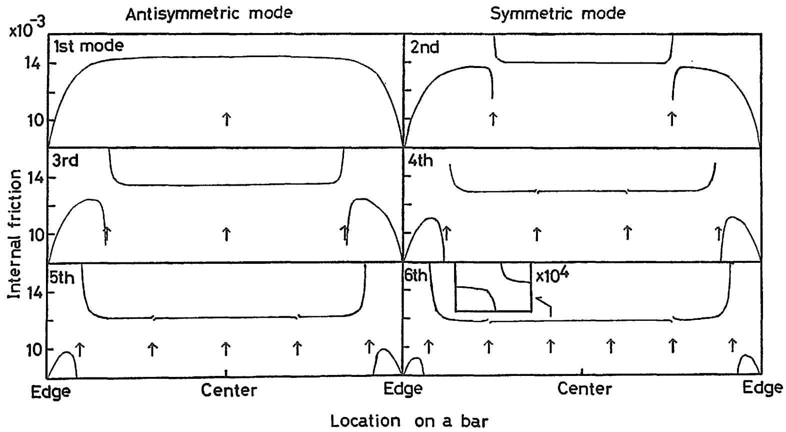

Fig. 1. Internal friction of an anisotropic, viscoelastic bar at each mode and location.
Note :Arrows show nodal points.

Table 1. Resonant frequencies and internal friction for an anisotropic, viscoelastic bar.

Mode	Resonant frequency (Hz)		Internal friction ($\times 10^{-3}$)	
	Elementary	Present	Present	Approximate
1st	1, 000	1, 009	14. 36	14. 40
2nd	2, 000	2, 070	13. 98	14. 10
3rd	2, 999	3, 231	13. 45	13. 67
4th	3, 999	4, 531	12. 86	13. 17
5th	4, 999	6, 013	12. 27	12. 62
6th	5, 999	7, 700	11. 72	12. 10

Table 1 shows the values of the internal friction by eq. (9) and the internal friction around the center of the bar in Fig. 1. The values of the resonant frequencies mentioned above and the frequencies by elementary St. Venant theory ignoring the bending moment are also listed in Fig. 1. The two types of internal friction values almost agree with each other, although the analytical values are somewhat smaller than the approximate values and this tendency is same as that for viscoelastic Timoshenko theory.[2,4,6] Furthermore, at higher modes, the values approach to the value of $\tan \delta_E$ from $\tan \delta_G$. This corresponds to the increase of the difference between the two types of resonant frequency values due to the occurance of bending moment effect.

Viscoelastic, warping torsion theory is useful to predict the internal friction values of torsionally vibrating bars at higher modes. However, the analytically estimated values are inconsistent at the bar ends and nodal points for the anisotropic, viscoelastic beams. We should give further considerations when anisotropic viscousity are introduced into the authorized elastic vibration theory such as the above warping torsion theory and Timoshenko theory.[2]

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Appendix

The energy method to evaluate internal friction is fundamentally depends on the following relations[3,4,7] :

$$\omega_n^2 = \Sigma D_{ii} \lambda_{ii}^4 / \rho h \qquad \text{(a. 1)}$$

$$\tan \delta = \Sigma D_{ii} \tan \delta_{ii} \lambda_{ii}^4 / \Sigma D_{ii} \lambda_{ii}^4 \qquad \text{(a. 2)}$$

where D_{ii} is the rigidity for elastic modulus a_{ii} ; $\tan \delta_{ii}$ is the internal friction associated with a_{ii} ; λ_{ii} is the eigen value decided from the shape, vibration mode, and edge conditions of a vibrating body and has the dimension of length^{-1}. The most simple method to evaluate the internal friction is to calculate independently the two types of frequencies ω_n and ω_n' with the constants D_{ii} and $D_{ii} \tan \delta_{ii}$, respectively. Then the internal friction is evaluated from eq. (a. 2). This method is approximately correct when the two types of independently evaluated parameters λ in the numerator and denominator in eq. (a. 2) are identical. This condition is satisfied in the case of the vibration of a bar or a beam, and also in the case of that of a plate of which vibration mode corresponds to a beam mode. By using this method and a finite element method, the internal friction is evaluated for the specimen with an arbitrary shape.[3] However, this simple method is not applicable to the complicated vibration mode of a plate, that is, interaction of two beam modes.[4] Then the values of λ decided for the denominator in eq. (a. 2) should be substituted into the numerator.